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A COUNTEREXAMPLE TO A COMPACT EMBEDDING THEOREM FOR FUNCTIONS WITH VALUES IN A HILBERT SPACE

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Let V and H be two real separable Hilbert spaces with V densely and continuously embedded in H. Identifying H with its dual, we have a Gelfand triple (V,H,V'), where V' is the dual space to V. For a positive T, let $\mathcal{V} = L^2(0,T;V)$, $\mathcal{V}' = L^2(0,T;V')$ denote the spaces of the square summable functions defined on $(0,T)$ with values in V and V', respectively. Let us define $\mathcal{W} = \{v \in \mathcal{V}: v' \in \mathcal{V}'\}$, where the time derivative $u' = \frac{du}{dt}$ is understood in the weak sense. Equipped with the standard norm $\| v \|_{\mathcal{W}} = \left(\| v \|_{\mathcal{V}}^2 + \| v' \|_{\mathcal{V}'}^2 \right)^{\frac{1}{2}}$, $\mathcal{W}$ becomes a Hilbert space. It is known (cf. [2]) that the embedding

$$\mathcal{W} \subset C(0,T;H) \tag{1}$$

is continuous, where the space of continuous functions from $[0,T]$ into H, is endowed with the supremum norm. E. Nagy proved the following theorem in [3].

Theorem: *If V and H are infinite-dimensional separable Hilbert spaces such that $V \subset H$ densely, continuously and compactly, then embedding* (1) *is also compact.*

The purpose of the paper is to deliver that this result of Nagy was recently exploited in several papers in connection with the study of evolution equations and inclusions (see, for instance, [1]).

References

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