

AN APPLICATION OF HÖLDER'S INEQUALITY FOR CONVOLUTIONS

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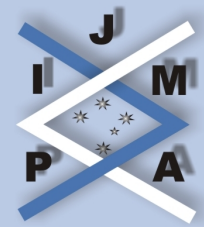
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Abstract: Let $\mathcal{A}_p(n)$ be the class of analytic and multivalent functions $f(z)$ in the open unit disk \mathbb{U} . Furthermore, let $\mathcal{S}_p(n, \alpha)$ and $\mathcal{T}_p(n, \alpha)$ be the subclasses of $\mathcal{A}_p(n)$ consisting of multivalent starlike functions $f(z)$ of order α and multivalent convex functions $f(z)$ of order α , respectively. Using the coefficient inequalities for $f(z)$ to be in $\mathcal{S}_p(n, \alpha)$ and $\mathcal{T}_p(n, \alpha)$, new subclasses $\mathcal{S}_p^*(n, \alpha)$ and $\mathcal{T}_p^*(n, \alpha)$ are introduced. Applying the Hölder inequality, some interesting properties of generalizations of convolutions (or Hadamard products) for functions $f(z)$ in the classes $\mathcal{S}_p^*(n, \alpha)$ and $\mathcal{T}_p^*(n, \alpha)$ are considered.



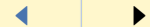
Hölder's Inequality for Convolution

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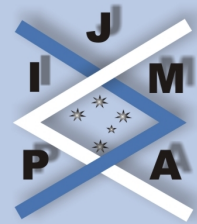
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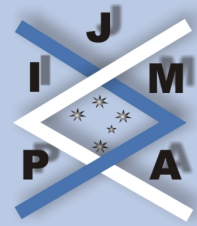
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1. Introduction

Let $\mathcal{A}_p(n)$ be the class of functions $f(z)$ of the form

$$f(z) = z^p + \sum_{k=p+n}^{\infty} a_k z^k$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} \mid |z| < 1\}$ for some natural numbers p and n . Let $\mathcal{S}_p(n, \alpha)$ be the subclass of $\mathcal{A}_p(n)$ consisting of functions $f(z)$ which satisfy

$$\operatorname{Re} \left(\frac{z f'(z)}{f(z)} \right) > \alpha \quad (z \in \mathbb{U})$$

for some α ($0 \leq \alpha < p$). Also let $\mathcal{T}_p(n, \alpha)$ be the subclass of $\mathcal{A}_p(n)$ consisting of functions $f(z)$ satisfying $z f'(z)/p \in \mathcal{S}_p(n, \alpha)$, that is,

$$\operatorname{Re} \left(1 + \frac{z f''(z)}{f'(z)} \right) > \alpha \quad (z \in \mathbb{U})$$

for some α ($0 \leq \alpha < p$). These classes, $\mathcal{A}_p(n)$, $\mathcal{S}_p(n, \alpha)$ and $\mathcal{T}_p(n, \alpha)$, were studied by Owa [3]. It is easy to derive the following lemmas, which provide the sufficient conditions for functions $f(z) \in \mathcal{A}_p(n)$ to be in the classes $\mathcal{S}_p(n, \alpha)$ and $\mathcal{T}_p(n, \alpha)$, respectively.

Lemma 1.1. *If $f(z) \in \mathcal{A}_p(n)$ satisfies*

$$(1.1) \quad \sum_{k=p+n}^{\infty} (k - \alpha) |a_k| \leq p - \alpha$$

for some α ($0 \leq \alpha < p$), then $f(z) \in \mathcal{S}_p(n, \alpha)$.

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Lemma 1.2. If $f(z) \in \mathcal{A}_p(n)$ satisfies

$$(1.2) \quad \sum_{k=p+n}^{\infty} k(k-\alpha)|a_k| \leq p(p-\alpha)$$

for some α ($0 \leq \alpha < p$), then $f(z) \in \mathcal{T}_p(n, \alpha)$.

Remark 1. We note that Silverman [4] has given Lemma 1.1 and Lemma 1.2 in the case of $p = 1$ and $n = 1$. Also, Srivastava, Owa and Chatterjea [5] have given the coefficient inequalities in the case of $p = 1$.

In view of Lemma 1.1 and Lemma 1.2, we introduce the subclass $\mathcal{S}_p^*(n, \alpha)$ consisting of functions $f(z)$ which satisfy the coefficient inequality (1.1), and the subclass $\mathcal{T}_p^*(n, \alpha)$ consisting of functions $f(z)$ which satisfy the coefficient inequality (1.2).

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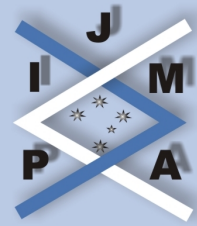
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2. Convolution Properties for the Classes $\mathcal{S}_p^*(n, \alpha)$ and $\mathcal{T}_p^*(n, \alpha)$

For functions $f_j(z) \in \mathcal{A}_p(n)$ given by

$$f_j(z) = z^p + \sum_{k=p+n}^{\infty} a_{k,j} z^k \quad (j = 1, 2, \dots, m),$$

we define

$$G_m(z) = z^p + \sum_{k=p+n}^{\infty} \left(\prod_{j=1}^m a_{k,j} \right) z^k$$

and

$$H_m(z) = z^p + \sum_{k=p+n}^{\infty} \left(\prod_{j=1}^m a_{k,j}^{p_j} \right) z^k \quad (p_j > 0).$$

Then $G_m(z)$ denotes the convolution of $f_j(z)$ ($j = 1, 2, \dots, m$). Therefore, $H_m(z)$ is the generalization of the convolutions. In the case of $p_j = 1$, we have $G_m(z) = H_m(z)$. The generalization of the convolution was considered by Choi, Kim and Owa [1].

In the present paper, we discuss an application of the Hölder inequality for $H_m(z)$ to be in the classes $\mathcal{S}_p^*(n, \alpha)$ and $\mathcal{T}_p^*(n, \alpha)$.

For $f_j(z) \in \mathcal{A}_p(n)$, the Hölder inequality is given by

$$\sum_{k=p+n}^{\infty} \left(\prod_{j=1}^m |a_{k,j}| \right) \leq \prod_{j=1}^m \left(\sum_{k=p+n}^{\infty} |a_{k,j}|^{p_j} \right)^{\frac{1}{p_j}},$$

where $p_j > 1$ and $\sum_{j=1}^m \frac{1}{p_j} \geq 1$.

Recently, Nishiwaki, Owa and Srivastava [2] have given some results of Hölder-type inequalities for a subclass of uniformly starlike functions.

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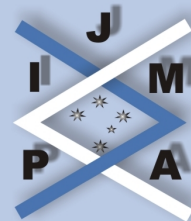
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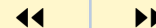
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Theorem 2.1. If $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $H_m(z) \in \mathcal{S}_p^*(n, \beta)$ with

$$\beta = \inf_{k \geq p+n} \left\{ p - \frac{(k-p) \prod_{j=1}^m (p - \alpha_j)^{p_j}}{\prod_{j=1}^m (k - \alpha_j)^{p_j} - \prod_{j=1}^m (p - \alpha_j)^{p_j}} \right\},$$

where $p_j \geq \frac{1}{q_j}$, $q_j > 1$ and $\sum_{j=1}^m \frac{1}{q_j} \geq 1$.

Proof. For $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$, Lemma 1.1 gives us that

$$\sum_{k=p+n}^{\infty} \left(\frac{k - \alpha_j}{p - \alpha_j} \right) |a_{k,j}| \leq 1 \quad (j = 1, 2, \dots, m),$$

which implies

$$\left\{ \sum_{k=p+n}^{\infty} \left(\frac{k - \alpha_j}{p - \alpha_j} \right) |a_{k,j}| \right\}^{\frac{1}{q_j}} \leq 1$$

with $q_j > 1$ and $\sum_{j=1}^m \frac{1}{q_j} \geq 1$. Applying the Hölder inequality, we have:

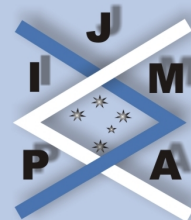
$$\sum_{k=p+n}^{\infty} \left\{ \prod_{j=1}^m \left(\frac{k - \alpha_j}{p - \alpha_j} \right)^{\frac{1}{q_j}} |a_{k,j}|^{\frac{1}{q_j}} \right\} \leq 1.$$

Note that we have to find the largest β such that

$$\sum_{k=p+n}^{\infty} \left(\frac{k - \beta}{p - \beta} \right) \left(\prod_{j=1}^m |a_{k,j}|^{p_j} \right) \leq 1,$$

that is,

$$\sum_{k=p+n}^{\infty} \left(\frac{k - \beta}{p - \beta} \right) \left(\prod_{j=1}^m |a_{k,j}|^{p_j} \right) \leq \sum_{k=p+n}^{\infty} \left\{ \prod_{j=1}^m \left(\frac{k - \alpha_j}{p - \alpha_j} \right)^{\frac{1}{q_j}} |a_{k,j}|^{\frac{1}{q_j}} \right\}.$$



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Therefore, we need to find the largest β such that

$$\left(\frac{k-\beta}{p-\beta}\right)\left(\prod_{j=1}^m|a_{k,j}|^{p_j}\right)\leq\prod_{j=1}^m\left(\frac{k-\alpha_j}{p-\alpha_j}\right)^{\frac{1}{q_j}}|a_{k,j}|^{\frac{1}{q_j}},$$

which is equivalent to

$$\left(\frac{k-\beta}{p-\beta}\right)\left(\prod_{j=1}^m|a_{k,j}|^{p_j-\frac{1}{q_j}}\right)\leq\prod_{j=1}^m\left(\frac{k-\alpha_j}{p-\alpha_j}\right)^{\frac{1}{q_j}}$$

for all $k\geq p+n$. Since

$$\prod_{j=1}^m\left(\frac{k-\alpha_j}{p-\alpha_j}\right)^{p_j-\frac{1}{q_j}}|a_{k,j}|^{p_j-\frac{1}{q_j}}\leq 1\quad\left(p_j-\frac{1}{q_j}\geq 0\right),$$

we see that

$$\prod_{j=1}^m|a_{k,j}|^{p_j-\frac{1}{q_j}}\leq\frac{1}{\prod_{j=1}^m\left(\frac{k-\alpha_j}{p-\alpha_j}\right)^{p_j-\frac{1}{q_j}}}.$$

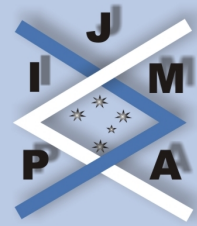
This implies that

$$\frac{k-\beta}{p-\beta}\leq\prod_{j=1}^m\left(\frac{k-\alpha_j}{p-\alpha_j}\right)^{p_j}$$

for all $k\geq p+n$. Therefore, β should be

$$\beta\leq p-\frac{(k-p)\prod_{j=1}^m(p-\alpha_j)^{p_j}}{\prod_{j=1}^m(k-\alpha_j)^{p_j}-\prod_{j=1}^m(p-\alpha_j)^{p_j}}\quad(k\geq p+n).$$

This completes the proof of the theorem. □



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Taking $p_j = 1$ in Theorem 2.1, we obtain

Corollary 2.2. *If $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $G_m(z) \in \mathcal{S}_p^*(n, \beta)$ with*

$$\beta = p - \frac{n \prod_{j=1}^m (p - \alpha_j)}{\prod_{j=1}^m (p + n - \alpha_j) - \prod_{j=1}^m (p - \alpha_j)}.$$

Proof. In view of Theorem 2.1, we have

$$\beta \leq \inf_{k \geq p+n} \left\{ p - \frac{(k-p) \prod_{j=1}^m (p - \alpha_j)}{\prod_{j=1}^m (k - \alpha_j) - \prod_{j=1}^m (p - \alpha_j)} \right\}.$$

Let $F(k; m)$ be the right hand side of the above inequality. Further, let us define $G(k; m)$ by the numerator of $F'(k; m)$. When $m = 2$,

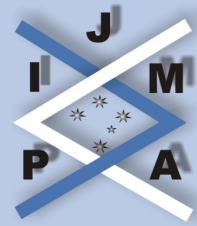
$$\begin{aligned} G(k; 2) &= -(p - \alpha_1)(p - \alpha_2) \{ (k - \alpha_1)(k - \alpha_2) - (p - \alpha_1)(p - \alpha_2) \} \\ &\quad + (k - p)(p - \alpha_1)(p - \alpha_2) \{ (k - \alpha_1) + (k - \alpha_2) \} \\ &= (p - \alpha_1)(p - \alpha_2)(k - p)^2 > 0. \end{aligned}$$

Since $F(k; 2)$ is an increasing function of k , we see that

$$\begin{aligned} (2.1) \quad F(k; 2) &\geq F(p + n; 2) \\ &= p - \frac{n(p - \alpha_1)(p - \alpha_2)}{(p + n - \alpha_1)(p + n - \alpha_2) - (p - \alpha_1)(p - \alpha_2)}. \end{aligned}$$

Therefore, the corollary is true for $m = 2$. Let us suppose that $G_{m-1}(z) \in \mathcal{S}_p^*(n, \beta^*)$ and $f_m(z) \in \mathcal{S}_p^*(n, \alpha_m)$, where

$$\beta^* = p - \frac{n \prod_{j=1}^{m-1} (p - \alpha_j)}{\prod_{j=1}^{m-1} (p + n - \alpha_j) - \prod_{j=1}^{m-1} (p - \alpha_j)}.$$



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Then replacing α_1 by β^* and α_2 by α_m from (2.1), we see that

$$\begin{aligned}\beta &= p - \frac{n(p - \beta^*)(p - \alpha_m)}{(p + n - \beta^*)(p + n - \alpha_m) - (p - \beta^*)(p - \alpha_m)} \\ &= p - \frac{n \prod_{j=1}^m (p - \alpha_j)}{\prod_{j=1}^m (p + n - \alpha_j) - \prod_{j=1}^m (p - \alpha_j)}.\end{aligned}$$

Therefore, the corollary is true for the integer m . Using mathematical induction, we complete the proof of the corollary. \square

Taking $\alpha_j = \alpha$ in Theorem 2.1, we have:

Corollary 2.3. *If $f_j(z) \in \mathcal{S}_p^*(n, \alpha)$ for all $j = 1, 2, \dots, m$, then $H_m(z) \in \mathcal{S}_p^*(n, \beta)$ with*

$$\beta = p - \frac{n(p - \alpha)^s}{(p + n - \alpha)^s - (p - \alpha)^s},$$

where

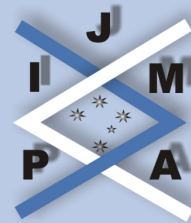
$$s = \sum_{j=1}^m p_j \geq 1 + \frac{p - \alpha}{n}, \quad p_j \geq \frac{1}{q_j}, \quad q_j > 1 \quad \text{and} \quad \sum_{j=1}^m \frac{1}{q_j} \geq 1.$$

Proof. By means of Theorem 2.1, we obtain that

$$\beta \leq p - \frac{(k - p)(p - \alpha)^s}{(k - \alpha)^s - (p - \alpha)^s} \quad (k \geq p + n).$$

Let us define $F(k)$ by

$$F(k) = p - \frac{(k - p)(p - \alpha)^s}{(k - \alpha)^s - (p - \alpha)^s} \quad (k \geq p + n).$$



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Then the numerator of $F'(k)$ can be written as

$$(p - \alpha)^s (k - \alpha)^s \{s(k - p) - (k - \alpha)\} + (p - \alpha)^s.$$

Since $s \geq 1 + \frac{p-\alpha}{n}$, we easily see that the numerator of $F'(k)$ is positive for $k \geq p+n$. Therefore, $F(k)$ is increasing for $k \geq p+n$. This gives the value of β in the corollary. \square

We consider the example for Corollary 2.3.

Example 2.1. Let us define $f_j(z)$ by

$$f_j(z) = z^p + \frac{p - \alpha}{p + n - \alpha} \epsilon z^{p+n} + \frac{p - \alpha}{p + n + j - \alpha} \epsilon_j z^{p+n+j} \quad (|\epsilon| + |\epsilon_j| \leq 1)$$

for each $j = 1, 2, \dots, m$, which is equivalent to $f_j(z) \in \mathcal{S}_p^*(n, \alpha)$. Then $H_m(z) \in \mathcal{S}_p^*(n, \beta)$ with

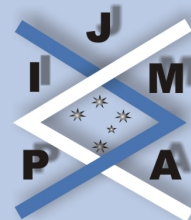
$$\beta = p - \frac{n(p - \alpha)^s}{(p + n - \alpha)^s - (p - \alpha)^s}.$$

Because, for functions

$$(2.2) \quad f_j(z) = z^p + \frac{p - \alpha}{p + n - \alpha} \epsilon z^{p+n} + \frac{p - \alpha}{p + n + j - \alpha} \epsilon_j z^{p+n+j}$$

for each $j = 1, 2, \dots, m$, we have

$$\begin{aligned} \sum_{k=p+n}^{\infty} \frac{k - \alpha}{p - \alpha} |a_k| &= \frac{p + n - \alpha}{p - \alpha} |\epsilon| |a_{p+n}| + \frac{p + n + j - \alpha}{p - \alpha} |\epsilon_j| |a_{p+n+j}| \\ &= |\epsilon| + |\epsilon_j| \leq 1 \end{aligned}$$



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from Lemma 1.1 which implies $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$. From (2.2), we see that

$$H_m(z) = z^p + \left(\frac{p - \alpha}{p + n - \alpha} \epsilon \right)^s z^{p+n}.$$

Therefore $H_m(z) \in \mathcal{S}_p^*(n, \beta)$.

We also derive other results about \mathcal{S}_p^* and \mathcal{T}_p^* .

Theorem 2.4. *If $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $H_m(z) \in \mathcal{T}_p^*(n, \beta)$ with*

$$\beta = \inf_{k \geq p+n} \left\{ p - \frac{k(k-p) \prod_{j=1}^m (p - \alpha_j)^{p_j}}{p \prod_{j=1}^m (k - \alpha_j)^{p_j} - k \prod_{j=1}^m (p - \alpha_j)^{p_j}} \right\},$$

where $p_j \geq \frac{1}{q_j}$, $q_j > 1$ and $\sum_{j=1}^m \frac{1}{q_j} \geq 1$.

Proof. Using the same method as the proof in Theorem 2.1, we have to find the largest β such that

$$\left(\frac{k(k - \beta)}{p(p - \beta)} \right) \left(\prod_{j=1}^m |a_{k,j}|^{p_j} \right) \leq \prod_{j=1}^m \left(\frac{k - \alpha_j}{p - \alpha_j} \right)^{\frac{1}{q_j}} |a_{k,j}|^{\frac{1}{q_j}},$$

which implies that

$$\beta \leq p - \frac{k(k-p) \prod_{j=1}^m (p - \alpha_j)^{p_j}}{p \prod_{j=1}^m (k - \alpha_j)^{p_j} - k \prod_{j=1}^m (p - \alpha_j)^{p_j}}$$

for all $k \geq p + n$. □



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Corollary 2.5. If $f_j(z) \in \mathcal{S}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $G_m(z) \in \mathcal{T}_p^*(n, \beta)$ with

$$\beta = p - \frac{n(p+n) \prod_{j=1}^m (p - \alpha_j)}{p \prod_{j=1}^m (p+n - \alpha_j) - (p+n) \prod_{j=1}^m (p - \alpha_j)}.$$

Theorem 2.6. If $f_j(z) \in \mathcal{T}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $H_m(z) \in \mathcal{T}_p^*(n, \beta)$ with

$$\beta = \inf_{k \geq p+n} \left\{ p - \frac{k(k-p) \prod_{j=1}^m p^{p_j} (p - \alpha_j)^{p_j}}{p \prod_{j=1}^m k^{p_j} (k - \alpha_j)^{p_j} - k \prod_{j=1}^m p^{p_j} (p - \alpha_j)^{p_j}} \right\},$$

where $p_j \geq \frac{1}{q_j}$, $q_j > 1$ and $\sum_{j=1}^m \frac{1}{q_j} \geq 1$.

Proof. To prove the theorem, we have to find the largest β such that

$$\frac{k(k-\beta)}{p(p-\beta)} \left(\prod_{j=1}^m |a_{k,j}|^{p_j} \right) \leq \prod_{j=1}^m \left(\frac{k(k-\alpha_j)}{p(p-\alpha_j)} \right)^{\frac{1}{q_j}} |a_{k,j}|^{\frac{1}{q_j}}$$

for all $k \geq p+n$. □

Corollary 2.7. If $f_j(z) \in \mathcal{T}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $G_m(z) \in \mathcal{T}_p^*(n, \beta)$ with

$$\beta = p - \frac{np^{m-1} \prod_{j=1}^m (p - \alpha_j)}{(p+n)^{m-1} \prod_{j=1}^m (p+n - \alpha_j) - p^{m-1} \prod_{j=1}^m (p - \alpha_j)}.$$



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Theorem 2.8. If $f_j(z) \in \mathcal{T}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $H_m(z) \in \mathcal{S}_p^*(n, \beta)$ with

$$\beta = \inf_{k \geq p+n} \left\{ p - \frac{(k-p) \prod_{j=1}^m p^{p_j} (p - \alpha_j)^{p_j}}{\prod_{j=1}^m k^{p_j} (k - \alpha_j)^{p_j} - \prod_{j=1}^m p^{p_j} (p - \alpha_j)^{p_j}} \right\},$$

where $p_j \geq \frac{1}{q_j}$, $q_j > 1$ and $\sum_{j=1}^m \frac{1}{q_j} \geq 1$.

Proof. We note that, we need to find the largest β such that

$$\frac{k - \beta}{p - \beta} \left(\prod_{j=1}^m |a_{k,j}|^{p_j} \right) \leq \prod_{j=1}^m \left(\frac{k(k - \alpha_j)}{p(p - \alpha_j)} \right)^{\frac{1}{q_j}} |a_{k,j}|^{\frac{1}{q_j}}$$

for all $k \geq p + n$. □

Corollary 2.9. If $f_j(z) \in \mathcal{T}_p^*(n, \alpha_j)$ for each $j = 1, 2, \dots, m$, then $G_m(z) \in \mathcal{S}_p^*(n, \beta)$ with

$$\beta = p - \frac{np^m \prod_{j=1}^m (p - \alpha_j)}{(p+n)^m \prod_{j=1}^m (p+n - \alpha_j) - p^m \prod_{j=1}^m (p - \alpha_j)}.$$

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