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Research Article

Further Results on Derivations of Ranked Bigroupoids

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Further properties on $(X, *, \&)$-self-(co)derivations of ranked bigroupoids are investigated, and conditions for an $(X, *, \&)$-self-(co)derivation to be regular are provided. The notion of ranked $*$-subsystems is introduced, and related properties are investigated.

1. Introduction

Several authors [1–4] have studied derivations in rings and near rings. Jun and Xin [5] applied the notion of derivation in ring and near-ring theory to BCI-algebras, and as a result they introduced a new concept, called a (regular) derivation, in BCI-algebras. Zhan and Liu [6] studied f-derivations in BCI-algebras. Alshehri [7] applied the notion of derivations to incline algebras. Alshehri et al. [8] introduced the notion of ranked bigroupoids and discussed $(X, *, \&)$-self-(co)derivations. In this paper, we investigate further properties on $(X, *, \&)$-self-(co)derivations and provide conditions for an $(X, *, \&)$-self-(co)derivation to be regular. We introduce the notion of ranked $*$-subsystems and investigate related properties.

2. Preliminaries

In a nonempty set X with a constant 0 and a binary operation $*$, we consider the following axioms:

(a1) $((x * y) * (x * z)) * (z * y) = 0$,

(a2) $(x * (x * y)) * y = 0$,

(a3) $x * x = 0$,

(a4) $x * y = 0$ and $y * x = 0$ imply $x = y$,

(b1) $x * 0 = x$,

(b2) $(x * y) * z = (x * z) * y$,

(b3) $((x * z) * (y * z)) * (x * y) = 0$,

(b4) $x * (x * (x * y)) = x * y$.

If X satisfies axioms (a1), (a2), (a3), and (a4), then we say that $(X, *, 0)$ is a BCI-algebra. Note that a BCI-algebra $(X, *, 0)$ satisfies conditions (b1), (b2), (b3), and (b4) (see [9]).

In a p-semisimple BCI-algebra X, the following hold:

(b5) $(x * z) * (y * z) = x * y$,

(b6) $0 * (0 * x) = x$.

3. Derivations on Ranked Bigroupoids

A *ranked bigroupoid* (see [8]) is an algebraic system $(X, *, \bullet)$ where X is a non-empty set and "$*$" and "$\bullet$" are binary operations defined on X. We may consider the first binary operation $*$ as the major operation and the second binary operation $\bullet$ as the minor operation.

Given a ranked bigroupoid $(X, *, \&)$, a map $d : X \to X$ is called an $(X, *, \&)$-*self-derivation* (see [8]) if for all $x, y \in X$,

$$d(x * y) = (d(x) * y)\&(x * d(y)). \tag{3.1}$$

In the same setting, a map $d : X \to X$ is called an $(X, *, \&)$-*self-coderivation* (see [8]) if for all $x, y \in X$,

$$d(x * y) = (x * d(y))\&(d(x) * y). \tag{3.2}$$

Note that if $(X, *)$ is a commutative groupoid, then $(X, *, \&)$-self-derivations are $(X, *, \&)$-self-coderivations. A map $d : X \to X$ is called an *abelian*-$(X, *, \&)$-*self-derivation* (see [8]) if it is both an $(X, *, \&)$-self-derivation and an $(X, *, \&)$-self-coderivation.

Proposition 3.1. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$.*

(1) *Assume that X satisfies axioms (b1), (b2), (b3), (a3), and (a4). If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then $d(x) = d(x)\&x$ for all $x \in X$.*

(2) *If X satisfies two axioms (b1) and (a3) and a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then the following are equivalent:*

(2.1) $d(0) = 0$;

(2.2) $(\forall x \in X)(d(x) = x\&d(x))$.

Proof. (1) Let $x \in X$. Using (b1) and (b2), we have

$$\begin{aligned} d(x) = d(x * 0) &= (d(x) * 0)\&(x * d(0)) \\ &= d(x)\&(x * d(0)) \\ &= (x * d(0)) * ((x * d(0)) * d(x)) \\ &= (x * d(0)) * ((x * d(x)) * d(0)). \end{aligned} \tag{3.3}$$

It follows from (b3) that

$$d(x) * (d(x)\&x) = ((x * d(0)) * ((x * d(x)) * d(0))) * (d(x)\&x) = 0. \tag{3.4}$$

Using (b2) and (a3), we have $(d(x)\&x) * d(x) = 0$, and so $d(x) = d(x)\&x$ for all $x \in X$ by (a4).

(2) Let d be an $(X, *, \&)$-self-coderivation. If $d(0) = 0$, then

$$d(x) = d(x * 0) = (x * d(0))\&(d(x) * 0) = x\&d(x) \tag{3.5}$$

for all $x \in X$. Assume that $d(x) = x\&d(x)$ for all $x \in X$. Taking $x = 0$ implies that $d(0) = 0\&d(0) = 0$. □

Corollary 3.2. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$.*

(1) *If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then $d(x) = d(x)\&x$ for all $x \in X$.*

(2) *If a map $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then the following are equivalent:*

(2.1) $d(0) = 0$;

(2.2) $(\forall x \in X)\ (d(x) = x\&d(x))$.

Lemma 3.3. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which three axioms (b2),(a3), and (a4) are valid and the minor operation & is defined by $x\&y = y*(y*x)$ for all $x, y \in X$.*

(1) *For every $x \in X$ with $x\&0 = x$, one has*

$$(\forall y \in X) \quad (y * x = 0 \Longrightarrow y = x). \tag{3.6}$$

(2) *If an element a of X satisfies $a\&0 = a$, then $a\&x = a$ for all $x \in X$.*

Proof. (1) Let $y \in X$ be such that $y * x = 0$. Then

$$\begin{aligned} x * y = (x\&0) * y &= (0 * y) * (0 * x) \\ &= ((y * x) * y) * (0 * x) = (0 * x) * (0 * x) = 0, \end{aligned} \tag{3.7}$$

and so $y = x$ by (a4).

(2) Since $(a\&x) * a = 0$, it follows from (3.6) that $a\&x = a$ for all $x \in X$. □

Corollary 3.4. *Let* $(X, *, \&)$ *be a ranked bigroupoid in which* $(X, *, 0)$ *is a BCI-algebra and the minor operation* $\&$ *is defined by* $x\&y = y * (y * x)$ *for all* $x, y \in X$.

(1) *For every* $x \in X$ *with* $x\&0 = x$, *one has*

$$(\forall y \in X) \quad (y * x = 0 \Longrightarrow y = x). \tag{3.8}$$

(2) *If an element* a *of* X *satisfies* $a\&0 = a$, *then* $a\&x = a$ *for all* $x \in X$.

Proposition 3.5. *Let* $(X, *, \&)$ *be a ranked bigroupoid with distinguished element* 0 *in which four axioms* (b2), (b4), (a3), *and* (a4) *are valid and the minor operation* $\&$ *is defined by* $x\&y = y * (y * x)$ *for all* $x, y \in X$. *If a map* $d : X \to X$ *is an* $(X, *, \&)$-*self-coderivation, then* $0 * d(x) = d(x)$ *for all* $x \in X$ *with* $0 * x = x$.

Proof. Let $x \in X$ be such that $0 * x = x$. Since $(0 * d(x))\&0 = 0 * d(x)$, it follows from Lemma 3.3(2) that $d(x) = d(0 * x) = (0 * d(x))\&(d(0) * x) = 0 * d(x)$. □

Corollary 3.6. *Let* $(X, *, \&)$ *be a ranked bigroupoid in which* $(X, *, 0)$ *is a BCI-algebra and the minor operation* $\&$ *is defined by* $x\&y = y * (y * x)$ *for all* $x, y \in X$. *If a map* $d : X \to X$ *is an* $(X, *, \&)$-*self-coderivation, then* $0 * d(x) = d(x)$ *for all* $x \in X$ *with* $0 * x = x$.

Using Proposition 3.5, we can find an $(X, *, \&)$-self-derivation which is not an $(X, *, \&)$-self-coderivation.

Example 3.7. Let $(\mathbb{Z}, -, \&)$ be a ranked bigroupoid where $\mathbb{Z}$ is the set of all integers with the minus operation "−" and the minor operation "&" defined by $x\&y = y-(y-x)$ for all $x, y \in \mathbb{Z}$. Let d be a self map of $\mathbb{Z}$ given by $d(x) = x-1$ for all $x \in \mathbb{Z}$. Then d is a $(\mathbb{Z}, -, \&)$-self-derivation since

$$\begin{aligned} d(x-y) &= (x-y)-1 = (x-y+1)-2 \\ &= (x-y-1)\&(x-y+1) = ((x-1)-y)\&(x-(y-1)) \\ &= (d(x)-y)\&(x-d(y)). \end{aligned} \tag{3.9}$$

Note that $0 - d(0) = 0 - (0-1) = 1 \neq -1 = 0 - 1 = d(0)$. Hence d is not a $(\mathbb{Z}, -, \&)$-self-coderivation by Proposition 3.5.

Proposition 3.8. *Let* $(X, *, \&)$ *be a ranked bigroupoid with distinguished element* 0 *and the minor operation* $\&$ *is defined by* $x\&y = y*(y*x)$ *for all* $x, y \in X$. *For an* $(X, *, \&)$-*self-derivation* $d : X \to X$, *if* $(X, *, 0)$ *satisfies axioms* (b2),(b5), *and* (b6), *then* $d(x) = d(0) * (0 * x)$ *for all* $x \in X$. *Moreover, if* $d(0) = 0$, *then* d *is an identity map.*

Proof. Assume that $(X, *, 0)$ satisfies axioms (b2), (b5), and (b6). Then

$$\begin{aligned} d(x) = d(x\&0) &= (d(0) * (0 * x))\&(0 * d(0 * x)) \\ &= (0 * d(0 * x)) * ((0 * d(0 * x)) * (d(0) * (0 * x))) \\ &= (0 * d(0 * x)) * ((0 * (d(0) * (0 * x))) * d(0 * x)) \\ &= 0 * (0 * (d(0) * (0 * x))) \\ &= d(0) * (0 * x), \end{aligned} \tag{3.10}$$

for all $x \in X$. Moreover, if $d(0) = 0$ then $d(x) = d(0) * (0 * x) = x\&0 = x$ for all $x \in X$, and so d is an identity map. □

Corollary 3.9. *Let $(X, *, \&)$ be a ranked bigroupoid in which $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If a map $d : X \to X$ is an $(X, *, \&)$-self-derivation, then*

(1) $d(0) = d(0)\&0$;

(2) *if $(X, *, 0)$ is p-semisimple, then $d(x) = d(0) * (0 * x)$ for all $x \in X$;*

(3) *if $(X, *, 0)$ is p-semisimple and $d(0) = 0$, then d is an identity map.*

Definition 3.10. Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0. A self map d of $(X, *, \&)$ is said to be *regular* if $d(0) = 0$.

Example 3.11. Consider a ranked bigroupoid $(X, *, \&)$ in which $X = \{0, a, b, c, d, e\}$ and binary operations "$*$" and "&" are defined by

$$x * y = \begin{cases} 0 & \text{if } (x, y) \in \{(0, a), (b, d), (c, e)\} \cup \{(z, z) \mid z \in X\}, \\ a & \text{if } (x, y) \in \{(a, 0), (d, b), (e, c)\}, \\ b & \text{if } (x, y) \in \{(b, 0), (0, c), (0, e), (a, e), (b, a), (c, b), (c, d), (d, a), (e, d)\}, \\ c & \text{if } (x, y) \in \{(c, 0), (c, a), (e, a), (0, b), (b, c), (0, d), (a, d), (b, e), (d, e)\}, \\ d & \text{if } (x, y) \in \{(d, 0), (e, b), (a, c)\}, \\ e & \text{if } (x, y) \in \{(a, b), (d, c), (e, 0)\} \end{cases} \tag{3.11}$$

&	0	a	b	c	d	e
0	0	0	0	0	0	0
a	0	a	0	0	a	a
b	b	b	0	b	b	b
c	c	c	c	c	c	c
d	b	d	b	b	d	d
e	c	e	c	c	e	e.

Define a map $d : X \to X$ by

$$d(x) = \begin{cases} 0 & \text{if } x \in \{0, a\}, \\ b & \text{if } x \in \{b, d\}, \\ c & \text{if } x \in \{c, e\}. \end{cases} \tag{3.12}$$

Then d is an abelian $(X, *, \&)$-self-derivation which is regular.

Proposition 3.12. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$ and $0 * x = 0$ for all $x \in X$. Then every $(X, *, \&)$-self-derivation is regular. Moreover, if X satisfies the axioms (b1) and (a3) then every $(X, *, \&)$-self-coderivation is regular.*

Proof. Let d be an $(X, *, \&)$-self-derivation. Then

$$d(0) = d(0 * x) = (d(0) * x)\&(0 * d(x)) = (d(0) * x)\&0 = 0. \tag{3.13}$$

If d is an $(X, *, \&)$-self-coderivation, then

$$d(0) = d(0 * x) = (0 * d(x))\&(d(0) * x) = 0\&(d(0) * x) = 0. \tag{3.14}$$

Hence every $(X, *, \&)$-self-(co)derivation is regular. □

Proposition 3.13. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$ and two axioms $(a3)$ and $(b1)$ are satisfied. Let d be a self map of X and $a \in X$ such that $d(x) * a = 0$ (resp., $a * d(x) = 0$) for all $x \in X$. If d is an $(X, *, \&)$-self-derivation (resp., $(X, *, \&)$-self-coderivation), then it is regular.*

Proof. Assume that d is an $(X, *, \&)$-self-derivation. For any $x \in X$, we have

$$0 = d(x * a) * a = ((d(x) * a)\&(x * d(a))) * a = (0\&(x * d(a))) * a = 0 * a, \tag{3.15}$$

which implies that

$$d(0) = d(0 * a) = (d(0) * a)\&(0 * d(a)) = 0\&(0 * d(a)) = 0. \tag{3.16}$$

Hence d is regular. Now, let d be an $(X, *, \&)$-self-coderivation such that $a * d(x) = 0$ for all $x \in X$. Then

$$0 = a * d(a * x) = a * ((a * d(x))\&(d(a) * x)) = a * (0\&(d(a) * x)) = a * 0, \tag{3.17}$$

and so

$$d(0) = d(a * 0) = (a * d(0))\&(d(a) * 0) = 0\&(d(a) * 0) = 0\&d(a) = 0. \tag{3.18}$$

Therefore d is regular. □

Definition 3.14. Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0. Let d be a self map of $(X, *, \&)$. A subset A of X is called a ranked $*$-subsystem of X if it satisfies the following:

(r1) $0 \in A$,

(r2) $(\forall x, y \in X)(x \in A, y * x \in A \Rightarrow y \in A)$.

Moreover, if a ranked $*$-subsystem A of X satisfies $d(A) \subseteq A$, then we say that A is *ranked* d-invariant.

Example 3.15. Consider a ranked bigroupoid $(X, *, \&)$ in which $X = \{0, a, b, c, d, e\}$ and binary operations "$*$" and "&" are defined by

$$x * y = \begin{cases} 0 & \text{if } (x, y) \in \{(0, a), (b, c), (b, d), (b, e), (c, d), (c, e)\} \cup \{(z, z) \mid z \in X\}, \\ a & \text{if } (x, y) \in \{(a, 0), (c, b), (d, b), (e, b), (d, c), (e, c), (e, d), (d, e)\}, \\ c & \text{if } (x, y) = (c, 0), \\ d & \text{if } (x, y) = (d, 0), \\ e & \text{if } (x, y) = (e, 0), \\ b & \text{otherwise}, \end{cases} \tag{3.19}$$

and $x\&y = y * (y * x)$ for all $x, y \in X$. Define a map $d: X \to X$ by

$$d(x) = \begin{cases} b & \text{if } x \in \{0, a\} \\ 0 & \text{otherwise}. \end{cases} \tag{3.20}$$

Then d is an abelian $(X, *, \&)$-self-derivation which is not regular. It is easily check that $A = \{0, a\}$ is a ranked $*$-subsystem of X. Since $d(A) = \{b\} \not\subseteq A$, d is not ranked d-invariant.

Example 3.16. In Example 3.11, $A = \{0, a\}$ is a ranked d-invariant $*$-subsystem of X.

Theorem 3.17. *Let $(X, *, \&)$ be a ranked bigroupoid with distinguished element 0 in which three axioms $(b1)$,$(b2)$, and $(a3)$ are valid and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. For an $(X, *, \&)$-self-coderivation d, if d is regular then every ranked $*$-subsystem of X is ranked d-invariant.*

Proof. Assume that d is regular and let A be a ranked $*$-subsystem of X. Then $d(x) = x\&d(x)$ for all $x \in X$ by Proposition 3.1(2). Let $y \in d(A)$. Then $y = d(a)$ for some $a \in A$. Thus $y * a = d(a) * a = (a\&d(a)) * a = 0 \in A$, and so $y \in A$ by (r2). Hence $d(A) \subseteq A$ and A is ranked d-invariant. □

Corollary 3.18. *Let d be an $(X, *, \&)$-self-coderivation where $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. If d is regular, then every ideal of X is ranked d-invariant.*

Example 3.15 shows that Theorem 3.17 is not true if we drop the regularity of d.

We consider the converse of Theorem 3.17.

Theorem 3.19. *Let d be an $(X, *, \&)$-self-coderivation where $(X, *, \&)$ is a ranked bigroupoid with distinguished element 0 in which the minor operation & is defined by $x\&y = y*(y*x)$ for all $x, y \in X$ and there does not exist a nonzero element x of X such that $x * 0 = 0$. If every ranked $*$-subsystem of X is ranked d-invariant, then d is regular.*

Proof. Assume that every ranked $*$-subsystem of X is ranked d-invariant. Note that $A = \{0\}$ is a ranked $*$-subsystem of X. Thus $d(A) = d(\{0\}) \subseteq \{0\}$, and therefore $d(0) = 0$, that is, d is regular. □

Corollary 3.20. *Let d be an $(X, *, \&)$-self-coderivation where $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. Then d is regular if and only if every ranked $*$-subsystem of X is ranked d-invariant.*

Proposition 3.21. *Let $(X, *, \&)$ be a ranked bigroupoid where $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. For any $\alpha \in X$, let d_α be a self map of X defined by $d_\alpha(x) = x * \alpha$ for all $x \in X$. If X satisfies the following conditions:*

(1) $((x * y) * z) * (x * (y * z)) = 0$ *for all* $x, y, z \in X$,

(2) $(\forall x, y \in X)\ (x * y = 0 \Rightarrow x = y)$,

then d_α is an abelian $(X, *, \&)$-self-derivation.

Proof. If X satisfies two given conditions, then the following identity is valid (see [9]):

$$(\forall x, y, z \in X)\big((x * y) * z = x * (y * z)\big). \tag{3.21}$$

It follows from (b1), (a3), and (b2) that

$$\begin{aligned} d_\alpha(x * y) &= (x * y) * \alpha = (x * (y * \alpha)) * 0 \\ &= (x * (y * \alpha)) * ((x * (y * \alpha)) * (x * (y * \alpha))) \\ &= (x * (y * \alpha)) * ((x * (y * \alpha)) * ((x * \alpha) * y)) \\ &= (d_\alpha(x) * y)\&(x * d_\alpha(y)). \end{aligned} \tag{3.22}$$

Hence d_α is an $(X, *, \&)$-self-derivation. Similarly, we can verify that d_α is an $(X, *, \&)$-self-coderivation. □

Corollary 3.22. *Let $(X, *, \&)$ be a ranked bigroupoid where $(X, *, 0)$ is a BCI-algebra and the minor operation & is defined by $x\&y = y * (y * x)$ for all $x, y \in X$. For any $\alpha \in X$, let d_α be a self map of X defined by $d_\alpha(x) = x * \alpha$ for all $x \in X$. If X satisfies (b1) and the following conditions:*

(1) $((x * y) * z) * (x * (y * z)) = 0$ *for all* $x, y, z \in X$,

(2) $(x * y) * (x * z) = z * y$ *for all* $x, y, z \in X$,

then d_α is an abelian $(X, *, \&)$-self-derivation.

Proof. If X satisfies both (b1) and the second condition, then X is a p-semisimple BCI-algebra (see [9]). Hence the second condition of Proposition 3.21 is valid. Therefore d_α is an abelian $(X, *, \&)$-self-derivation. □

4. Conclusion

Alshehri et al. [8] introduced the notion of ranked bigroupoids and discussed $(X, *, \&)$-self-(co)derivations.

A nonempty set X together with maps $* : X \times X \rightarrow X$ and $\& : X \times X \rightarrow X$ is called a *ranked bigroupoid*. For a ranked bigroupoid $(X, *, \&)$, a map $d : X \rightarrow X$ is called:

(1) an $(X, *, \&)$-*self-derivation* if

$$d(x * y) = (d(x) * y)\&(x * d(y)) \tag{4.1}$$

for all $x, y \in X$;

(2) an $(X, *, \&)$-*self-coderivation* if

$$d(x * y) = (x * d(y))\&(d(x) * y) \tag{4.2}$$

for all $x, y \in X$.

In this paper, we have investigated further properties on $(X, *, \&)$-self-(co)derivations and have provided conditions for an $(X, *, \&)$-self-(co)derivation to be regular. We have introduced the notion of ranked $*$-subsystems and have investigated related properties.

In general, there are many kind of derivations (generalized derivations, biderivations, triderivations, etc.) in algebraic structures, for example, (near) rings, prime rings, semiprime rings, Γ-near-rings, incline algebras, Banach algebras, lattices, MV-algebras, and BCK/BCI-algebras.

Based on this paper together with related papers on derivations, we will consider several kind of derivations in ranked bigroupoids.

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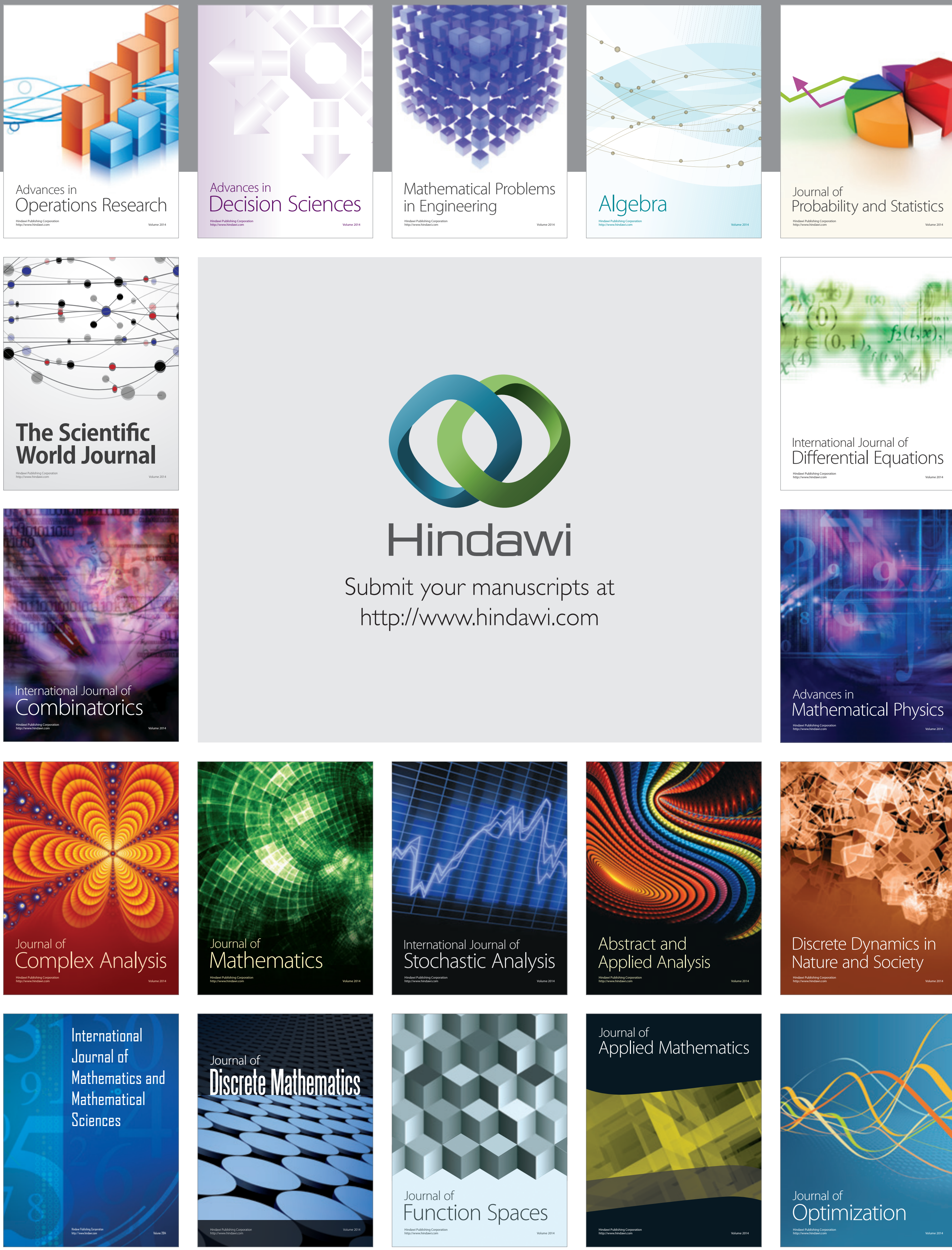
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