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Research Article

Global Convergence of a Modified Spectral Conjugate Gradient Method

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A modified spectral PRP conjugate gradient method is presented for solving unconstrained optimization problems. The constructed search direction is proved to be a sufficiently descent direction of the objective function. With an Armijo-type line search to determinate the step length, a new spectral PRP conjugate algorithm is developed. Under some mild conditions, the theory of global convergence is established. Numerical results demonstrate that this algorithm is promising, particularly, compared with the existing similar ones.

1. Introduction

Recently, it is shown that conjugate gradient method is efficient and powerful in solving large-scale unconstrained minimization problems owing to its low memory requirement and simple computation. For example, in [1–17], many variants of conjugate gradient algorithms are developed. However, just as pointed out in [2], there exist many theoretical and computational challenges to apply these methods into solving the unconstrained optimization problems. Actually, 14 open problems on conjugate gradient methods are presented in [2]. These problems concern the selection of initial direction, the computation of step length, and conjugate parameter based on the values of the objective function, the influence of accuracy of line search procedure on the efficiency of conjugate gradient algorithm, and so forth.

The general model of unconstrained optimization problem is as follows:

$$\min f(x), \quad x \in R^n, \tag{1.1}$$

where $f : R^n \to R$ is continuously differentiable such that its gradient is available. Let $g(x)$ denote the gradient of f at x, and let x_0 be an arbitrary initial approximate solution of (1.1). Then, when a standard conjugate gradient method is used to solve (1.1), a sequence of solutions will be generated by

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, \ldots, \tag{1.2}$$

where α_k is the steplength chosen by some line search method and d_k is the search direction defined by

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -g_k + \beta_k d_{k-1} & \text{if } k > 0, \end{cases} \tag{1.3}$$

where β_k is called conjugacy parameter and g_k denotes the value of $g(x_k)$. For a strictly convex quadratical programming, β_k can be appropriately chosen such that d_k and d_{k-1} are conjugate with respect to the Hessian matrix of the objective function. If β_k is taken by

$$\beta_k = \beta_k^{\mathrm{PRP}} = \frac{g_k^T (g_k - g_{k-1})}{\|g_{k-1}\|^2}, \tag{1.4}$$

where $\|\cdot\|$ stands for the Euclidean norm of vector, then (1.2)–(1.4) are called Polak-Ribiére-Polyak (PRP) conjugate gradient method (see [8, 18]).

It is well known that PRP method has the property of finite termination when the objective function is a strong convex quadratic function combined with the exact line search. Furthermore, in [7], for a twice continuously differentiable strong convex objective function, the global convergence has also been proved. However, it seems to be nontrivial to establish the global convergence theory under the condition of inexact line search, especially for a general nonconvex minimization problem. Quite recently, it is noticed that there are many modified PRP conjugate gradient methods studied (see, e.g., [10–13, 17]). In these methods, the search direction is constructed to possess the sufficient descent property, and the theory of global convergence is established with different line search strategy. In [17], the search direction d_k is given by

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -g_k + \beta_k^{\mathrm{PRP}} d_{k-1} - \theta_k y_{k-1} & \text{if } k > 0, \end{cases} \tag{1.5}$$

where

$$\theta_k = \frac{g_k^T d_{k-1}}{\|g_{k-1}\|^2}, \qquad y_{k-1} = g_k - g_{k-1}, \qquad s_{k-1} = x_k - x_{k-1}. \tag{1.6}$$

Similar to the idea in [17], a new spectral PRP conjugate gradient algorithm will be developed in this paper. On one hand, we will present a new spectral conjugate gradient direction, which also possess the sufficiently descent feature. On the other hand, a modified Armijo-type line search strategy is incorporated into the developed algorithm. Numerical experiments will be used to make a comparison among some similar algorithms.

The rest of this paper is organized as follows. In the next section, a new spectral PRP conjugate gradient method is proposed. Section 3 will be devoted to prove the global convergence. In Section 4, some numerical experiments will be done to test the efficiency, especially in comparison with the existing other methods. Some concluding remarks will be given in the last section.

2. New Spectral PRP Conjugate Gradient Algorithm

In this section, we will firstly study how to determine a descent direction of objective function.

Let x_k be the current iterate. Let d_k be defined by

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -\theta_k g_k + \beta_k^{\mathrm{PRP}} d_{k-1} & \text{if } k > 0, \end{cases} \tag{2.1}$$

where β_k^{PRP} is specified by (1.4) and

$$\theta_k = \frac{d_{k-1}^T y_{k-1}}{\|g_{k-1}\|^2} - \frac{d_{k-1}^T g_k g_k^T g_{k-1}}{\|g_k\|^2 \|g_{k-1}\|^2}. \tag{2.2}$$

It is noted that d_k given by (2.1) and (2.2) is different from those in [3, 16, 17], either for the choice of θ_k or for that of β_k.

We first prove that d_k is a sufficiently descent direction.

Lemma 2.1. *Suppose that d_k is given by* (2.1) *and* (2.2). *Then, the following result*

$$g_k^T d_k = -\|g_k\|^2 \tag{2.3}$$

holds for any $k \geq 0$.

Proof. Firstly, for $k = 0$, it is easy to see that (2.3) is true since $d_0 = -g_0$.

Secondly, assume that

$$d_{k-1}^T g_{k-1} = -\|g_{k-1}\|^2 \tag{2.4}$$

holds for $k-1$ when $k \geq 1$. Then, from (1.4), (2.1), and (2.2), it follows that

$$\begin{aligned} g_k^T d_k &= -\theta_k \|g_k\|^2 + \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2} d_{k-1}^T g_k \\ &= -\frac{d_{k-1}^T(g_k - g_{k-1})}{\|g_{k-1}\|^2} g_k^T g_k + \frac{d_{k-1}^T g_k g_k^T g_{k-1}}{\|g_k\|^2 \|g_{k-1}\|^2} g_k^T g_k + \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2} d_{k-1}^T g_k \\ &= \frac{d_{k-1}^T g_{k-1}}{\|g_{k-1}\|^2} g_k^T g_k = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}\left(-\|g_{k-1}\|^2\right) = -\|g_k\|^2. \end{aligned} \tag{2.5}$$

Thus, (2.3) is also true with $k-1$ replaced by k. By mathematical induction method, we obtain the desired result. □

From Lemma 2.1, it is known that d_k is a descent direction of f at x_k. Furthermore, if the exact line search is used, then $g_k^T d_{k-1} = 0$; hence

$$\theta_k = \frac{d_{k-1}^T y_{k-1}}{\|g_{k-1}\|^2} - \frac{d_{k-1}^T g_k g_k^T g_{k-1}}{\|g_k\|^2 \|g_{k-1}\|^2} = -\frac{d_{k-1}^T g_{k-1}}{\|g_{k-1}\|^2} = 1. \tag{2.6}$$

In this case, the proposed spectral PRP conjugate gradient method reduces to the standard PRP method. However, it is often that the exact line search is time-consuming and sometimes is unnecessary. In the following, we are going to develop a new algorithm, where the search direction d_k is chosen by (2.1)-(2.2) and the stepsize is determined by Armijio-type inexact line search.

Algorithm 2.2 (Modified Spectral PRP Conjugate Gradient Algorithm). We have the following steps.

Step 1. Given constants $\delta_1, \rho \in (0,1)$, $\delta_2 > 0$, $\epsilon > 0$. Choose an initial point $x_0 \in R^n$. Let $k := 0$.

Step 2. If $\|g_k\| \leq \epsilon$, then the algorithm stops. Otherwise, compute d_k by (2.1)-(2.2), and go to Step 3.

Step 3. Determine a steplength $\alpha_k = \max\{\rho^j, j = 0, 1, 2, \ldots\}$ such that

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta_1 \alpha_k g_k^T d_k - \delta_2 \alpha_k^2 \|d_k\|^2. \tag{2.7}$$

Step 4. Set $x_{k+1} := x_k + \alpha_k d_k$, and $k := k+1$. Return to Step 2.

Since d_k is a descent direction of f at x_k, we will prove that there must exist j_0 such that $\alpha_k = \rho^{j_0}$ satisfies the inequality (2.7).

Proposition 2.3. *Let $f : R^n \to R$ be a continuously differentiable function. Suppose that d is a descent direction of f at x. Then, there exists j_0 such that*

$$f(x + \alpha d) \le f(x) + \delta_1 \alpha g^T d - \delta_2 \alpha^2 \|d\|^2, \tag{2.8}$$

where $\alpha = \rho^{j_0}$, g is the gradient vector of f at x, $\delta_1, \rho \in (0,1)$ and $\delta_2 > 0$ are given constant scalars.

Proof. Actually, we only need to prove that a step length α is obtained in finitely many steps. If it is not true, then for all sufficiently large positive integer m, we have

$$f(x + \rho^m d) - f(x) > \delta_1 \rho^m g^T d - \delta_2 \rho^{2m} \|d\|^2. \tag{2.9}$$

Thus, by the mean value theorem, there is a $\theta \in (0,1)$ such that

$$\rho^m g(x + \theta \rho^m d)^T d > \delta_1 \rho^m g^T d - \delta_2 \rho^{2m} \|d\|^2. \tag{2.10}$$

It reads

$$\left(g(x + \theta \rho^m d) - g\right)^T d > (\delta_1 - 1) g^T d - \delta_2 \rho^m \|d\|^2. \tag{2.11}$$

When $m \to \infty$, it is obtained that

$$(\delta_1 - 1) g^T d < 0. \tag{2.12}$$

From $\delta_1 \in (0,1)$, it follows that $g^T d > 0$. This contradicts the condition that d is a descent direction. □

Remark 2.4. From Proposition 2.3, it is known that Algorithm 2.2 is well defined. In addition, it is easy to see that more descent magnitude can be obtained at each step by the modified Armijo-type line search (2.7) than the standard Armijo rule.

3. Global Convergence

In this section, we are in a position to study the global convergence of Algorithm 2.2. We first state the following mild assumptions, which will be used in the proof of global convergence.

Assumption 3.1. The level set $\Omega = \{x \in R^n \mid f(x) \le f(x_0)\}$ is bounded.

Assumption 3.2. In some neighborhood N of Ω, f is continuously differentiable and its gradient is Lipschitz continuous, namely, there exists a constant $L > 0$ such that

$$\|g(x) - g(y)\| \le L \|x - y\|, \quad \forall x, y \in N. \tag{3.1}$$

Since $\{f(x_k)\}$ is decreasing, it is clear that the sequence $\{x_k\}$ generated by Algorithm 2.2 is contained in a bounded region from Assumption 3.1. So, there exists a convergent subsequence of $\{x_k\}$. Without loss of generality, it can be supposed that $\{x_k\}$ is convergent. On the other hand, from Assumption 3.2, it follows that there is a constant $\gamma_1 > 0$ such that

$$\|g(x)\| \leq \gamma_1, \quad \forall x \in \Omega. \tag{3.2}$$

Hence, the sequence $\{g_k\}$ is bounded.

In the following, we firstly prove that the stepsize α_k at each iteration is large enough.

Lemma 3.3. *With Assumption 3.2, there exists a constant $m > 0$ such that the following inequality*

$$\alpha_k \geq m \frac{\|g_k\|^2}{\|d_k\|^2} \tag{3.3}$$

holds for all k sufficiently large.

Proof. Firstly, from the line search rule (2.7), we know that $\alpha_k \leq 1$.

If $\alpha_k = 1$, then we have $\|g_k\| \leq \|d_k\|$. The reason is that $\|g_k\| > \|d_k\|$ implies that

$$\|g_k\|^2 > \|g_k\| \|d_k\| \geq -g_k^T d_k, \tag{3.4}$$

which contradicts (2.3). Therefore, taking $m = 1$, the inequality (3.3) holds.

If $0 < \alpha_k < 1$, then the line search rule (2.7) implies that $\rho^{-1}\alpha_k$ does not satisfy the inequality (2.7). So, we have

$$f\left(x_k + \rho^{-1}\alpha_k d_k\right) - f(x_k) > \delta_1 \alpha_k \rho^{-1} g_k^T d_k - \delta_2 \rho^{-2} \alpha_k^2 \|d_k\|^2. \tag{3.5}$$

Since

$$\begin{aligned} f\left(x_k + \rho^{-1}\alpha_k d_k\right) - f(x_k) &= \rho^{-1}\alpha_k g\left(x_k + t_k \rho^{-1}\alpha_k d_k\right)^T d_k \\ &= \rho^{-1}\alpha_k g_k^T d_k + \rho^{-1}\alpha_k \left(g\left(x_k + t_k \rho^{-1}\alpha_k d_k\right) - g_k\right)^T d_k \\ &\leq \rho^{-1}\alpha_k g_k^T d_k + L\rho^{-2}\alpha_k^2 \|d_k\|^2, \end{aligned} \tag{3.6}$$

where $t_k \in (0,1)$ satisfies $x_k + t_k \rho^{-1}\alpha_k d_k \in N$ and the last inequality is from (3.2), it is obtained that

$$\delta_1 \alpha_k \rho^{-1} g_k^T d_k - \delta_2 \rho^{-2} \alpha_k^2 \|d_k\|^2 < \rho^{-1}\alpha_k g_k^T d_k + L\rho^{-2}\alpha_k^2 \|d_k\|^2 \tag{3.7}$$

due to (3.5) and (3.1). It reads

$$(1 - \delta_1)\alpha_k \rho^{-1} g_k^T d_k + (L + \delta_2)\rho^{-2}\alpha_k^2 \|d_k\|^2 > 0, \tag{3.8}$$

that is,

$$(L+\delta_2)\rho^{-1}\alpha_k\|d_k\|^2 > (\delta_1-1)g_k^T d_k. \tag{3.9}$$

Therefore,

$$\alpha_k > \frac{(\delta_1-1)\rho g_k^T d_k}{(L+\delta_2)\|d_k\|^2}. \tag{3.10}$$

From Lemma 2.1, it follows that

$$\alpha_k > \frac{\rho(1-\delta_1)\|g_k\|^2}{(L+\delta_2)\|d_k\|^2}. \tag{3.11}$$

Taking

$$m = \min\left\{1, \frac{\rho(1-\delta_1)}{L+\delta_2}\right\}, \tag{3.12}$$

then the desired inequality (3.3) holds. □

From Lemmas 2.1 and 3.3 and Assumption 3.1, we can prove the following result.

Lemma 3.4. *Under Assumptions 3.1 and 3.2, the following results hold:*

$$\sum_{k\geq 0}\frac{\|g_k\|^4}{\|d_k\|^2} < \infty, \tag{3.13}$$

$$\lim_{k\to\infty}\alpha_k^2\|d_k\|^2 = 0. \tag{3.14}$$

Proof. From the line search rule (2.7) and Assumption 3.1, there exists a constant M such that

$$\sum_{k=0}^{n-1}\left(-\delta_1\alpha_k g_k^T d_k + \delta_2\alpha_k^2\|d_k\|^2\right) \leq \sum_{k=0}^{n-1}\left(f(x_k) - f(x_{k+1})\right) = f(x_0) - f(x_n) < 2M. \tag{3.15}$$

Then, from Lemma 2.1, we have

$$\begin{aligned} 2M &\geq \sum_{k=0}^{n-1}\left(-\delta_1\alpha_k g_k^T d_k + \delta_2\alpha_k^2\|d_k\|^2\right) \\ &= \sum_{k=0}^{n-1}\left(\delta_1\alpha_k\|g_k\|^2 + \delta_2\alpha_k^2\|d_k\|^2\right) \end{aligned}$$

$$\geq \sum_{k=0}^{n-1}\left(\delta_1 m \frac{\|g_k\|^2}{\|d_k\|^2}\|g_k\|^2 + \delta_2 \cdot m^2 \cdot \frac{\|g_k\|^4}{\|d_k\|^4}\cdot \|d_k\|^2\right)$$
$$= \sum_{k=0}^{n-1}(\delta_1 + \delta_2 m)\frac{\|g_k\|^4}{\|d_k\|^2}\cdot m. \tag{3.16}$$

Therefore, the first conclusion is proved.

Since

$$2M \geq \sum_{k=0}^{n-1}\left(\delta_1\alpha_k\|g_k\|^2 + \delta_2\alpha_k^2\|d_k\|^2\right) \geq \delta_2\sum_{k=0}^{n-1}\alpha_k^2\|d_k\|^2, \tag{3.17}$$

the series

$$\sum_{k=0}^{\infty}\alpha_k^2\|d_k\|^2 \tag{3.18}$$

is convergent. Thus,

$$\lim_{k\to\infty}\alpha_k^2\|d_k\|^2 = 0. \tag{3.19}$$

The second conclusion (3.14) is obtained. □

In the end of this section, we come to establish the global convergence theorem for Algorithm 2.2.

Theorem 3.5. *Under Assumptions 3.1 and 3.2, it holds that*

$$\lim_{k\to\infty}\inf\|g_k\| = 0. \tag{3.20}$$

Proof. Suppose that there exists a positive constant $\epsilon > 0$ such that

$$\|g_k\| \geq \epsilon \tag{3.21}$$

for all k. Then, from (2.1), it follows that

$$\|d_k\|^2 = d_k^T d_k$$
$$= \left(-\theta_k g_k^T + \beta_k^{\mathrm{PRP}} d_{k-1}^T\right)\left(-\theta_k g_k + \beta_k^{\mathrm{PRP}} d_{k-1}\right)$$

$$\begin{aligned}
&= \theta_k^2 \|g_k\|^2 - 2\theta_k \beta_k^{\mathrm{PRP}} d_{k-1}^T g_k + \left(\beta_k^{\mathrm{PRP}}\right)^2 \|d_{k-1}\|^2 \\
&= \theta_k^2 \|g_k\|^2 - 2\theta_k \left(d_k^T + \theta_k g_k^T\right) g_k + \left(\beta_k^{\mathrm{PRP}}\right)^2 \|d_{k-1}\|^2 \\
&= \theta_k^2 \|g_k\|^2 - 2\theta_k d_k^T g_k - 2\theta_k^2 \|g_k\|^2 + \left(\beta_k^{\mathrm{PRP}}\right)^2 \|d_{k-1}\|^2 \\
&= \left(\beta_k^{\mathrm{PRP}}\right)^2 \|d_{k-1}\|^2 - 2\theta_k d_k^T g_k - \theta_k^2 \|g_k\|^2.
\end{aligned} \tag{3.22}$$

Dividing by $(g_k^T d_k)^2$ in the both sides of this equality, then from (1.4), (2.3), (3.1), and (3.21), we obtain

$$\begin{aligned}
\frac{\|d_k\|^2}{\|g_k\|^4} &= \frac{(\beta_k^{\mathrm{PRP}})^2 \|d_{k-1}\|^2 - 2\theta_k d_k^T g_k - \theta_k^2 \|g_k\|^2}{\|g_k\|^4} \\
&= \frac{\left(g_k^T (g_k - g_{k-1})\right)^2}{\|g_{k-1}\|^4} \frac{\|d_{k-1}\|^2}{\|g_k\|^4} - \frac{(\theta_k - 1)^2}{\|g_k\|^2} + \frac{1}{\|g_k\|^2} \\
&\le \frac{\|g_k - g_{k-1}\|^2}{\|g_{k-1}\|^4} \frac{\|d_{k-1}\|^2}{\|g_k\|^2} - \frac{(\theta_k - 1)^2}{\|g_k\|^2} + \frac{1}{\|g_k\|^2} \\
&\le \frac{\|g_k - g_{k-1}\|^2}{\|g_k\|^2} \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} + \frac{1}{\|g_k\|^2} \\
&< \frac{L^2 \alpha_{k-1}^2 \|d_{k-1}\|^2}{\epsilon^2} \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} + \frac{1}{\|g_k\|^2}.
\end{aligned} \tag{3.23}$$

From (3.14) in Lemma 3.4, it follows that

$$\lim_{k \to \infty} \alpha_{k-1}^2 \|d_{k-1}\|^2 = 0. \tag{3.24}$$

Thus, there exists a sufficient large number k_0 such that for $k \ge k_0$, the following inequalities

$$0 \le \alpha_{k-1}^2 \|d_{k-1}\|^2 < \frac{\epsilon^2}{L^2} \tag{3.25}$$

hold.

Therefore, for $k \geq k_0$,

$$\frac{\|d_k\|^2}{\|g_k\|^4} \leq \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} + \frac{1}{\|g_k\|^2} \leq \cdots \leq \frac{\|d_{k_0}\|^2}{\|g_{k_0}\|^4} + \sum_{i=k_0+1}^{k} \frac{1}{\|g_i\|^2} < \frac{C_0}{\epsilon^2} + \sum_{i=k_0+1}^{k} \frac{1}{\epsilon^2} = \frac{C_0 + k - k_0}{\epsilon^2}, \tag{3.26}$$

where $C_0 = \epsilon^2 \|d_{k_0}\|^2 / \|g_{k_0}\|^2$ is a nonnegative constant.

The last inequality implies

$$\sum_{k\geq 1} \frac{\|g_k\|^4}{\|d_k\|^2} \geq \sum_{k>k_0} \frac{\|g_k\|^4}{\|d_k\|^2} > \epsilon^2 \sum_{k>k_0} \frac{1}{C_0 + k - k_0} = \infty, \tag{3.27}$$

which contradicts the result of Lemma 3.4.

The global convergence theorem is established. □

4. Numerical Experiments

In this section, we will report the numerical performance of Algorithm 2.2. We test Algorithm 2.2 by solving the 15 benchmark problems from [19] and compare its numerical performance with that of the other similar methods, which include the standard PRP conjugate gradient method in [6], the modified FR conjugate gradient method in [16], and the modified PRP conjugate gradient method in [17]. Among these algorithms, either the updating formula or the line search rule is different from each other.

All codes of the computer procedures are written in MATLAB 7.0.1 and are implemented on PC with 2.0 GHz CPU processor, 1 GB RAM memory, and XP operation system.

The parameters are chosen as follows:

$$\epsilon = 10^{-6}, \qquad \rho = 0.75, \qquad \delta_1 = 0.1, \qquad \delta_2 = 1. \tag{4.1}$$

In Tables 1 and 2, we use the following denotations:

Dim: the dimension of the objective function;

GV: the gradient value of the objective function when the algorithm stops;

NI: the number of iterations;

NF: the number of function evaluations;

CT: the run time of CPU;

mfr: the modified FR conjugate gradient method in [16];

prp: the standard PRP conjugate gradient method in [6];

Table 1: Comparison of efficiency with the other methods.

Function	Algorithm	Dim	GV	NI	NF	CT(s)
Rrosenbrock	mfr	2	$8.8818e-007$	328	7069	0.2970
	prp	2	$9.2415e-007$	760	41189	1.4370
	mprp	2	$8.6092e-007$	124	2816	0.0940
	msprp	2	$6.9643e-007$	122	2597	0.1400
Freudenstein and Roth	mfr	2	$5.5723e-007$	236	5110	0.2190
	prp	2	$7.1422e-007$	331	18798	0.6250
	mprp	2	$2.4666e-007$	67	1904	0.0940
	msprp	2	$8.6967e-007$	62	1437	0.0780
Brown badly	mfr	2	—	—	—	—
	prp	2	—	—	—	—
	mprp	2	$7.9892e-007$	105	10279	0.2030
	msprp	2	$7.6029e-007$	70	7117	0.2660
Beale	mfr	2	$6.1730e-007$	74	714	0.0780
	prp	2	$8.2455e-007$	292	12568	0.4370
	mprp	2	$6.2257e-007$	130	1539	0.0940
	msprp	2	$8.7861e-007$	91	877	0.0470
Powell singular	mfr	4	$9.9827e-007$	4122	10578	0.6870
	prp	4	—	—	—	—
	mprp	4	$9.6909e-007$	13565	218964	5.2660
	msprp	4	$9.8512e-007$	11893	169537	7.2500
Wood	mfr	4	$7.7937e-007$	263	5787	0.2660
	prp	4	$9.9841e-007$	1284	69501	2.3440
	mprp	4	$9.6484e-007$	280	6432	0.1720
	msprp	4	$7.9229e-007$	404	9643	0.4070
Extended Powell singular	mfr	4	$9.9827e-007$	4122	10578	0.6800
	prp	4	—	—	—	—
	mprp	4	$9.6909e-007$	13565	218964	5.5310
	msprp	4	$9.8512e-007$	11893	169537	7.4070
Broyden tridiagonal	mfr	4	$4.8451e-007$	53	784	0.0630
	prp	4	$6.6626e-007$	87	4460	0.1180
	mprp	4	$5.8166e-007$	39	430	0.0320
	msprp	4	$9.7196e-007$	52	785	0.0780

msprp: the modified PRP conjugate gradient method in [17];

mprp: the new algorithm developed in this paper.

From the above numerical experiments, it is shown that the proposed algorithm in this paper is promising.

5. Conclusion

In this paper, a new spectral PRP conjugate gradient algorithm has been developed for solving unconstrained minimization problems. Under some mild conditions, the global

Table 2: Comparison of efficiency with the other methods.

Function	Algorithm	Dim	GV	NI	NF	CT(s)
Kowalik and Osborne	mfr	4	—	—	—	—
	prp	4	$8.9521e-007$	833	26191	1.2970
	mprp	4	$9.9698e-007$	6235	35425	3.5940
	msprp	4	$9.9560e-007$	7059	37976	4.9850
Broyden banded	mfr	6	$8.9469e-007$	40	505	0.0780
	prp	6	$8.4684e-007$	268	9640	0.4840
	mprp	6	$8.9029e-007$	102	1319	0.0940
	msprp	6	$9.3276e-007$	44	556	0.0940
Discrete boundary	mfr	6	$9.1531e-007$	107	509	0.0780
	prp	6	$7.8970e-007$	269	11449	0.4690
	mprp	6	$8.28079e-007$	157	1473	0.0930
	msprp	6	$9.9436e-007$	165	1471	0.1410
Variably dimensioned	mfr	8	$7.3411e-007$	57	1233	0.1250
	prp	8	$7.3411e-007$	113	7403	0.3290
	mprp	8	$9.0900e-007$	69	1544	0.0780
	msprp	8	$7.3411e-007$	57	1233	0.1100
Broyden tridiagonal	mfr	9	$9.1815e-007$	129	2173	0.1250
	prp	9	$6.4584e-007$	113	5915	0.2500
	mprp	9	$7.3529e-007$	187	2967	0.1250
	msprp	9	$9.2363e-007$	82	1304	0.1100
Linear-rank1	mfr	10	$9.7462e-007$	84	3762	0.1720
	prp	10	$4.5647e-007$	98	6765	0.2810
	mprp	10	$6.9140e-007$	51	2216	0.0780
	msprp	10	$6.6630e-007$	50	2162	0.1250
Linear-full rank	mfr	12	$7.6919e-007$	9	36	0.0160
	prp	12	$8.2507e-007$	47	1904	0.1090
	mprp	12	$7.6919e-007$	9	36	0.0630
	msprp	12	$7.6919e-007$	9	36	0.0150

convergence has been proved with an Armijo-type line search rule. Compared with the other similar algorithms, the numerical performance of the developed algorithm is promising.

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