Research Article

Delay-Dependent Robust Exponential Stability for Uncertain Neutral Stochastic Systems with Interval Time-Varying Delay

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This paper discusses the mean-square exponential stability of uncertain neutral linear stochastic systems with interval time-varying delays. A new augmented Lyapunov-Krasovskii functional (LKF) has been constructed to derive improved delay-dependent robust mean-square exponential stability criteria, which are forms of linear matrix inequalities (LMIs). By free-weight matrices method, the usual restriction that the stability conditions only bear slow-varying derivative of the delay is removed. Finally, numerical examples are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Dynamical systems, such as the distributed networks, electric power systems, and communication systems, can be efficiently modeled by neutral stochastic functional differential equations, which have been extensively studied in recent years, see [1–8] and references therein. Basically, sufficient conditions on stability of time delay systems are divided into two categories: delay-dependent and delay-independent cases. It is well known that the latter is less conservative than the former, especially when the delay is very small. Therefore, increasing attention has been focused on delay-dependent stability analysis of delay systems in recent years, see [1–25].

There are unstable systems without delay that can be stabilized with proper nonzero delay, see [26]. Therefore, it is of great significance to consider the stability of systems with interval time-varying delay, it is a time delay that varies in an interval, in which the lower bound is not restricted to 0. Recently, interval time-varying delay was introduced and investigated, see [11–17].

In the past years, as an effective approach of improving the performance of delay-dependent stability criteria, free-weighting matrices method has attracted much attention, see [2–5, 10–12]. The mean-square exponential stability was studied for neutral stochastic systems with fixed delays in [2, 3]. The global asymptotic stability for a class of neutral stochastic neural networks was studied in [4]. Improved delay-dependent robust stability criteria of uncertain stochastic systems with interval time-varying delay were proposed in [12].

The usual restriction on derivative of the time-varying delay that $\dot{\tau}(t) < 1$ in [5–7, 18] brings conservatism, which shows that the resulting conditions can only bear slow-varying delay. In order to remove this limitation, fast-varying rate condition was studied in [4, 12, 23].

Stability of stochastic differential delay systems with nonlinear impulsive effects is studied in [24, 25], the equivalent relations are established between the stability of the stochastic differential delay equations with impulsive effects and that of a corresponding stochastic differential delay equations without impulses.

To our best knowledge, few works on the robust delay-dependent exponential stability analysis have been reported for uncertain neutral stochastic distributed system with fast-varying interval time-varying delay. This paper focuses on the stability analysis of uncertain neutral stochastic distributed system with interval time-varying delay. By Lyapunov-Krasovskii functional theory and free-weighting matrices method, a new delay-dependent mean-square exponential stability criterion is formulated in terms of LMI, and the usual restriction of $\dot{\tau}(t) < 1$ is removed. Finally, three numerical example are given to illustrate the effectiveness of the proposed method.

Notation 1. Throughout this paper, the notations are standard. If A is a vector or matrix, its transpose is denoted by A^T. $P > 0$ means that P is a symmetric positive definite matrix. $*$ denotes the symmetry part of a symmetry matrix. $\star$ indicates terms that can be induced by symmetry, for example, $A + A^T = A + \star$ and $MPM^T = MP\star$. Denote by $\lambda_M\{\cdot\}$ and $\lambda_m\{\cdot\}$ the maximum and minimum eigenvalue of a matrix, respectively. Let $\rho(\cdot)$ denote the spectral norm of a matrix, while $|\cdot|$ refers to the Euclidean vector norm. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space relative to an increasing family $\{\mathcal{F}\}_{t \geq 0}$ of σ algebras $\mathcal{F}_t \subset \mathcal{F}$ and $\mathbb{E}\{\cdot\}$ the mathematical expectation operator with respect to the probability measure $\mathcal{P}$. $\mathcal{L}_2[0, \infty)$ denotes the space of square integrable vector functions over $[0, \infty)$. Let $\tau > 0$ and $C([-\tau, 0], \mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$-valued functions ϕ on $[-\tau, 0]$ with the norm $\|\phi\| = \sup\{|\phi(\theta)| : -\tau \leqslant \theta \leq 0\}$. Let $\mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0], \mathbb{R}^n)$-valued random variables $\varphi = \{\varphi(\theta) : -\tau \leqslant \theta \leqslant 0\}$.

2. Preliminaries

Consider the following uncertain Itô-type neutral stochastic system with both discrete and distributed interval time-varying delays

$$
\begin{aligned}
(\Sigma) : \mathrm{d}[x(t) - Dx(t_\tau)] \\
= \left\{ [A_0 + \Delta A_0(t)]x(t) + [A_1 + \Delta A_1(t)]x(t_\tau) + [A_2 + \Delta A_2(t)] \int_{t_\tau}^t x(s)\mathrm{d}s \right\} \\
+ \left\{ [B_0 + \Delta B_0(t)]x(t) + [B_1 + \Delta B_1(t)]x(t_\tau) + [B_2 + \Delta B_2(t)] \int_{t_\tau}^t x(s)\mathrm{d}s \right\} \mathrm{d}B(t),
\end{aligned}
\tag{2.1}
$$

$$
x(t) = \phi(t), \quad t \in [-\tau_2, 0],
\tag{2.2}
$$

where $x(t) \in \mathrm{R}^n$ is the state vector; $B(t)$ is a standard scalar Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, we assume $\mathbb{E}\{\mathrm{d}B(t)\} = 0, \mathbb{E}\{\mathrm{d}^2 B(t)\} = \mathrm{d}t$. $\phi(t)$ is any given initial data in $\mathcal{L}_{\mathcal{F}_0}^2([-\tau, 0]; \mathrm{R}^n)$. Denote $\eta(t) = x(t) - Dx(t_\tau), t_\tau = t - \tau(t), t_{\tau_1} = t - \tau_1, t_{\tau_2} = t - \tau_2$, $\tau(t)$ is time-varying delays, satisfying

$$\dot{\tau}(t) \leqslant \overline{\tau} < +\infty, \quad 0 \leqslant \tau_1 \leqslant \tau(t) \leqslant \tau_2 < +\infty, \qquad \delta = \tau_2 - \tau_1. \tag{2.3}$$

Remark 2.1. It should be noted that system (2.1)-(2.2) encompasses many state space models of neutral delay systems, which can be used to represent many important physical systems such as a large class of distributed networks containing lossless transmission lines, population ecology, heat exchangers, wind tunnel, and water resources systems (see [27–29] and the references therein). For example, signal transformation cannot be finished in time due to the finite signal propagation speed in distributed networks containing lossless transmission lines, or to the finite switching speed of amplifiers in electronic networks, so the models only depending on the discrete time delays are not complete, the more exact models should include the distributed time delays, and the delays are found both in the states and in the derivatives of the states.

Remark 2.2. It is worth pointing out that, when a time-varying delay appears, it is usually assumed that $\dot{\tau}(t) < 1$ is satisfied and the lower bound of the delay is restricted to be zero in the literature [5, 7, 8] and so forth. However, in this paper, we only require $\dot{\tau}(t) \leqslant \overline{\tau}$, in addition, the range of delay may vary in a range for which the lower bound is not restricted to be zero. Therefore, the time-varying delay in this paper is more general.

$A_i, B_j, i, j = 0, 1, 2$, are known real constant matrices of appropriate dimensions; $\Delta A_i, \Delta B_j, i, j = 0, 1, 2$, are unknown matrices representing time-varying parameter uncertainties. For the sake of convenience, we assume that the uncertainties are norm-bounded and can be described as

$$(\Delta A_i(t) \quad \Delta B_i(t)) = GF(t)(S_i \quad T_i), \tag{2.4}$$

where $S_i, T_i, i = 0, 1, 2$, are known real constant matrices, $F(t)$ is an unknown real time-varying function with appropriate dimension satisfying

$$F^{\mathrm{T}}(t)F(t) \leqslant I. \tag{2.5}$$

It is assumed that the elements of $F(t)$ are Lebesgue measurable. Throughout this paper, the following assumption, definitions, and lemmas will be used to develop our results.

Assumption 2.3. The difference operator matrix D satisfies $\|D\| < 1$.

Definition 2.4 (see [1]). The neutral stochastic delay system (2.1)-(2.2) is said to be mean-square exponentially stable if there is a pair of positive constants α, β such that

$$\mathbb{E}\left\{x^{\mathrm{T}}(t)x(t)\right\} \leqslant \alpha \mathrm{e}^{-\beta t} \sup_{-\tau_2 \leqslant \theta \leqslant 0} \mathbb{E}\left\{\overline{\phi}(\theta)\right\}. \tag{2.6}$$

Lemma 2.5 (see [30]). *If there exists a vector function $v(t)$: $[0, r] \rightarrow \mathbb{R}^n$ such that $\int_0^r v^{\mathrm{T}}(s)Pv(s)\,\mathrm{d}s$ and $\int_0^r v(s)\,\mathrm{d}s$ are well defined, then the following inequality:*

$$\left[\int_0^r v(s)\mathrm{d}s \right]^{\mathrm{T}} P \left[\int_0^r v(s)\mathrm{d}s \right] \leqslant r \int_0^r v^{\mathrm{T}}(s)Pv(s)\mathrm{d}s \qquad (2.7)$$

holds for any pair of symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ and $r > 0$.

Lemma 2.6 (see [31]). *For any vectors $x, y \in \mathbb{R}^n$, matrices A, D, E, P, and F are real matrices of appropriate dimensions with $P > 0$ and $F^T F \leqslant I$, the following inequalities hold:*

(1) $2x^{\mathrm{T}}DFEy \leqslant \epsilon^{-1}x^{\mathrm{T}}DD^{\mathrm{T}}x + \epsilon y^{\mathrm{T}}EE^{\mathrm{T}}y$;

(2) *for any scalar $\epsilon > 0$ such that $P - \epsilon DD^{\mathrm{T}} > 0$, then $(A + DFE)^{\mathrm{T}}P^{-1}(A + DFE) \leqslant \epsilon^{-1}E^{\mathrm{T}}E + A^{\mathrm{T}}(P - \epsilon DD^{\mathrm{T}})^{-1}A$;*

(3) $2x^{\mathrm{T}}y \leqslant x^{\mathrm{T}}P^{-1}x + y^{\mathrm{T}}Py$.

3. Main Results

In this section, a robustly stochastically exponentially stable criterion for the uncertain linear neutral stochastic distributed delayed system (Σ) will be established by applying the Lyapunov-Krasovskii theory and free-weighting matrices method.

For convenience, define a new state variable

$$f(t) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t_\tau) + (A_2 + \Delta A_2(t)) \int_{t_\tau}^t x(s)\mathrm{d}s \qquad (3.1)$$

and a new perturbation variable

$$g(t) = (B_0 + \Delta B_0(t))x(t) + (B_1 + \Delta B_1(t))x(t_\tau) + (B_2 + \Delta B_2(t)) \int_{t_\tau}^t x(s)\mathrm{d}s. \qquad (3.2)$$

Then, system (2.1) becomes

$$\mathrm{d}[x(t) - Dx(t_\tau)] = f(t)\mathrm{d}t + g(t)\mathrm{d}B(t). \qquad (3.3)$$

Applying Leibniz-Newton formula to (2.1), it yields the following zero equations which will be used in our main result:

$$2\xi^{\mathrm{T}}(t)N\left[x(t) - Dx(t_\tau) - x(t_\tau) + Dx(t_\tau - \tau(t_\tau)) - \int_{t_\tau}^t f(s)\mathrm{d}s - \int_{t_\tau}^t g(s)\mathrm{d}B(s)\right] = 0,$$

$$2\xi^{\mathrm{T}}(t)H\left[x(t_{\tau_1}) - Dx(t_{\tau_1} - \tau(t_{\tau_1})) - x(t_\tau) + Dx(t_\tau - \tau(t_\tau)) - \int_{t_\tau}^{t_{\tau_1}} f(s)\mathrm{d}s - \int_{t_\tau}^{t_{\tau_1}} g(s)\mathrm{d}B(s)\right] = 0,$$

$$2\xi^{\mathrm{T}}(t)M\left[x(t_\tau) - Dx(t_\tau - \tau(t_\tau)) - x(t_{\tau_2}) + Dx(t_{\tau_2} - \tau(t_{\tau_2})) - \int_{t_{\tau_2}}^{t_\tau} f(s)\mathrm{d}s - \int_{t_{\tau_2}}^{t_\tau} g(s)\mathrm{d}B(s)\right] = 0,$$

$$(3.4)$$

where N, H, and M are any matrices with appropriate dimensions, and

$$\xi^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t_\tau), \frac{1}{\tau_2}\int_{t_\tau}^t x^{\mathrm{T}}(s)\mathrm{d}s, x^{\mathrm{T}}(t_{\tau_1}), x^{\mathrm{T}}(t_{\tau_2}),\right.$$

$$\left. x^{\mathrm{T}}(t_{\tau_1} - \tau(t_{\tau_1})), x^{\mathrm{T}}(t_{\tau_2} - \tau(t_{\tau_2})), x^{\mathrm{T}}(t_\tau - \tau(t_\tau))\right]. \qquad (3.5)$$

With zero equations (3.4) above, we obtain the following theorem.

Theorem 3.1. *For given scalars $0 \leqslant \tau_1 \leqslant \tau_2$ and $\overline{\tau}$, system (Σ) is robustly stochastically exponentially stable for all time-varying delays satisfying (2.3) and for all admissible uncertainties satisfying (2.4) and (2.5), if there exist symmetric positive definite matrices $P > 0, R_i > 0, i = 0, 1, 2, \ldots, 10$, scalars $\epsilon_j, j = 1, 2, 3$ and any matrices N, H, M with appropriate dimensions, such that the following LMI (3.6) holds:*

$$\Phi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{pmatrix} < 0, \qquad (3.6)$$

where

$$\Phi_1 = \begin{pmatrix} \Theta & \overline{P}G & V_1^T Z & 0 & V_2^T U & 0 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & -Z & ZG & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 & 0 \\ * & * & * & * & -U & UG \\ * & * & * & * & * & -\epsilon_3 I \end{pmatrix},$$

$$\Phi_2 = \left(\overline{N} \ \ \overline{H} \ \ \overline{M} \ \ \overline{N} \ \ \overline{H} \ \ \overline{M}\right),$$

$$\Phi_3 = \text{diag} = \{-R_7 \ \ -R_8 \ \ -R_7 - R_8 \ \ -R_9 \ \ -R_{10} \ \ -R_9 - R_{10}\}.$$

$$\Theta = \Gamma + \Psi + \Psi^{\mathrm{T}}, \qquad V_1 = [A_0 \ \ A_1 \ \ A_2 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0], \qquad V_2 = [B_0 \ \ B_1 \ \ B_2 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0],$$

$$\overline{P} = [P \ \ -PD \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^{\mathrm{T}}, \qquad Z = \tau_2 R_7 + \delta R_8, \qquad U = P + \tau_2 R_9 + \delta R_{10},$$

$$\overline{N} = \left[N^{\mathrm{T}}, 0, 0, 0, 0, 0\right]^{\mathrm{T}}, \qquad \overline{H} = \left[H^{\mathrm{T}}, 0, 0, 0, 0, 0\right]^{\mathrm{T}}, \qquad \overline{M} = \left[M^{\mathrm{T}}, 0, 0, 0, 0, 0\right]^{\mathrm{T}},$$

$$(3.7)$$

with

$$\Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 & 0 & 0 & 0 & 0 \\ * & \Gamma_{22} & \Gamma_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_3 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_{66} & 0 & 0 \\ * & * & * & * & * & * & \Gamma_{77} & 0 \\ * & * & * & * & * & * & * & \Gamma_{88} \end{pmatrix},$$

$$\Psi = \left(N \ \ (M - H) - N(D + I) \ \ 0 \ \ H \ \ -M \ \ -HD \ \ MD \ \ (N + H - M)D\right), \qquad (3.8)$$

$$\Gamma_{11} = PA_0 + A_0^{\mathrm{T}}P + R_1 + R_2 + R_3 + \tau_2 R_0 + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_0 + \epsilon_3 T_0^{\mathrm{T}}T_0,$$

$$\Gamma_{12} = PA_1 - A_0^{\mathrm{T}}PD + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_1 + \epsilon_3 T_0^{\mathrm{T}}T_1, \qquad \Gamma_{13} = PA_2 + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_2 + \epsilon_3 T_0^{\mathrm{T}}T_2,$$

$$\Gamma_{22} = R_4 + R_5 + R_6 - (1 - \overline{\tau})R_2 - D^{\mathrm{T}}PA_1 - A_1^{\mathrm{T}}PD + (\epsilon_1 + \epsilon_2)S_1^{\mathrm{T}}S_1 + \epsilon_3 T_1^{\mathrm{T}}T_1,$$

$$\Gamma_{23} = -D^{\mathrm{T}}PA_2 + (\epsilon_1 + \epsilon_2)S_1^{\mathrm{T}}S_2 + \epsilon_3 T_1^{\mathrm{T}}T_2, \qquad \Gamma_{33} = -\tau_2 R_0 + (\epsilon_1 + \epsilon_2)S_2^{\mathrm{T}}S_2 + \epsilon_3 T_2^{\mathrm{T}}T_2,$$

$$\Gamma_{66} = R_4, \qquad \Gamma_{77} = R_6, \qquad \Gamma_{88} = (1 - \overline{\tau})R_5.$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(t, \xi_t) = \sum_{i=0}^{4} V_i(t, \xi_t), \qquad (3.9)$$

where

$$V_0(t, x_t) = [x(t) - Dx(t_\tau)]^{\mathrm{T}}P[x(t) - Dx(t_\tau)];$$

$$V_1(t, x_t) = \int_{t_{\tau_1}}^{t} \begin{pmatrix} x(s) \\ x(s - \tau(s)) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} R_1 & 0 \\ 0 & R_4 \end{pmatrix} \begin{pmatrix} x(s) \\ x(s - \tau(s)) \end{pmatrix} ds$$

$$+ \int_{t_\tau}^t \begin{pmatrix} x(s) \\ x(s-\tau(s)) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} R_2 & 0 \\ 0 & R_5 \end{pmatrix} \begin{pmatrix} x(s) \\ x(s-\tau(s)) \end{pmatrix} \mathrm{d}s$$

$$+ \int_{t_{\tau_2}}^t \begin{pmatrix} x(s) \\ x(s-\tau(s)) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} R_3 & 0 \\ 0 & R_6 \end{pmatrix} \begin{pmatrix} x(s) \\ x(s-\tau(s)) \end{pmatrix} \mathrm{d}s;$$

$$V_2(t, x_t) = \int_{t_{\tau_2}}^t [s - (t - \tau_2)] x^{\mathrm{T}}(s) R_0 x(s) \mathrm{d}s;$$

$$V_3(t, x_t) = \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t f^{\mathrm{T}}(s) R_7 f(s) \mathrm{d}s \, \mathrm{d}\theta + \delta \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t f^{\mathrm{T}}(s) R_8 f(s) \mathrm{d}s \, \mathrm{d}\theta;$$

$$V_4(t, x_t) = \int_{-\tau_2}^0 \int_{t+\theta}^t \mathrm{tr}\left(g^{\mathrm{T}}(s) R_9 g(s) \right) \mathrm{d}s \, \mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \mathrm{tr}\left(g^{\mathrm{T}}(s) R_{10} g(s) \right) \mathrm{d}s \, \mathrm{d}\theta.$$

$$(3.10)$$

According to Itô's differential formula [1], the stochastic differential is

$$dV(t, x_t) = LV(t, x_t)\mathrm{d}t + 2[x(t) - Dx(t_\tau)]^{\mathrm{T}} Pg(t)\mathrm{d}B(t), \qquad (3.11)$$

where

$$LV(t, x_t)\mathrm{d}t = 2[x(t) - Dx(t_\tau)]^{\mathrm{T}} Pf(t) + g(t)^{\mathrm{T}} Pg(t) + \sum_{i=1}^4 \dot{V}_i(t, x_t). \qquad (3.12)$$

Direct computations give

$$\dot{V}_1(t, x_t) \leqslant x^{\mathrm{T}}(t)(R_1 + R_2 + R_3)x(t) - x^{\mathrm{T}}(t_{\tau_1})R_1 x(t_{\tau_1}) - x^{\mathrm{T}}(t_{\tau_2})R_3 x(t_{\tau_2})$$

$$+ x^{\mathrm{T}}(t_\tau)(R_4 + R_5 + R_6 - (1 - \overline{\tau})R_2)x(t_\tau)$$

$$- x^{\mathrm{T}}(t_{\tau_1} - \tau(t_{\tau_1}))R_4 x(t_{\tau_1} - \tau(t_{\tau_1}))$$

$$- (1 - \overline{\tau})x^{\mathrm{T}}(t_\tau - \tau(t_\tau))R_5 x(t_\tau - \tau(t_\tau))$$

$$- x^{\mathrm{T}}(t_{\tau_2} - \tau(t_{\tau_2}))R_6 x(t_{\tau_2} - \tau(t_{\tau_2})) \quad \{by \ \dot{\tau}(t) \leqslant \overline{\tau}\}$$

$$\dot{V}_2(t, x_t) \leqslant \tau_2 x^{\mathrm{T}}(t) R_0 x(t) - \int_{t_\tau}^{t} x^{\mathrm{T}}(s) R_0 x(s) \mathrm{d}s$$

$$\leq \tau_2 x^{\mathrm{T}}(t) R_0 x(t) - \left(\frac{1}{\tau_2} \int_{t_\tau}^{t} x(s) \mathrm{d}s \right)^{\mathrm{T}} (\tau_2 R_0) \left(\frac{1}{\tau_2} \int_{t_\tau}^{t} x(s) \mathrm{d}s \right); \quad \text{by Lemma 2.5,}$$

$$\dot{V}_3(t, x_t) = f^{\mathrm{T}}(t)^{\mathrm{T}} \left(\tau_2^2 R_7 + \delta^2 R_8 \right) f(t)$$

$$- \tau_2 \int_{t_{\tau_2}}^{t} f^{\mathrm{T}}(s)^{\mathrm{T}} R_7 f(s) \mathrm{d}s - \delta \int_{t_{\tau_2}}^{t_{\tau_1}} f^{\mathrm{T}}(s)^{\mathrm{T}} R_8 f(s) \mathrm{d}s;$$

$$\dot{V}_4(t, x_t) = \mathrm{tr} \left(g^{\mathrm{T}}(t)^{\mathrm{T}} (\tau_2 R_9 + \delta R_{10}) g(t) \right)$$

$$- \int_{t_{\tau_2}}^{t} \mathrm{tr} \left(g^{\mathrm{T}}(s)^{\mathrm{T}} R_9 g(s) \right) \mathrm{d}s - \int_{t_{\tau_2}}^{t_{\tau_1}} \mathrm{tr} \left(g^{\mathrm{T}}(s)^{\mathrm{T}} R_{10} g(s) \right) \mathrm{d}s.$$

$$(3.13)$$

By (1) of Lemma 2.6, for any scalar $\epsilon_1 > 0$, we have

$$2[x(t) - Dx(t_\tau)]^{\mathrm{T}} P f(t) = 2\xi^{\mathrm{T}}(t) \overline{P} V_1 \xi(t) + 2\xi^{\mathrm{T}}(t) \overline{P} G F(t) V_3 \xi(t)$$

$$\leq 2\xi^{\mathrm{T}}(t) \overline{P} V_1 \xi(t) + \epsilon_1^{-1} \xi^{\mathrm{T}}(t) \overline{P} G \star + \epsilon_1 \xi^{\mathrm{T}}(t) V_3^{\mathrm{T}} \star,$$

$$(3.14)$$

and by (2) of Lemma 2.6, for any $\epsilon_2 > 0$ satisfying $[\tau_2^2 R_7 + \delta^2 R_8]^{-1} - \epsilon_2^{-1} G G^{\mathrm{T}} > 0$, we have

$$f^{\mathrm{T}}(t) \left[\tau_2^2 R_7 + \delta^2 R_8 \right] f(t) = \xi^{\mathrm{T}}(t) (V_1 + G F(t) V_3)^{\mathrm{T}} Z \star$$

$$\leq \xi^{\mathrm{T}}(t) V_1^{\mathrm{T}} \left[Z^{-1} - \epsilon_2^{-1} G G^{\mathrm{T}} \right]^{-1} \star + \epsilon_2 \xi^{\mathrm{T}}(t) V_3^{\mathrm{T}} \star.$$

$$(3.15)$$

For any scalar $\epsilon_3 > 0$ satisfying $[\tau_2 R_9 + \delta R_{10}]^{-1} - \epsilon_3^{-1} G G^{\mathrm{T}} > 0$, the following inequality holds:

$$\mathrm{tr} \left(g^{\mathrm{T}}(t) [P + \tau_2 R_9 + \delta R_{10}] g(t) \right) = g^{\mathrm{T}}(t) [P + \tau_2 R_9 + \delta R_{10}] g(t)$$

$$= \xi^{\mathrm{T}}(t) (V_2 + G F(t) V_4)^{\mathrm{T}} U \star$$

$$\leq \xi^{\mathrm{T}}(t) V_2^{\mathrm{T}} \left[U^{-1} - \epsilon_3^{-1} G G^{\mathrm{T}} \right]^{-1} \star + \epsilon_3 \xi^{\mathrm{T}}(t) V_4^{\mathrm{T}} \star.$$

$$(3.16)$$

In addition, it follows from (3.4), (3) of **Lemma 2.6** and **Lemma 2.5** that

$$-2\xi^{\mathrm{T}}(t)N\int_{t_\tau}^t f(s)\mathrm{d}s \leqslant \xi^{\mathrm{T}}(t)NR_7^{-1}\star + \left(\int_{t_\tau}^t f(s)\mathrm{d}s\right)^{\mathrm{T}} R_7\star$$

$$\leqslant \xi^{\mathrm{T}}(t)NR_7^{-1}\star + \tau_2\int_{t_\tau}^t f^{\mathrm{T}}(s)R_7 f(s)\mathrm{d}s,$$

$$-2\xi^{\mathrm{T}}(t)H\int_{t_\tau}^{t_{\tau_1}} f(s)\mathrm{d}s \leqslant \xi^{\mathrm{T}}(t)HR_8^{-1}\star + \left(\int_{t_\tau}^{t_{\tau_1}} f(s)\mathrm{d}s\right)^{\mathrm{T}} R_8\star$$

$$\leqslant \xi^{\mathrm{T}}(t)HR_8^{-1}\star + \delta\int_{t_\tau}^{t_{\tau_1}} f^{\mathrm{T}}(s)R_8 f(s)\mathrm{d}s,$$

$$-2\xi^{\mathrm{T}}(t)M\int_{t_{\tau_2}}^{t_\tau} f(s)\mathrm{d}s$$

$$\leqslant \xi^{\mathrm{T}}(t)M(R_7+R_8)^{-1}\star + \left(\int_{t_{\tau_2}}^{t_\tau} f(s)\mathrm{d}s\right)^{\mathrm{T}} (R_7+R_8)\star$$

$$\leq \xi^{\mathrm{T}}(t)M(R_7+R_8)^{-1}\star + \int_{t_{\tau_2}}^{t_\tau} f^{\mathrm{T}}(s)(\tau_2 R_7 + \delta R_8)f(s)\mathrm{d}s,$$

$$-2\xi^{\mathrm{T}}(t)N\int_{t_\tau}^t g(s)\mathrm{d}B(s) \leq \xi^{\mathrm{T}}(t)NR_9^{-1}\star + \left(\int_{t_\tau}^t g(s)\mathrm{d}B(s)\right)^{\mathrm{T}} R_9\star,$$

$$-2\xi^{\mathrm{T}}(t)H\int_{t_\tau}^{t_{\tau_1}} g(s)\mathrm{d}B(s) \leq \xi^{\mathrm{T}}(t)HR_{10}^{-1}\star + \left(\int_{t_\tau}^{t_{\tau_1}} g(s)\mathrm{d}B(s)\right)^{\mathrm{T}} R_{10}\star,$$

$$-2\xi^{\mathrm{T}}(t)M\int_{t_{\tau_2}}^{t_\tau} g(s)\mathrm{d}B(s) \leq \xi^{\mathrm{T}}(t)M(R_9+R_{10})^{-1}\star$$

$$+\left(\int_{t_{\tau_2}}^{t_\tau} g(s)\mathrm{d}B(s)\right)^{\mathrm{T}} (R_9+R_{10})\star.$$

$$(3.17)$$

Note that

$$\mathbb{E}\left\{\left(\int_{t_\tau}^t g(s)\mathrm{d}B(s)\right)^{\mathrm{T}} R_9\star\right\} = \int_{t_\tau}^t \mathrm{tr}\left(g^{\mathrm{T}}(s)R_9 g(s)\right)\mathrm{d}s,$$

$$\mathbb{E}\left\{\left(\int_{t_\tau}^{t_{\tau_1}} g(s)\mathrm{d}B(s)\right)^{\mathrm{T}} R_{10}\star\right\} = \int_{t_\tau}^{t_{\tau_1}} \left(g^{\mathrm{T}}(s)R_{10}g(s)\right)\mathrm{d}s, \qquad (3.18)$$

$$\mathbb{E}\left\{\left(\int_{t_{\tau_2}}^{t_\tau} g(s)\mathrm{d}B(s)\right)^{\mathrm{T}}(R_9 + R_{10})\star\right\} = \int_{t_{\tau_2}}^{t_\tau}\left(g^{\mathrm{T}}(s)(R_9 + R_{10})g(s)\right)\mathrm{d}s. \qquad (3.19)$$

Applying inequalities from (3.13) to (3.19) to (3.12), it yields

$$\mathcal{L}_\omega V(t, \xi_t) \leqslant \xi^{\mathrm{T}}(t)\Lambda\xi(t), \qquad (3.20)$$

where

$$\begin{aligned}
\Lambda &= \Theta + \epsilon_1^{-1}\overline{P}G\left(\overline{P}G\right)^{\mathrm{T}} + V_1^{\mathrm{T}}\left[Z^{-1} - \epsilon_2^{-1}GG^{\mathrm{T}}\right]^{-1}V_1 + V_2^{\mathrm{T}}\left[U^{-1} - \epsilon_3^{-1}GG^{\mathrm{T}}\right]^{-1}V_2 \\
&\quad + N\left(R_7^{-1} + R_9^{-1}\right)N^{\mathrm{T}} + H\left(R_8^{-1} + R_{10}^{-1}\right)H^{\mathrm{T}} + M\left((R_7 + R_8)^{-1} + (R_9 + R_{10})^{-1}\right)M^{\mathrm{T}}
\end{aligned} \qquad (3.21)$$

with Θ being defined in Theorem 3.1.

By applying the Schur complement to (3.6) results in $\Lambda < 0$, which implies

$$LV(t, x_t) \leqslant \xi^{\mathrm{T}}(t)\Lambda\xi(t) < 0 \qquad (3.22)$$

therefore,

$$LV(t, x_t) \leqslant -\nu\xi(t)\xi(t) \leqslant -\nu x^{\mathrm{T}}(t)x(t), \qquad (3.23)$$

where $\nu = \lambda_m\{-\Lambda\}$. Now, integrating both sides of (3.23) from 0 to $t > 0$, and then taking the expectation, we obtain

$$\mathbb{E}\left\{V(s, x_s)\,|_{s=0}^{s=t}\right\} = \mathbb{E}\left\{\int_0^t LV(s, x_s)\mathrm{d}s\right\} \leqslant -\nu\int_0^t \mathbb{E}\left\{x^{\mathrm{T}}(s)x(s)\right\}\mathrm{d}s. \qquad (3.24)$$

On the other hand, it follows from (3.9) that

$$\mathbb{E}\{V(t, x_t)\} \geq \mathbb{E}\left\{\eta^{\mathrm{T}}(t)P\eta(t)\right\} \geq \lambda_m\{P\}\mathbb{E}\left\{\eta^{\mathrm{T}}(t)\eta(t)\right\}. \qquad (3.25)$$

Therefore, by (3.24) and (3.25)

$$\lambda_m\{P\}\mathbb{E}\left\{\eta^{\mathrm{T}}(t)\eta(t)\right\} \leqslant \mathbb{E}\{V(0, x_0)\} - \nu\int_0^t \mathbb{E}\left\{x^{\mathrm{T}}(s)x(s)\right\}\mathrm{d}s. \qquad (3.26)$$

Since $\rho(D) < 1$, choose $\epsilon > 0$ such that $e^{\tau_2\epsilon}\rho^2(D) < 1$, which implies $\rho^2(D) < e^{-\tau_2\epsilon} < 1$, denote $\epsilon_0 = e^{-\tau_2\epsilon}$. Let σ be any nonnegative real number. For all $0 \leqslant t \leqslant \sigma$, it follows from (1) of Lemma 2.6 that

$$
\begin{aligned}
\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} &\geq \mathbb{E}x^\mathrm{T}(t)x(t) - 2\mathbb{E}\left\{x^\mathrm{T}(t)Dx(t_\tau)\right\} + \mathbb{E}x^\mathrm{T}(t_\tau)D^\mathrm{T}Dx(t_\tau) \\
&\geq (1 - \epsilon_0)\mathbb{E}x^\mathrm{T}(t)x(t) - \left(\epsilon_0^{-1} - 1\right)\mathbb{E}x^\mathrm{T}(t_\tau)D^\mathrm{T}Dx(t_\tau).
\end{aligned}
\tag{3.27}
$$

Hence

$$
\mathbb{E}x^\mathrm{T}(t)x(t) \leqslant \frac{1}{1 - \epsilon_0}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} + \frac{1}{\epsilon_0}\mathbb{E}\left\{x^\mathrm{T}(t_\tau)D^\mathrm{T}Dx(t_\tau)\right\}.
\tag{3.28}
$$

By (3.25), (3.27), and (3.28), we then derive that for all $0 \leqslant t \leqslant \sigma$,

$$
\begin{aligned}
\mathbb{E}x^\mathrm{T}(t)x(t) &\leqslant \frac{1}{1 - \epsilon_0}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} + \frac{1}{\epsilon_0}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t_\tau)D^\mathrm{T}Dx(t_\tau)\right\} \\
&\leqslant \frac{1}{(1 - \epsilon_0)}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} + \frac{\lambda_M\{D'D\}}{\epsilon_0}\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\} \\
&= \frac{1}{(1 - \epsilon_0)}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} + \frac{\|D\|^2}{\epsilon_0}\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\}.
\end{aligned}
\tag{3.29}
$$

However, this holds for all $-\tau \leqslant t \leqslant 0$ as well. Therefore,

$$
\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\} \leqslant \frac{1}{(1 - \epsilon_0)}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\} + \frac{\rho^2(D)}{\epsilon_0}\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\}.
\tag{3.30}
$$

Since $\rho^2(D) < \epsilon_0$ and $P > 0$, we obtain that

$$
\frac{(1 - \epsilon_0)(\epsilon_0 - \rho^2(D))}{\epsilon_0}\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\} \leqslant \sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\}.
\tag{3.31}
$$

By supremum property,

$$
\sup_{-\tau_2\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\} \geq \sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\}.
\tag{3.32}
$$

By (3.31) and (3.32),

$$
\frac{(1 - \epsilon_0)(\epsilon_0 - \rho^2(D))}{\epsilon_0}\sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{x^\mathrm{T}(t)x(t)\right\} \leqslant \sup_{0\leqslant t\leqslant\sigma}\mathbb{E}\left\{\eta^\mathrm{T}(t)\eta(t)\right\}.
\tag{3.33}
$$

For any $t \geq 0$, it follows from (3.26), (3.33) and σ being an arbitrary nonnegative real number that

$$\sup_{t \geq 0} \mathbb{E}\left\{x^{\mathrm{T}}(t)x(t)\right\} \leqslant \mu\mathbb{E}\{V(0,x_0)\} - \nu\mu \int_0^t \mathbb{E}\left\{x^{\mathrm{T}}(s)x(s)\right\}\mathrm{d}s. \tag{3.34}$$

Since σ is an arbitrary nonnegative number, we have

$$\mathbb{E}\left\{x^{\mathrm{T}}(t)x(t)\right\} \leqslant \mu\mathbb{E}\{V(0,x_0)\} - \nu\mu \int_0^t \mathbb{E}\left\{x^{\mathrm{T}}(s)x(s)\right\}\mathrm{d}s, \quad \forall t \geq 0. \tag{3.35}$$

Then, applying Gronwall-Bellman lemma to (3.35), it yields

$$\mathbb{E}\left\{x^{\mathrm{T}}(t)x(t)\right\} \leqslant \mu\mathbb{E}\{V(0,x_0)\}\mathrm{e}^{-\nu\mu t}. \tag{3.36}$$

Note that there exists a scalar $\alpha > 0$ such that

$$\mu\mathbb{E}\{V(0,x_0)\} \leqslant \alpha \sup_{-\tau_2 \leqslant \theta \leqslant 0} \mathbb{E}\{\phi(\theta)\}. \tag{3.37}$$

Denote $\mu = \epsilon_0[(1 - \epsilon_0)(\epsilon_0 - \rho^2(D))\lambda_m\{P\}]^{-1} > 0, \beta = \nu\mu > 0$, then we have

$$\mathbb{E}\left\{x^{\mathrm{T}}(t)x(t)\right\} \leqslant \alpha\mathrm{e}^{-\beta t} \sup_{-\tau_2 \leqslant \theta \leqslant 0} \mathbb{E}\{\phi(\theta)\}. \tag{3.38}$$

By Definition 2.4, system (Σ) is robustly stochastically mean-square exponentially stable. $\quad\square$

Remark 3.2. Theorem 3.1 provides a delay-dependent condition with a new Lyapunov-Krasovskii functional, which makes full use of the relationship among the time-varying delay, its upper and lower bounds, and their difference. It is noted that this condition is obtained by free-weighting matrices techniques; model transformation is not resorted to in the derivation of Theorem 3.1. Thus, the conservatism caused by model transformation will be reduced.

If $\tau_1 = \tau_2 = \tau_0$, then $\bar{\tau} = 0, \tau(t) = \tau_0$. Applying Leibniz-Newton formula to (2.1), it yields the following zero equations:

$$2\zeta^{\mathrm{T}}(t)L\left[x(t) - Dx(t - \tau_0) - x(t - \tau_0) + Dx(t - 2\tau_0) - \int_{t_{\tau_0}}^t f(s)\mathrm{d}s - \int_{t_{\tau_0}}^t g(s)\mathrm{d}B(s)\right] = 0, \tag{3.39}$$

where L is any matrices with appropriate dimension, and

$$\zeta^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t - \tau_0), \frac{1}{\tau_0}\int_{t-\tau_0}^t x^{\mathrm{T}}(s)\mathrm{d}s, x^{\mathrm{T}}(t - 2\tau_0)\right]. \tag{3.40}$$

With zero equations (3.39) above, we obtain the following corollary.

Corollary 3.3. *For given constant delay τ_0, system (Σ) is robustly stochastically exponentially stable for all admissible uncertainties satisfying (2.4) and (2.5), if there exist symmetric positive definite matrices $P_0 > 0, Z_0 > 0, Z_1 > 0, Z_2 > 0, Z_3 > 0, Z_4 > 0$, scalars ϵ_i, $i = 1, 2, 3$ and any matrices L with appropriate dimensions such that the following LMI (3.41) holds:*

$$
\Omega = \begin{pmatrix}
\Xi & \overline{P}_0 G & \tau_0^2 W_1^T Z_3 & 0 & \tau_0 W_2^T Z_4 & 0 & \overline{L} & \overline{L} \\
* & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\tau_0^2 Z_3 & \tau_0^2 Z_3 G & 0 & 0 & 0 & 0 \\
* & * & * & -\epsilon_2 I & 0 & 0 & 0 & 0 \\
* & * & * & * & -\tau_0 Z_4 & \tau_0 Z_4 G & 0 & 0 \\
* & * & * & * & * & -\epsilon_3 I & 0 & 0 \\
* & * & * & * & * & * & -Z_3 & 0 \\
* & * & * & * & * & *I & 0 & -Z_4
\end{pmatrix} < 0, \tag{3.41}
$$

where

$$
\Xi = \Pi + \Upsilon + \Upsilon^T, \qquad W_1 = \begin{bmatrix} A_0 & A_1 & A_2 & 0 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} B_0 & B_1 & B_2 & 0 \end{bmatrix},
$$
$$
\overline{P}_0 = \begin{bmatrix} P_0 & -DP_0 & 0 & 0 \end{bmatrix}^T, \qquad \overline{L} = \begin{bmatrix} L^T & 0 & 0 & 0 \end{bmatrix}^T, \tag{3.42}
$$

with

$$
\Pi = \begin{pmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & 0 \\
* & \Pi_{22} & \Pi_{23} & 0 \\
* & * & \Pi_{33} & 0 \\
* & * & * & -Z_2
\end{pmatrix}, \qquad \Upsilon = \begin{pmatrix} L & -L(D+I) & 0 & LD \end{pmatrix},
$$

$$
\Pi_{11} = P_0 A_0 + A_0^T P_0 + Z_1 + Z_2 + \tau_0 Z_0 + (\epsilon_1 + \epsilon_2) S_0^T S_0 + \epsilon_3 T_0^T T_0,
$$
$$
\Pi_{12} = P_0 A_1 - A_0^T D P_0 + (\epsilon_1 + \epsilon_2) S_0^T S_1 + \epsilon_3 T_0^T T_1, \tag{3.43}
$$
$$
\Pi_{13} = P_0 A_2 + (\epsilon_1 + \epsilon_2) S_0^T S_2 + \epsilon_3 T_0^T T_2,
$$
$$
\Pi_{22} = -Z_1 - P_0 D^T A_1 - A_1^T D P_0 + (\epsilon_1 + \epsilon_2) S_1^T S_1 + \epsilon_3 T_1^T T_1,
$$
$$
\Pi_{23} = -P_0 D^T A_2 + (\epsilon_1 + \epsilon_2) S_1^T S_2 + \epsilon_3 T_1^T T_2, \qquad \Pi_{33} = -\tau_0 Z_0 + (\epsilon_1 + \epsilon_2) S_2^T S_2 + \epsilon_3 T_2^T T_2.
$$

If $A_2 = B_2 = 0, \Delta A_2(t) = \Delta B_2(t) = 0, D = 0$, then, system (2.1) becomes

$$
dx(t) = \overline{f}(t)dt + \overline{g}(t)dB(t), \tag{3.44}
$$

where

$$
\overline{f}(t) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t_\tau),
$$
$$
\overline{g}(t) = (B_0 + \Delta B_0(t))x(t) + (B_1 + \Delta B_1(t))x(t_\tau). \tag{3.45}
$$

Applying Leibniz-Newton formula to (3.44), it yields the following zero equations which will be used in our main result

$$2\tilde{\xi}^{\mathrm{T}}(t)\widehat{N}\left[x(t) - x(t_\tau) - \int_{t_\tau}^{t}\overline{f}(s)\mathrm{d}s - \int_{t_\tau}^{t}\overline{g}(s)\mathrm{d}B(s)\right] = 0,$$

$$2\tilde{\xi}^{\mathrm{T}}(t)\widehat{H}\left[x(t_{\tau_1}) - x(t_\tau) - \int_{t_\tau}^{t_{\tau_1}}\overline{f}(s)\mathrm{d}s - \int_{t_\tau}^{t_{\tau_1}}\overline{g}(s)\mathrm{d}B(s)\right] = 0, \qquad (3.46)$$

$$2\tilde{\xi}^{\mathrm{T}}(t)\widehat{M}\left[x(t_\tau) - x(t_{\tau_2}) - \int_{t_{\tau_2}}^{t_\tau}\overline{f}(s)\mathrm{d}s - \int_{t_{\tau_2}}^{t_\tau}\overline{g}(s)\mathrm{d}B(s)\right] = 0,$$

where N, H, and M are any matrices with appropriate dimensions, and

$$\tilde{\xi}^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t_\tau), x^{\mathrm{T}}(t_{\tau_1}), x^{\mathrm{T}}(t_{\tau_2})\right]. \qquad (3.47)$$

With zero equation (3.46) above, we obtain the following corollary.

Corollary 3.4. *For given scalars $0 \leqslant \tau_1 \leqslant \tau_2$ and $\overline{\tau}$, system (3.44) is robustly stochastically exponentially stable for all time-varying delays satisfying (2.3) and for all admissible uncertainties satisfying (2.4) and (2.5), if there exist symmetric positive definite matrices $P > 0$, $R_i > 0$, $i = 1, 2, \ldots, 10$, scalars $\epsilon_j, j = 1, 2, 3$ and any matrices $\widehat{N}, \widehat{H}, \widehat{M}$ with appropriate dimensions, such that the following LMI (3.48) holds:*

$$\tilde{\Phi} = \begin{pmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 \\ * & \tilde{\Phi}_3 \end{pmatrix} < 0, \qquad (3.48)$$

where

$$\tilde{\Phi}_1 = \begin{pmatrix} \Theta & \tilde{P}G & \tilde{V}_1^T Z & 0 & \tilde{V}_2^T U & 0 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & -Z & ZG & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 & 0 \\ * & * & * & * & -U & UG \\ * & * & * & * & * & -\epsilon_3 I \end{pmatrix},$$

$$\tilde{\Phi}_2 = \begin{pmatrix} \widetilde{N} & \widetilde{H} & \widetilde{M} & \widetilde{N} & \widetilde{H} & \widetilde{M} \end{pmatrix}, \qquad (3.49)$$

$$\tilde{\Phi}_3 = \mathrm{diag} = \{-R_7 \ -R_8 \ -R_7 \ -R_8 \ -R_9 \ -R_{10} \ -R_9 \ -R_{10}\},$$

$$\tilde{\Theta} = \tilde{\Gamma} + \tilde{\Psi} + \tilde{\Psi}^{\mathrm{T}}, \qquad \tilde{V}_1 = \begin{bmatrix} A_0 & A_1 & 0 & 0 \end{bmatrix}, \qquad \tilde{V}_2 = \begin{bmatrix} B_0 & B_1 & 0 & 0 \end{bmatrix},$$

$$\tilde{P} = \begin{bmatrix} P & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \qquad Z = \tau_2 R_7 + \delta R_8, \qquad U = P + \tau_2 R_9 + \delta R_{10},$$

$$\widetilde{N} = \begin{bmatrix} \widehat{N}^{\mathrm{T}}, 0, 0, 0, 0, 0 \end{bmatrix}^{\mathrm{T}}, \qquad \widetilde{H} = \begin{bmatrix} \widehat{H}^{\mathrm{T}}, 0, 0, 0, 0, 0 \end{bmatrix}^{\mathrm{T}}, \qquad \widetilde{M} = \begin{bmatrix} \widehat{M}^{\mathrm{T}}, 0, 0, 0, 0, 0 \end{bmatrix}^{\mathrm{T}},$$

with

$$\widetilde{\Gamma} = \begin{pmatrix} \widetilde{\Gamma}_{11} & \widetilde{\Gamma}_{12} & 0 & 0 \\ * & \widetilde{\Gamma}_{22} & 0 & 0 \\ * & * & -R_1 & 0 \\ * & * & * & -R_3 \end{pmatrix},$$

$$\widetilde{\Psi} = \begin{pmatrix} \widehat{N} & \widehat{M} - \widehat{H} - \widehat{N} & \widehat{H} & -\widehat{M} \end{pmatrix},$$

$$\widetilde{\Gamma}_{11} = PA_0 + A_0^T P + R_1 + R_2 + R_3 + (\epsilon_1 + \epsilon_2)S_0^T S_0 + \epsilon_3 T_0^T T_0,$$

$$\widetilde{\Gamma}_{12} = PA_1 + (\epsilon_1 + \epsilon_2)S_0^T S_1 + \epsilon_3 T_0^T T_1, \qquad \widetilde{\Gamma}_{22} = -(1 - \overline{\tau})R_2 + (\epsilon_1 + \epsilon_2)S_1^T S_1 + \epsilon_3 T_1^T T_1.$$

(3.50)

If $\tau_1 = \tau_2 = \tau_0$, $A_2 = B_2 = 0$, $\Delta A_i(t) = \Delta B_i(t) = 0$, $i = 0, 1, 2$, then $\overline{\tau} = 0$, $\tau(t) = \tau_0$ and (2.1) becomes

$$d[x(t) - Dx(t - \tau_0)] = \widehat{f}(t)dt + \widehat{g}(t)dB(t),$$

(3.51)

where

$$\widehat{f}(t) = A_0 x(t) + A_1 x(t - \tau_0), \qquad \widehat{g}(t) = B_0 x(t) + B_1 x(t - \tau_0).$$

(3.52)

Applying Leibniz-Newton formula to (3.51), it yields the following zero equations

$$2\varsigma^T(t)\widehat{L}\left[x(t) - Dx(t - \tau_0) - x(t - \tau_0) + Dx(t - 2\tau_0) - \int_{t_{\tau_0}}^{t} \widehat{f}(s)ds - \int_{t_{\tau_0}}^{t} \widehat{g}(s)dB(s)\right] = 0,$$

(3.53)

where $\widehat{L}$ is any matrices with appropriate dimension, and

$$\varsigma^T(t) = \left[x^T(t), x^T(t - \tau_0), x^T(t - 2\tau_0)\right].$$

(3.54)

With zero equations (3.53) above, we obtain the following corollary.

Corollary 3.5. *For given constant delay τ_0, system (3.51) is robustly stochastically exponentially stable, if there exist symmetric positive definite matrices $P_0 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, $Z_4 > 0$, any matrices $\widehat{L}$ with appropriate dimensions, such that the following LMI (3.41) holds:*

$$\widetilde{\Omega} = \begin{pmatrix} \widetilde{\Xi} & \tau_0^2 \widetilde{W}_1^T Z_3 & \tau_0 \widetilde{W}_2^T Z_4 & \widetilde{L} & \widetilde{L} \\ * & -\tau_0^2 Z_3 & 0 & 0 & 0 \\ * & * & -\tau_0 Z_4 & 0 & 0 \\ * & * & * & -Z_3 & 0 \\ * & * & * & * & -Z_4 \end{pmatrix} < 0,$$

(3.55)

where

$$\widetilde{\Xi} = \widetilde{\Pi} + \widetilde{\Upsilon} + \widetilde{\Upsilon}^{\mathrm{T}}, \qquad \widetilde{W}_1 = [A_0 \ \ A_1 \ \ 0], \qquad \widetilde{W}_2 = [B_0 \ \ B_1 \ \ 0], \qquad \widetilde{L} = \begin{bmatrix} \widehat{L}^{\mathrm{T}} & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \quad (3.56)$$

with

$$\widetilde{\Pi} = \begin{pmatrix} \widetilde{\Pi}_{11} & \widetilde{\Pi}_{12} & 0 \\ * & \widetilde{\Pi}_{22} & 0 \\ * & * & -Z_2 \end{pmatrix}, \qquad \widetilde{\Upsilon} = \begin{pmatrix} \widehat{L} & -\widehat{L}(D+I) & \widehat{L}D \end{pmatrix},$$

$$\widetilde{\Pi}_{11} = P_0 A_0 + A_0^{\mathrm{T}} P_0 + Z_1 + Z_2, \qquad \widetilde{\Pi}_{12} = P_0 A_1 - A_0^{\mathrm{T}} D P_0,$$

$$\widetilde{\Pi}_{22} = -Z_1 - P_0 D^{\mathrm{T}} A_1 - A_1^{\mathrm{T}} D P_0. \tag{3.57}$$

Deterministic systems may be regarded as special class of stochastic systems, let $B_i = 0, \Delta B_i = 0, i = 0, 1, 2$, then system (2.1)-(2.2) becomes the following uncertain neutral system with both discrete and distributed interval time-varying delays

$$(\Sigma_0) : \dot{x}(t) - D\dot{x}(t_\tau) = [A_0 + \Delta A_0(t)]x(t) + [A_1 + \Delta A_1(t)]x(t_\tau) + [A_2 + \Delta A_2(t)] \int_{t_\tau}^{t} x(s)\mathrm{d}s, \tag{3.58}$$

$$x(t) = \phi(t), \quad t \in [-\tau_2, 0]. \tag{3.59}$$

Applying Leibniz-Newton formula to (3.58), it yields the following zero equations which will be used in our main result:

$$2\xi^{\mathrm{T}}(t)N\left[x(t) - Dx(t_\tau) - x(t_\tau) + Dx(t_\tau - \tau(t_\tau)) - \int_{t_\tau}^{t} f(s)\mathrm{d}s\right] = 0,$$

$$2\xi^{\mathrm{T}}(t)H\left[x(t_{\tau_1}) - Dx(t_{\tau_1} - \tau(t_{\tau_1})) - x(t_\tau) + Dx(t_\tau - \tau(t_\tau)) - \int_{t_\tau}^{t_{\tau_1}} f(s)\mathrm{d}s\right] = 0, \qquad (3.60)$$

$$2\xi^{\mathrm{T}}(t)M\left[x(t_\tau) - Dx(t_\tau - \tau(t_\tau)) - x(t_{\tau_2}) + Dx(t_{\tau_2} - \tau(t_{\tau_2})) - \int_{t_{\tau_2}}^{t_\tau} f(s)\mathrm{d}s\right] = 0,$$

where N, H, and M are any matrices with appropriate dimensions, and $\xi(t)$ is defined in (3.5). With zero equations (3.60) above, we obtain

Corollary 3.6. *For given scalars* $0 \leqslant \tau_1 \leqslant \tau_2$ *and* $\overline{\tau}$, *system* (Σ_0) *is robustly stochastically exponentially stable for all time-varying delays satisfying* (2.3) *and for all admissible uncertainties satisfying* (2.4) *and* (2.5), *if there exist symmetric positive definite matrices* $P > 0, R_i > 0,$

$i = 0, 1, 2, \ldots, 8$, *scalars ϵ_j, $j = 1, 2, 3$ and any matrices N, H, M with appropriate dimensions, such that the following LMI (3.61) holds:*

$$\Phi = \begin{pmatrix} \Theta & \overline{P}G & V_1^{\mathrm{T}}Z & 0 & N & H & M \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -Z & ZG & 0 & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -R_7 & 0 & 0 \\ * & * & * & * & * & -R_8 & 0 \\ * & * & * & * & * & * & -R_7 - R_8 \end{pmatrix} < 0, \tag{3.61}$$

where

$$\Theta = \Gamma + \Psi + \Psi^{\mathrm{T}}, \qquad V_1 = \begin{bmatrix} A_0 & A_1 & A_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad V_2 = \begin{bmatrix} B_0 & B_1 & B_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{P} = \begin{bmatrix} P & -PD & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \qquad Z = \tau_2 R_7 + \delta R_8, \qquad U = P + \tau_2 R_9 + \delta R_{10},$$

$$\tag{3.62}$$

with

$$\Gamma = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 0 & 0 & 0 & 0 & 0 \\ * & \Gamma_{22} & \Gamma_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_3 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_{66} & 0 & 0 \\ * & * & * & * & * & * & \Gamma_{77} & 0 \\ * & * & * & * & * & * & * & \Gamma_{88} \end{pmatrix},$$

$$\Psi = \begin{pmatrix} N & (M - H) - N(D + I) & 0 & H & -M & -HD & MD & (N + H - M)D \end{pmatrix}, \tag{3.63}$$

$$\Gamma_{11} = PA_0 + A_0^{\mathrm{T}}P + R_1 + R_2 + R_3 + \tau_2 R_0 + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_0 + \epsilon_3 T_0^{\mathrm{T}}T_0,$$

$$\Gamma_{12} = PA_1 - A_0^{\mathrm{T}}PD + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_1 + \epsilon_3 T_0^{\mathrm{T}}T_1, \qquad \Gamma_{13} = PA_2 + (\epsilon_1 + \epsilon_2)S_0^{\mathrm{T}}S_2 + \epsilon_3 T_0^{\mathrm{T}}T_2,$$

$$\Gamma_{22} = R_4 + R_5 + R_6 - (1 - \overline{\tau})R_2 - D^{\mathrm{T}}PA_1 - A_1^{\mathrm{T}}PD + (\epsilon_1 + \epsilon_2)S_1^{\mathrm{T}}S_1 + \epsilon_3 T_1^{\mathrm{T}}T_1,$$

$$\Gamma_{23} = -D^{\mathrm{T}}PA_2 + (\epsilon_1 + \epsilon_2)S_1^{\mathrm{T}}S_2 + \epsilon_3 T_1^{\mathrm{T}}T_2, \qquad \Gamma_{33} = -\tau_2 R_0 + (\epsilon_1 + \epsilon_2)S_2^{\mathrm{T}}S_2 + \epsilon_3 T_2^{\mathrm{T}}T_2,$$

$$\Gamma_{66} = -R_4, \qquad \Gamma_{77} = -R_6, \qquad \Gamma_{88} = -(1 - \overline{\tau})R_5.$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(t, \xi_t) = \sum_{i=0}^{3} V_i(t, \xi_t), \tag{3.64}$$

where $V_i(t, \xi_t)$, $i = 0, 1, 2, 3$ are defined in (3.9). It can be concluded from Theorem 3.1. □

If $\tau_1 = \tau_2 = \tau_0$, $A_2 = 0$, $B_i = \Delta A_i(t) = \Delta B_i(t) = 0$, $i = 0, 1, 2$, then $\bar{\tau} = 0$, $\tau(t) = \tau_0$ and (2.1) becomes

$$\dot{x}(t) - D\dot{x}(t - \tau_0) = A_0 x(t) + A_1 x(t - \tau_0). \tag{3.65}$$

Applying Leibniz-Newton formula to (3.65), it yields the following zero equations:

$$2\varsigma^{\mathrm{T}}(t)\widehat{L}\left[x(t) - Dx(t - \tau_0) - x(t - \tau_0) + Dx(t - 2\tau_0) - \int_{t_{\tau_0}}^{t} \widehat{f}(s)\mathrm{d}s\right] = 0, \tag{3.66}$$

where $\widehat{L}$ is any matrices with appropriate dimension, $\widehat{f}(t) = A_0 x(t) + A_1 x(t - \tau_0)$, and

$$\varsigma^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t - \tau_0), x^{\mathrm{T}}(t - 2\tau_0)\right]. \tag{3.67}$$

With zero equations (3.66) above, we obtain the following corollary.

Corollary 3.7. *For given constant delay τ_0, system (3.65) is robustly stochastically exponentially stable, if there exist symmetric positive definite matrices $P_0 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, $Z_4 > 0$, and any matrices $\widehat{L}$ with appropriate dimensions, such that the following LMI (3.68) holds:*

$$\widetilde{\Omega} = \begin{pmatrix} \widetilde{\Xi} & \tau_0^2 \widetilde{W}_1^{\mathrm{T}} Z_3 & \check{L} & \check{L} \\ * & -\tau_0^2 Z_3 & 0 & 0 \\ * & * & -Z_3 & 0 \\ * & * & * & -Z_4 \end{pmatrix} < 0, \tag{3.68}$$

where

$$\widetilde{\Xi} = \widetilde{\Pi} + \widetilde{\Upsilon} + \widetilde{\Upsilon}^{\mathrm{T}}, \qquad \widetilde{W}_1 = \begin{bmatrix} A_0 & A_1 & 0 \end{bmatrix}, \qquad \check{L} = \begin{bmatrix} \widehat{L}^{\mathrm{T}} & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{3.69}$$

with

$$\widetilde{\Pi} = \begin{pmatrix} \widetilde{\Pi}_{11} & \widetilde{\Pi}_{12} & 0 \\ * & \widetilde{\Pi}_{22} & 0 \\ * & * & -Z_2 \end{pmatrix}, \qquad \widetilde{\Upsilon} = \begin{pmatrix} \widehat{L} & -\widehat{L}(D + I) & \widehat{L}D \end{pmatrix},$$

$$\widetilde{\Pi}_{11} = P_0 A_0 + A_0^{\mathrm{T}} P_0 + Z_1 + Z_2, \qquad \widetilde{\Pi}_{12} = P_0 A_1 - A_0^{\mathrm{T}} D P_0, \tag{3.70}$$

$$\widetilde{\Pi}_{22} = -Z_1 - P_0 D^{\mathrm{T}} A_1 - A_1^{\mathrm{T}} D P_0.$$
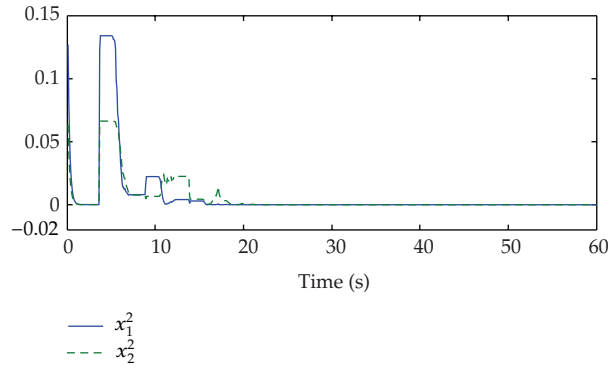
Figure 1: Mean-square exponential stability of the neutral stochastic system with interval time-varying delay.

Table 1: Allowable lower bounds and upper bounds for different $\bar{\tau}$.

$\bar{\tau}$	0.3	0.6	1.0	1.2	1.5	1.52
τ_2	1.5970	1.6724	1.8719	1.9717	2.1215	2.1315
τ_1	0.1500	0.3000	0.5000	0.6000	0.7500	0.7600
δ	1.4470	1.3724	1.3719	1.3717	1.3715	1.3715

4. Numerical Examples

Example 4.1. Consider the uncertain neutral linear stochastic time-varying delay system (Σ) with

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -1.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}, \qquad A_2 = -I, \qquad D = -0.5I,$$

$$B_0 = -0.1\,I, \qquad B_1 = 0.2I, \qquad B_2 = 0.1I, \qquad G = 0.1I, \qquad S_i = T_i = 0.1I, \qquad i = 0, 1, 2,$$

$$(4.1)$$

and $\tau(t) = \alpha \exp(-1/(1 + t)^2), 0 \leqslant \alpha \leqslant 2.1315$, set $\alpha = 1$, in this situation, $\tau_1 = 0.3679, \tau_2 = 1, \bar{\tau} = 0.7358, \delta = 0.6321$. Set $\phi(\theta) = [x_1^T(\theta), x_2^T(\theta)]^T$, where $x_1(\theta), x_2(\theta)$ are random initial values with zero mean and variance 1.0, and $F(t) = \sin(t)$, the trajectories of $x_1(t)$ and $x_2(t)$ are shown in Figure 1. According to Theorem 3.1, the lower bounds and the upper bounds on the time delay to guarantee the system is robustly stochastically mean-square exponentially stable are listed in Table 1.

Example 4.2. Consider the uncertain linear stochastic system (3.44) with

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix},$$

$$B_0 = B_1 = 0, \qquad G = 0.2I, \qquad S_i = T_i = I, \quad i = 0, 1, 2.$$

$$(4.2)$$

Table 2: Allowable upper bounds of τ_2 for different $\bar{\tau}$.

$\bar{\tau}$	0.3	0.5	0.9
Yan et al. [15]	0.7288	0.5252	0.1489
Corollary 3.4	0.9012	0.7498	0.5640

Table 3: Allowable upper bounds of τ_2 for different c.

c	0	0.1	0.3	0.5	0.7	0.9
Fridman and Shaked [19]	4.47	3.49	2.06	1.14	0.54	0.13
Corollary 3.7	4.47	4.05	3.29	2.58	1.85	0.92

According to Corollary 3.4, for $\tau_1 = 0$, the upper bounds on the time delay to guarantee the system is robustly stochastically mean-square exponentially stable are listed in Table 2. At the same time, Table 2 also lists the upper bounds obtained from the criterion in [15].

Example 4.3. Consider the uncertain neutral linear stochastic system (3.51) with

$$
A_0 = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.3 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix},
$$
$$
D = \begin{bmatrix} -0.2 & 0.0 \\ -0.95 & 0.2 \end{bmatrix}, \qquad B_0 = 0.2I, \qquad B_1 = 0.3I.
$$
(4.3)

If Corollary 3.5 in this paper is applied, the maximal admissible delay τ_M of this example is $\tau_M = 0.6001$.

Example 4.4. Consider the uncertain neutral linear stochastic system (3.65) with

$$
A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}, \qquad D = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \qquad 0 \leqslant c < 1. \tag{4.4}
$$

The maximum upper bound τ_M for this system in case of different cs is listed in Table 3, which shows that the results obtained by the methods proposed in this paper are less conservative than that in [19].

5. Conclusion

The mean-square exponential stability for uncertain neutral stochastic system with both discrete and distributed time-varying delays has been investigated in this paper. Sufficient conditions have been established without ignoring any terms in the weak infinitesimal operator of Lyapunov-Krasovskii functional by considering the relationship among the time-varying delay, its upper bounds, and their difference. The usual restriction that $\dot{\tau}(t) < 1$ has been removed by free-weight matrices method. According to the numerical examples, it has been shown that the proposed results improve some existing ones.

Acknowledgments

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