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Research Article

An Analytical Approach to the Analysis of Guard-Channel-Based Call Admission Control in Wireless Cellular Networks

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We develop an analytical approach to the performance analysis and optimization of wireless cellular networks for which different types of calls are prioritized based on a channel reservation scheme. We assume that the channel occupancy time differs for new and handover calls. We obtain simple formulas for calculating quality of service (QoS) metrics and solve some problems related to finding the optimal values of guard channels as well as present the results of numerical experiments.

1. Introduction

In wireless cellular networks, when a subscriber crosses the boundary of a cell (while on a call), the subscriber releases this cell's channel and requests an empty channel in a neighboring cell. This process is called a handover. If a neighboring cell has at least one empty channel, then such a handover call (h-call) is delivered continuously and almost transparently to the subscriber; otherwise, the call is dropped. Usually, dropping an ongoing call from a different cell is less desirable than blocking new attempts originating within the cell (o-calls). Thus, h-calls are considered to be more important (have a higher priority) than o-calls.

There are various prioritization schemes for h-calls in wireless networks. Efficient methods for minimizing the rate of dropped h-calls include reserving a number of channels (guard channels) specifically for h-calls, imposing a threshold for o-calls, and queuing h-calls. A detailed review of papers investigating these methods can be found in [1–3].

In this paper, we consider a model with call admission control (CAC) based on guard channels schemes (GC schemes) and without queuing of *h*-calls. In almost all existing papers, it has been assumed that *h*-calls and *o*-calls have identical channel occupancy times. Due to this assumption, the function of a cell is described by a one-dimensional Markov chain, and the authors managed to find simple recursive formulas for calculating the probabilities of dropping *h*-calls and of blocking *o*-calls. The classical guard channels (CGC) reservation scheme [4], as well as a generalized reservation scheme called fractional guard channels (FGC) and its own special case, that is, uniform fractional guard channels (UFGC) [5], was considered. However, the identity channel occupancy times assumption for calls of different types is rather limiting and unrealistic [6–8]. Therefore, one-dimensional Markov chains do not adequately describe such networks, and further research on multidimensional models is required. However, when using multidimensional models, we face the famous "curse of dimensionality". To address this problem, a few papers have suggested various approximation schemes. A detailed review of approximate methods can be found in [9].

In this paper, we develop an analytical approach to the calculation of models without *h*-call queues, in which *o*- and *h*-calls are not assumed to have identical channel occupancy times. First, we briefly review the results of well-known papers that study such models. One of the first works to investigate such models was [6]. This paper examined models featuring various reservation strategies for *h*-calls or establishing thresholds for *o*-calls. For the latter type of model, simple formulas are easily developed for calculating the loss probability of *h*-calls as well as the blocking probability of *o*-calls. The development of simple formulas is possible because there is an explicit solution to an appropriate set of global equilibrium equations (SGEE) for the state probabilities. However, there is no closed form solution to the corresponding equations for models with CAC based on the GC scheme; therefore, the authors suggested approximation formulas for QoS calculations based on some heuristic considerations. The accuracy of the suggested formulas was determined by simulation. The authors also suggested another approximation scheme, which they dubbed traditional and in which different occupancy times are replaced by a single time by following a defined procedure. It was shown that the traditional scheme gives a rough approximation, especially as the true occupancy times of different calls differ substantially. The common idea underlying both approximation methods is the unification of the channel occupancy times of polytypic calls with appropriate schemes. A similar approximation approach based on this idea is proposed in [7], and another approach is suggested in paper [8], which offers high accuracy in models for which the parameters for different call types differ substantially. For models with identical parameters, however, its results are less accurate.

In this paper, we develop an analytical approach for calculating models of a wireless network with nonidentical channel occupancy times for *h*- and *o*-calls in which the CAC are based on CGC, FGC, and UFGC schemes. We prove that the formulas suggested in [6, 7] are not merely approximations but are exact if the model satisfies a local balance condition (for more information about this condition, see [10]); otherwise, these formulas are approximations. We use these formulas to find the optimal number of guard channels for CAC based on CGC scheme.

Even though we consider a monoservice network for the sake of simplifying both our models and any intermediate calculations, the results that we achieve can be easily adapted for multiservice networks.

This paper is organized as follows. In Section 2, well-known QoS calculation algorithms are given for the CGC scheme. In Section 3, a new analytical approach is developed for the study of models with the FGC scheme, and, as a particular case, we obtain results for the CGC scheme. Section 4 contains the results of selecting the optimal number of guard channels for the CAC based on CGC scheme. In Section 5, we provide some concluding remarks. Finally, in an appendix, we provide proofs of all required facts.

2. Approximate Methods for CAC Based on the CGC Scheme

First, we will analyze the model of a cell belonging to a homogeneous wireless network with a CAC based on the CGC scheme. Henceforth, a homogeneous network is defined as one for which the traffic parameters of all cells within the network are statistically identical; that is, we can study the function of a representative cell in isolation. This assumption is true for practically all networks with small cells (e.g., networks with microcells).

The representative cell contains N channels, $1 < N < \infty$, which are used by Poisson flows of o- and h-calls. The intensity of an x-call is denoted by λ_x, where $x \in \{o, h\}$. If at least one free channel exists upon the arrival of an h-call, then the call seizes one of the free channels; otherwise, the call is dropped. A new arriving call is accepted only when the number of busy channels is less than m, for some fixed m satisfying $1 \le m \le N$ (in the case where $m = N$, there are no restrictions on new calls); otherwise, the new call is blocked.

The distribution functions corresponding to the channel occupancy times of both types of calls are exponential, but their parameters differ by the intensity of handling of n- and h-calls, which are given by μ_o and μ_h, respectively. Generally speaking, $\mu_o \neq \mu_h$.

Remark 2.1. The abovementioned channel occupancy times are defined not only in terms of required service times for different call types but also in terms of the subscriber mobility within cells. In all existing works, it has been assumed that the required service times of x-calls and the time spent by subscribers within the cell have exponential distributions with parameters τ_x and γ_x, $x \in \{o, h\}$, respectively. Thus, the channel occupancy time of x-calls is defined as the minimum of two exponentially distributed stochastic variables; that is, for an x-call, it has an exponential form with parameter $\mu_x = \tau_x + \gamma_x,\ \ x \in \{o, h\}$. Based on practical considerations, the residence duration may differ greatly between different types of calls; hence, the channel occupancy times will also differ.

Cell functionality is described by a two-dimensional Markov chain (MC); that is, the state of the given system at an arbitrary moment in time is described by a two-dimensional vector $\mathbf{k} = (k_0,\ k_h)$, where $k_o(k_h)$ indicates the number of n-calls (h-calls) in the cell. Thus, the state space of the corresponding MC is determined as follows:

$$S := \{\mathbf{k} : k_o = 0, 1, \ldots, m;\ \ k_h = 0, 1, \ldots, N;\ \ k_o + k_h \le N\}. \tag{2.1}$$

Let us denote the stationary probability of state $\mathbf{k} \in S$ as $p(\mathbf{k})$. Stationary probabilities are found from the appropriate SGEE and then used to calculate all the required QoS metrics of the model. The primary QoS metrics are the loss probability of h-calls (P_h) and the blocking probability of o-calls (P_o). They are defined as follows when using the above-mentioned CGC admission control:

$$\begin{aligned} P_h &= \sum_{\mathbf{k} \in S} p(\mathbf{k}) I(k_o + k_h = N), \\ P_o &= \sum_{\mathbf{k} \in S} p(\mathbf{k}) I(k_o + k_h \ge m). \end{aligned} \tag{2.2}$$

Henceforth, let $I(A)$ denote the indicator function of event A. From Formulas (2.2), we obtain $P_o = P_h$ when $m = N$.

We should note that for practical networks, this SGEE has large dimension, which explains why its exact solution is computationally difficult. In [8], for the case $\lambda_o \ll \lambda_h$, $\mu_o \ll \mu_h$ the following algorithm for approximating the values of the QoS metrics from (2.2) is developed based on strict mathematical justifications.

Step 1. Calculate the following parameters:

$$\rho_i(j) = \left(\nu_h^j / j!\right)\rho_i(0), \quad i = \overline{0, m}, \quad j = \overline{0, N-i}, \tag{2.3}$$

where $\rho_i(0) = \left(\sum_{j=0}^{m}(\nu_h^j / j!)\right)^{-1}$,

$$\pi(\langle i \rangle) = \left(\nu_o^i / i!\right)\prod_{j=1}^{i}\Lambda(j)\pi(\langle 0 \rangle), \quad i = \overline{1, m}\ , \tag{2.4}$$

where $(\langle 0 \rangle) = (1 + \sum_{i=1}^{m}(\nu_o^i / i!)\prod_{j=1}^{i}\Lambda(j))$, $\Lambda(i+1) = \rho_i(0)\sum_{j=0}^{m-1-i}(\nu_h^j / j!)$, $i = \overline{0, m-1}$.

Step 2. Calculate the approximate values of the QoS metrics in (2.2):

$$P_h \approx \sum_{i=0}^{m} E_B(\nu_h, N-i)\pi(\langle i \rangle), \tag{2.5}$$

$$P_o \approx \sum_{i=0}^{m}\sum_{j=m-i}^{N-i}\rho_i(j)\pi(\langle i \rangle), \tag{2.6}$$

where $E_B(\nu, s)$ is the Erlang B-formula, that is, $E_B(\nu, s) = (\nu^s / s!)\left(\sum_{i=0}^{s}(\nu^i / i!)\right)^{-1}$.

In the above algorithm, the following notation is used: $\nu_x = \lambda_x / \mu_x$, $x \in \{o, h\}$. The results achieved with this algorithm nearly coincide with those from exact calculations under the indicated above conditions: $\lambda_o \ll \lambda_h$, $\mu_o \ll \mu_h$. Under these conditions, the exact values of the QoS metrics for a model of moderate dimensions are calculated with SGEE.

The authors in [6] also introduced an approximation algorithm for solving this problem, which was based on the following heuristic considerations. Because the QoS metrics P_h and P_o increase in proportion to the incoming traffic load ν_h and ν_o, a one-dimensional birth and death process is used as a research model. The different channel occupation times are replaced with their averages, and the traffic intensities are replaced by their loads; that is, the following approximations are made: $\mu_o = \mu_h = 1$, $\lambda_o = \nu_o$, $\lambda_h = \nu_h$. The authors in [6]

Table 1: Comparison of different algorithms for the case $N = 10$, $\lambda_o = 1$, $\lambda_h = 5$, $\mu_o = 1$, $\mu_h = 5$.

m	P_o			P_h		
	[8]	[6]	Traditional	[8]	[6]	Traditional
1	0.7310586	0.7746003	0.8374876	$3.4676E-07$	$1.24228E-07$	$8.88776E-06$
2	0.4621171	0.4891989	0.5577866	$1.1128E-06$	$1.87684E-07$	$9.67381E-06$
3	0.2516532	0.2588479	0.2998047	$3.1089E-06$	$3.26787E-07$	$1.10285E-05$
4	0.1163676	0.1153544	0.1327018	$7.2415E-06$	$6.15877E-07$	$1.29415E-05$
5	0.0453591	0.0435004	0.0492806	$1.3815E-05$	$1.20496E-06$	$1.54023E-05$
6	0.0149759	0.0140255	0.0156344	$2.1738E-05$	$2.39303E-06$	$1.84346E-05$
7	0.0042446	0.0039216	0.0043088	$2.8994E-05$	$4.77667E-06$	$2.21056E-05$
8	0.0010474	0.0009644	0.0010449	$3.4104E-05$	$9.54873E-06$	$2.65221E-05$
9	0.0002249	0.0002101	0.0002227	$3.6928E-05$	$1.90954E-05$	$3.18253E-05$

proposed the following formulas for approximating the QoS metrics in (2.2):

$$\begin{gathered} P_h \approx \rho_N, \\ P_o \approx \sum_{i=m}^{N} \rho_i, \end{gathered} \tag{2.7}$$

where

$$\begin{gathered} \rho_i = \begin{cases} \dfrac{\nu^i}{i!} \cdot \rho_0, & \text{if } i \le m, \\ \dfrac{\nu^m \cdot \nu_h^{i-m}}{i!} \cdot \rho_0, & \text{if } m+1 \le i \le N, \end{cases} \\ \rho_0 = \left(\sum_{i=0}^{m} \frac{\nu^i}{i!} + \sum_{i=m+1}^{N} \frac{\nu^m \cdot \nu_h^{i-m}}{i!} \right)^{-1}, \qquad \nu = \nu_o + \nu_h. \end{gathered} \tag{2.8}$$

The authors noted that it is impossible to measure the accuracy of their proposed formulas analytically; therefore, they demonstrated the high accuracy of these formulas with simulations.

Another way of transforming a two-dimensional model to an approximate one-dimensional model proceeds by replacing different average channel occupation times with the weighted average $\mu = \nu/\lambda$, where $\lambda = \lambda_o + \lambda_h$. Such an approach to the problem solution is called traditional [6]. In this case, QoS parameter calculations require the use of formulas (2.7); however, in formulas (2.8), parameters ν_h and ν_o are defined as follows: $\nu_h = \lambda_h/\mu$, $\nu_o = \lambda_o/\mu$.

The authors in [6] also proved the low accuracy of this traditional approach, especially when μ_o and μ_h vary greatly. As was expected, the results of the last two approaches coincide at $\mu_o = \mu_h$.

Tables 1, 2, and 3 contain the results of numerical experiments for the above three methods. Comparison of these methods shows that the results contained in [6], do not differ significantly, even when the condition $\lambda_o \ll \lambda_h$, $\mu_o \ll \mu_h$ does not hold (see Tables 2 and 3).

Table 2: Comparison of different algorithms for the case $N = 10$, $\lambda_o = 5$, $\lambda_h = 1$, $\mu_o = 5$, $\mu_h = 1$.

m	P_o			P_h		
	[8]	[6]	Traditional	[8]	[6]	Traditional
1	0.73105860	0.77460030	0.7035873	$3.4676E-07$	$1.24228E-07$	$8.29989E-12$
2	0.46211714	0.48919887	0.42770452	$1.1128E-06$	$1.87684E-07$	$3.20498E-11$
3	0.25165324	0.25884799	0.22508059	$3.1089E-06$	$3.26787E-07$	$1.5623E-10$
4	0.11636757	0.11535437	0.10127703	$7.2415E-06$	$6.15877E-07$	$8.58268E-10$
5	0.04535909	0.04350037	0.03875433	$1.3815E-05$	$1.20496E-06$	$4.9833E-09$
6	0.01497587	0.01402552	0.01267733	$2.1738E-05$	$2.39303E-06$	$2.95839E-08$
7	0.00424458	0.00392164	0.00358918	$2.8994E-05$	$4.77667E-06$	$1.76973E-07$
8	0.00104741	0.00096442	0.00089234	$3.4104E-05$	$9.54873E-06$	$1.06105E-06$
9	0.00022489	0.00021005	0.00019732	$3.6928E-05$	$1.90954E-05$	$6.36523E-06$

Table 3: Comparison of different algorithms for the case $N = 10$, $\lambda_o = \lambda_h = 5$, $\mu_o = \mu_h = 5$.

m	P_o			P_h		
	[8]	[6]	Traditional	[8]	[6]	Traditional
1	0.73105860	0.77460030	0.7746003	$3.4676E-07$	$1.24228E-07$	$1.24228E-07$
2	0.46211714	0.48919887	0.48919887	$1.1128E-06$	$1.87684E-07$	$1.87684E-07$
3	0.25165324	0.25884799	0.25884799	$3.1089E-06$	$3.26787E-07$	$3.26787E-07$
4	0.11636757	0.11535437	0.11535437	$7.2415E-06$	$6.15877E-07$	$6.15877E-07$
5	0.04535909	0.04350037	0.04350037	$1.3815E-05$	$1.20496E-06$	$1.20496E-06$
6	0.01497587	0.01402552	0.01402552	$2.1738E-05$	$2.39303E-06$	$2.39303E-06$
7	0.00424458	0.00392164	0.00392164	$2.8994E-05$	$4.77667E-06$	$4.77667E-06$
8	0.00104741	0.00096442	0.00096442	$3.4104E-05$	$9.54873E-06$	$9.54873E-06$
9	0.00022489	0.00021005	0.00021005	$3.6928E-05$	$1.90954E-05$	$1.90954E-05$

3. An Analytical Method for CAC Based on FGC and UFGC Schemes

In this method, as in the CGC scheme, an h-call is accepted if there is at least one free channel upon its arrival; otherwise, the call is dropped. An arriving o-call is accepted with probability β_i if upon its arrival the number of busy channels equals i, $i = 0, 1, \ldots, N-1$; thus, the call is blocked with probability $1 - \beta_i$. Obviously, the CGC scheme is a special case of this scheme. Indeed, by setting $\beta_i = 1$ for all $i = 0, 1, \ldots, m-1$ and $\beta_i = 0$ for all $i = m, m+1, \ldots, N-1$ in the FGC scheme, we obtain the CAC based on the CGC scheme; by setting $\beta_i = \alpha$ for all $i = 0, 1, \ldots, N-1$, the FGC scheme reduces to the UFGC scheme with admission probability α [5].

The stationary distribution of state probabilities is defined by solving the appropriate SGEE. The solution of an SGEE again presents problems with exact calculation for high-dimensional state spaces (2.1). To overcome this problem, the authors in [6] applied the above-mentioned heuristic assumption and proposed the following algorithm for the approximate calculation of QoS metrics:

$$P_h \approx \rho_N, \qquad P_0 \approx \sum_{i=0}^{N} (1-\beta_i)\rho_i, \qquad \beta_N = 0, \tag{3.1}$$

where

$$\rho_i = \frac{1}{i!} \cdot \prod_{j=0}^{i-1} (\beta_j \nu_o + \nu_h) \cdot \rho_0, \qquad \rho_0 = \left(\sum_{i=0}^{N} \frac{1}{i!} \cdot \prod_{j=0}^{i-1} (\beta_j \nu_o + \nu_h) \right)^{-1}. \tag{3.2}$$

Here, we offer strict analysis of these formulas; moreover, we will prove that the formulas are exact if the system satisfies the local balance condition.

In the FGC scheme, as in the CGC scheme, the cell state at any time is described by a bidimensional vector $\mathbf{k} = (k_o, k_h)$, where $k_o(k_h)$ denotes the number of n-calls (h-calls) occupying channels. Additionally, the state space of the model is defined by (2.1). The intensities $q(\mathbf{k}, \mathbf{k}')$, $\mathbf{k}$, $\mathbf{k}' \in S$ of transitions between states of this two-dimensional MC are defined as follows:

$$q(\mathbf{k}, \mathbf{k}') = \begin{cases} \lambda_o \beta_i, & \text{if } k_o + k_h = i, \ \mathbf{k}' = \mathbf{k} + \mathbf{e}_1, \\ \lambda_h, & \text{if } \mathbf{k}' = \mathbf{k} + \mathbf{e_2}, \\ k_o \mu_h, & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e}_1, \\ k_h \mu_h, & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{e}_2, \\ 0 & \text{otherwise}, \end{cases} \tag{3.3}$$

where $\mathbf{e}_1 = (1,0)$, $\mathbf{e_2} = (0,1)$.

Because the loss probability of an h-call and the blocking probability of an o-call are determined by the number of busy channels (see Formulas (2.2)), consider the following partition of the state space (2.1):

$$S = \bigcup_{i=0}^{N} S_i, \qquad S_i \bigcap S_j = \emptyset, \qquad i \neq j, \tag{3.4}$$

where $S = \{\mathbf{k} \in S : k_o + k_h = i\}$.

Let us denote the probability of the merged state S_i by $\pi(i)$, that is,

$$\pi(i) := \sum_{\mathbf{k} \in S_i} p(\mathbf{k}). \tag{3.5}$$

Thus,

$$\sum_{i=0}^{N} \pi(i) = 1. \tag{3.6}$$

Then, the initial QoS metrics from (2.2) are defined via the stationary probabilities of the merged states as follows:

$$\begin{aligned} P_h &= \pi(N), \\ P_o &= \sum_{i=0}^{N} (1-\beta_i)\pi(i) \ . \end{aligned} \tag{3.7}$$

Hence, to calculate the QoS metrics in (2.2), it is sufficient to find the merged state probabilities $\pi(i)$ rather than calculating the probabilities of all states $\mathbf{k} \in S$.

Proposition 3.1. *If the system satisfies the local balance condition, then the merged state probabilities are defined as follows:*

$$\pi(j) = \frac{1}{j!}\prod_{i=0}^{j-1}(\nu_o\beta_i + \nu_h)\pi(0), \quad j = 1, \ldots, N, \tag{3.8}$$

where $\pi(0) = \left(\sum_{j=0}^{N}(1/j!)\prod_{i=0}^{j-1}(\nu_o\beta_i + \nu_h)\right)^{-1}$.

The proof is provided in the appendix.

Thus, taking (3.7) into account, we deduce that the proposed algorithm (8)–(11) in [6] is accurate and not merely an approximation as long as the system satisfies the local balance condition.

By applying these formulas to the special cases, we obtain exact formulas for the QoS metrics (2.2) when using the CAC based on both the CGC scheme and the UFGC scheme. Thus, for the CGC scheme we have that

$$\begin{aligned} P_h &= \pi(0) \cdot \left(\frac{\nu}{\nu_h}\right)^m \cdot \frac{\nu_h^N}{N!}, \\ P_o &= \pi(0) \cdot \left(\frac{\nu}{\nu_h}\right)^m \cdot \sum_{i=m+1}^{N} \frac{\nu_h^i}{i!}, \end{aligned} \tag{3.9}$$

where

$$\pi(0) = \left(\sum_{i=0}^{m} \frac{\nu^i}{i!} + \left(\frac{\nu}{\nu_h}\right)^m \cdot \sum_{i=m+1}^{N} \frac{\nu_h^i}{i!}\right)^{-1}. \tag{3.10}$$

For the UFGC scheme, the following formulas are obtained:

$$P_h = \frac{A^N}{N!} \cdot \pi(0),$$
$$P_o = 1 - \alpha, \tag{3.11}$$

where $A := \alpha\nu_o + \nu_h$, $\pi(0) = \left(\sum_{j=0}^{N}(A^j/j!)\right)^{-1}$.

Note that closed-form solutions (3.9)–(22) might be considered refined approximations for the similar model that features identical channel occupancy times for heterogeneous calls [5], that is, when $\mu_o = \mu_h$. Indeed, formulas (3.11) completely coincide with the exact results given in [5] in the case that the admission probability α is equal to 1; small deviations arise when α is close to 1.

4. Selection of Optimal Values for the Parameters of CAC Based on the CGC Scheme

The problem of how to provide a given level of handling quality for different call types is scientifically and practically very interesting. The solution of such problems requires some regulated parameters. Thus, in some networks, for which the distribution of channels between cells is fixed, only call admission control parameters can be regulated because controlling loads present a difficult task and one that is sometimes practically impossible.

It is extremely difficult to develop a method for finding the optimal parameter values for the FGC scheme, as this strategy contains a large number (exactly N) of controlled parameters (i.e., β_i, $i = 0, 1, \ldots, N-1$). However, even if it was possible to calculate the optimal values of β_i, $i = 0, 1, \ldots, N-1$, the practical realization of the resulting randomized call admission strategies would still include methodological difficulties. Thus, we address several problems arising from trying to find optimal parameter values for the CGC scheme to meet the required QoS level. The following scheme has only one controlled parameter (m).

First, we will study the problem of organizing the fair handling of different call types. As stated above, as m increases, the probability of h-call loss increases, whereas the probability of o-call blocking decreases; thus, at $m = N$, we obtain an equilibrium of $P_o = P_h$. In other words, at $m = N$, we have absolutely fair handling.

However, in real life, absolutely fair handling is not required, and so we may choose to introduce the concept of ε-fair handling, which denotes handling at which $P_o - P_h < \varepsilon$, for a given number $\varepsilon > 0$. Thus, the problem of finding ε-fair handling can be defined mathematically as follows:

$$m^* = \arg\min_m \{P_o(m) - P_h(m) < \varepsilon\}. \tag{4.1}$$

Considering that $P_o\ (N) = P_h\ (N)$ and assuming the monotonicity of $P_o(m)$ and $P_h(m)$, we can apply the dichotomy method to solve the equation in (4.1). We now describe the general algorithm. The initial uncertainty interval $[1, N]$ is split in half; that is, we set $b = \mathrm{int}(N+1)/2$, where $\mathrm{int}(x)$ is the integer part of x. If condition (4.1) is satisfied at this point, then we next consider the interval $[1, b]$; otherwise, we consider the interval $[b,\ N]$. The algorithm correctly terminates by locating an interval of unitary length whose right limit satisfies condition (4.1) but whose left limit does not. As a result, this right limit will be

Table 4: Solutions for the problem (4.1).

N	10	10	10	20	20	20	20	20	20	20
ν_o	10	10	10	15	15	15	15	15	15	15
ν_h	2/3	2/3	2/3	2/3	2/3	2/3	2	2	2	2
ε	0.4	0.3	≤0.2	0.2	0.3	0.4	0.4	0.3	0.2	≤0.1
m^*	8	9	10	16	13	11	13	15	17	20

the required value of m^*. This algorithm has a finite number of steps because by the condition that $P_o\ (N) = P_h\ (N)$, condition (4.1) is satisfied in the worst case when $m = N$. The results of solving (4.1) are given in Table 4.

Now let the QoS for different call types be measured by the limit values of the loss and blocking probabilities; that is, the limitations for their upper bounds are set as follows:

$$\begin{aligned} P_o(m) &\le \varepsilon_o, \\ P_h(m) &\le \varepsilon_h, \end{aligned} \tag{4.2}$$

where $\varepsilon_o > 0$ and $\varepsilon_h > 0$ are given parameters.

Then, the optimization problem is formulated as follows: find the extreme values of the parameter m satisfying (4.2). Let us first consider the following problem:

$$\underline{m} = \arg \min_m \{P_o(m) \le \varepsilon_o, P_h(m) \le \varepsilon_h\}. \tag{4.3}$$

Given the monotonicity of $P_o(m)$ and $P_h(m)$, we propose the following algorithm for solving problem (4.3).

Step 1. If $\varepsilon_o < P_o(N)$ or $\varepsilon_h < P_h(1)$, then there is no solution.

Step 2. If $\varepsilon_o > P_o(1)$ and $\varepsilon_h > P_h(1)$, then $\underline{m} = 1$.

Step 3. If $P_o(N) < \varepsilon_o < P_o(1)$ and $P_h(1) < \varepsilon_h < P_o(N)$, then solve the following problem:

$$m_1 = \arg \min_m \{P_o(m) \le \varepsilon_o\} \tag{4.4}$$

.

Step 4. If $P_h(m_1) > \varepsilon_h$, then there is no solution; otherwise the solution is given by $\underline{m} = m_1$.

Note that the dichotomy method can be used to solve the problem in Step 3 due to the monotonicity of $P_o(m)$.

A similar method is developed for solving the following problem:

$$\overline{m} = \arg \max_m \{P_o(m) \le \varepsilon_o, P_h(m) \le \varepsilon_h\}. \tag{4.5}$$

Table 5: Solutions for the problem (4.3), (4.5) in the case of $N = 10$.

ν_o	0.1	0.1	0.1	0.1	0.5	0.5	0.5	0.5	0.5	1.0
ν_h	4/3	4/3	4/3	1.0	1.0	1.0	1.0	1.0	1.0	2.0
ε_o	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
ε_h	10^{-4}	10^{-5}	10^{-6}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-7}
$[\underline{m}, \overline{m}]$	[6, 10]	[6, 10]	$\emptyset$	[5, 10]	[6, 10]	[6, 10]	[6, 10]	[6, 10]	[6, 6]	$\emptyset$

Combining the solutions of problems (4.3) and (4.5), we obtain a range of values for the parameter m meeting conditions (4.2). The results of the solutions of these problems are given in Table 5, where the symbol $\emptyset$ indicates no solution.

It is possible to pose other similar QoS optimization problems; however, due to space limitations, they are not considered here.

5. Conclusion

We have provided exact and simple formulas for the calculation and optimization of QoS metrics for traditional wireless network models, assuming that the two fundamentally different types of calls (new and handover) do not have identical channel occupancy times. It is important to note that the adaptation of the proposed approach for next generation networks is straightforward.

Appendix

To prove the above-mentioned proposition, we must first prove the following lemmas.

Lemma A.1. *If the system satisfies the local balance condition, then the following equations hold:*

$$\nu_o \beta_{i-1} \pi(i-1) = E(k_o \mid i) \pi(i), \tag{A.1}$$

$$\nu_h \pi(i-1) = E(k_h \mid i) \pi(i), \qquad i = 1, \ldots, N, \tag{A.2}$$

where $E(\cdot \mid \cdot)$ denotes conditional mathematical expectation.

Lemma A.2. *If the system satisfies the local balance condition, then the following equations hold:*

$$(\nu_o \beta_{i-1} + \nu_h) \pi(i-1) = i\pi(i), \qquad i = 1, \ldots, N. \tag{A.3}$$

Proof of Lemma A.1. We use a technique proposed in [11], and we begin with proving the first equation. From (3.3), we conclude that the set of local equilibrium equations (SLEE) for the states $\mathbf{k} \in S_i$, $i = 1, \ldots, N$ has the form

$$\nu_o \beta_{i-1} p(k_o - 1, k_h) I(k_o \geq 1) = k_o p(k_o, k_h). \tag{A.4}$$

Summing both parts of (A.4) for all possible $\mathbf{k} \in S_i$, we obtain

$$\nu_o \beta_{i-1} \sum_{\mathbf{k}\in S_i} p(k_o - 1, k_h) I(k_o \geq 1) = \sum_{\mathbf{k}\in S_i} k_o p(k_o, k_h) \ . \tag{A.5}$$

From (3.5), we conclude that

$$\sum_{\mathbf{k}\in S_i} p(k_o - 1, k_h) I(k_o \geq 1) = \pi(i-1). \tag{A.6}$$

Let us rearrange the right hand side of (A.5) as follows:

$$\sum_{\mathbf{k}\in S_i} k_o p(\mathbf{k}) = \sum_{\mathbf{k}\in S_i} k_o \cdot \frac{p(\mathbf{k})}{\pi(i)} \cdot \pi(i). \tag{A.7}$$

From the definition of conditional probability, we obtain that

$$P(\mathbf{k} \mid i) = P(\mathbf{k} \mid k_o + k_h = i) = \begin{cases} \dfrac{p(\mathbf{k})}{\pi(i)}, & \text{if } \mathbf{k} \in S_i, \\ 0, & \text{otherwise.} \end{cases} \tag{A.8}$$

Then, in light of (A.8), we deduce the following from (A.7):

$$\sum_{\mathbf{k}\in S_i} k_o p(\mathbf{k}) = \left(\sum_{\mathbf{k}\in S_i} k_o P(\mathbf{k} \mid i) \right) \pi(i) = E(k_o \mid i) \pi(i). \tag{A.9}$$

Finally, using (A.6) and (A.9), we obtain

$$\nu_o \beta_{i-1} \pi(i-1) = E(k_o \mid i) \pi(i). \tag{A.10}$$

The second equation is proved similarly. □

Proof of Lemma A.2. Summing over the parts of the equation in Lemma A.1, we obtain

$$(\nu_o \beta_{i-1} + \nu_h) \pi(i-1) = (E(k_o \mid i) + E(k_k \mid i)) \pi(i) = E(k_o + k_h \mid i) \pi(i) = i \pi(i). \tag{A.11}$$

Thus, Lemma A.2 also follows.

Consequently, from Lemma A.2, taking into account the normalization condition in (3.6), we conclude that the proposition holds.

Now let us suppose that the system does not satisfy the local balance condition; one can use the following approach. Taking into account the relations in (3.3), we conclude that the SGEE for the states $\mathbf{k} \in S_i$ has the following form:

$$\begin{aligned} (\lambda_o \beta_{i-1} + \lambda_h + k_o \mu_o + k_h \mu_h) p(\mathbf{k}) = \lambda_o \beta_{i-2} p(\mathbf{k} - \mathbf{e}_1) + \lambda_h p(\mathbf{k} - \mathbf{e}_2) + (k_o + 1) \mu_o p(\mathbf{k} + \mathbf{e}_1) \\ + (k_h + 1) \mu_h p(\mathbf{k} + \mathbf{e}_2). \end{aligned} \tag{A.12}$$

For notational simplicity, we assume that the states $\mathbf{k}$, $\mathbf{k}+\mathbf{e}_i$, $\mathbf{k}-\mathbf{e}_i$, $i = 1, 2$ appearing in (A.12) are in the state space (2.1); otherwise, the corresponding terms are zeroed. Summing both sides of (A.12) over all possible $\mathbf{k} \in S_i$, after collecting similar terms and accounting for (3.5), one obtains

$$\left(\lambda_o\beta_{i-1} + \lambda_h\right)\pi(i-1) = \sum_{\mathbf{k}\in S_i}\left(k_o\mu_o + k_h\mu_h\right)p(\mathbf{k}). \tag{A.13}$$

Now, various schemes may be used for the unification of channel occupancy times. If we use the heuristic from [6], that is, we set $\mu_o = \mu_h = 1$ and $\lambda_o = \nu_o$, $\lambda_h = \nu_h$ then from (A.13), we obtain

$$\left(\nu_o\beta_{i-1} + \nu_h\right)\pi(i-1) = \sum_{\mathbf{k}\in S_i}(k_o + k_h)p(\mathbf{k}). \tag{A.14}$$

By using the proof schemes from our lemmas, we conclude that right side of (A.14) is equal to $i\pi(i)$; that is, we obtain equation from Lemma A.2. □

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