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COMPARISON THEOREMS FOR DIFFERENTIAL EQUATIONS WITH SEVERAL DEVIATIONS. THE CASE OF PROPERTY A

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To the memory of T. Chanturia

1. Introduction

It is well known that comparison theorems play an important role in studying oscillatory properties of solutions of ordinary differential equations. V. Kondrat'ev [1] obtained the first comparison theorem for higher order linear differential equations in the case of Property A. Its integral generalization was later obtained by T. Chanturia [2].

In the present paper we consider the following differential equations

$$u^{(n)}(t)+\sum_{i=1}^{m}p_i(t)u(\tau_i(t))=0, \tag{1.1}$$

$$v^{(n)}(t)+\sum_{i=1}^{k}q_i(t)v(\sigma_i(t))=0, \tag{1.2}$$

where $n;m;k\in N$, $n\geq 2$, $p_i;q_j\in L_{loc}(R_+;R_+)$, $\tau_i;\sigma_j\in C(R_+;R+)$, $\lim\limits_{t\to+\infty}\tau_i(t)=\lim\limits_{t\to+\infty}\sigma_j(t)=+\infty$, σ_j are nondecreasing functions ($i=1,\dots,m$; $j=1,\dots,k$).

Definition 1.1. We say that the equation (1.1) has Property A if any of its solutions is oscillatory when n is even and either is oscillatory or satisfies $|u^{(i)}(t)|\downarrow 0$ as $t\uparrow+\infty$ ($i=0,\dots,n-1$) when n is odd.

Below we give comparison theorems allowing to deduce Property A of the equation (1.1) from Property A of the equation (1.2). Some of these results are generalizations of integral comparison theorems obtained by the author in [3] for the case of one deviation (when $m=k=1$), while some of them (specifically, those contained in Theorems 2.1 and 2.2) are new even in the latter case. In its turn. the results presented in [3] generalize to differential equations with one deviation T. Chanturia's integral comparison theorems for ordinary differential equations [2], some of them being new even for ordinary differential equations.

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2. Comparison theorems

Theorem 2.1. *Let $k = m$, $\tau_i(t) \le t$ $(i = 1, \dots, m)$ and for large t* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \ge \sigma_i(t)$ $(i = 1, \dots, m)$ *and*

$$\int_t^{+\infty} p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_t^{+\infty} q_i(s)\sigma_i^{n-2}(s)\,ds \quad (i = 1, \dots, m); \tag{2.1}$$

2) $\tau_i(t) \le \sigma_i(t)$ $(i = 1, \dots, m)$ *and*

$$\int_t^{+\infty} p_i(s)\,\frac{\tau_i^{n-1}(s)}{\sigma_i(s)}\,ds \ge \int_t^{+\infty} \sigma_i^{n-2}(s)q_i(s)\,ds \quad (i = 1, \dots, m). \tag{2.2}$$

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.2. *Let $k = m$, $\tau_i(t) \ge t$ $(i = 1, \dots, m)$, and for large t* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \ge \sigma_i(t)$ $(i = 1, \dots, m)$ *and*
(i)

$$\int_t^{+\infty} s^{n-2}p_i(s)\,ds \ge \int_t^{+\infty} s^{n-2}q_i(s)\,ds \quad (i = 1, \dots, m)$$

if n is even,
(ii) (2.1) *holds,*

$$\int_t^{+\infty} s^{n-2}p_i(s)\,ds \ge \int_t^{+\infty} q_i(s)s^{n-3}\sigma_i(s)\,ds \quad (i = 1, \dots, m)$$

and

$$\int^{+\infty} s^{n-1}\sum_{i=1}^{m} p_i(t)dt = +\infty \tag{2.3}$$

if n is odd;

2) $\tau_i(t) \le \sigma_i(t)$ $(i = 1, \dots, m)$ *and*
(i)

$$\int_t^{+\infty} \frac{s^{n-2}}{\sigma_i(s)}\,\tau_i(s)p_i(s)\,ds \ge \int_t^{+\infty} s^{n-2}q_i(s)\,ds \quad (i = 1, \dots, m)$$

if n is even,
(ii) (2.2), (2.3) *hold and*

$$\int_t^{+\infty} \frac{s^{n-3}}{\sigma_i(s)}\,\tau_i^2(s)p_i(s)\,ds \ge \int_t^{+\infty} s^{n-3}\sigma_i(s)q_i(s)\,ds \quad (i = 1, \dots, m)$$

if n is odd.

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.3. *Let $k = m$, $\tau_i(t) \le t$ $(i = 1,\dots,m)$ and for any sufficiently large t_0 there exists $t_1 = t_1(t_0) \ge t_0$ such that* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \le \sigma_i(t)$ $(i = 1,\dots,m)$ *and*

$$\int_{t_0}^{t} s\tau_i^{n-1}(s)p_i(s)\,ds \ge \int_{t_0}^{t} s\sigma_i^{n-1}(s)q_i(s)\,ds \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m);$$

2) $\tau_i(t) \ge \sigma_i(t)$ $(i = 1,\dots,m)$ *and*

$$\int_{t_0}^{t} s\sigma_i(s)\tau_i^{n-2}(s)p_i(s)\,ds \ge \int_{t_0}^{t} s\sigma_i^{n-1}(s)q_i(s)\,ds \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m). \tag{2.4}$$

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.4. *Let $k = m$, $\tau_i(t) \ge t$, $t \in R_+$ $(i = 1,\dots,m)$ and for any sufficiently large t_0 there exists $t_1 = t_1(t_0) \ge t_0$ such that* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \le \sigma_i(t)$ $(i = 1,\dots,m)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1}\tau_i(s)p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-1}\sigma_i(s)q_i(s)\,ds, \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m) \tag{2.5}$$

if n is even,
(ii) (2.3), (2.4) *hold and*

$$\int_{t_0}^{t} s^{n-2}\tau_i^2(s)p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-2}\sigma_i^2(s)q_i(s)\,ds, \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m)$$

if n is odd;

2) $\tau_i(t) \ge \sigma_i(t)$ *for* $t \in R_+$ $(i = 1,\dots,m)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1}\tau_i(s)p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-1}\sigma_i(s)q_i(s)\,ds, \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m)$$

if n is even,
(ii) (2.3), (2.5) *hold and*

$$\int_{t_0}^{t} s^{n-2}\sigma_i(s)\tau_i(s)p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-2}\sigma_i^2(s)q_i(s)\,ds, \quad \textit{for} \quad t \ge t_1 \quad (i = 1,\dots,m)$$

if n is odd.

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Below we will make use of the following notation

$$\tau_*(t) = \min\{\tau_i(t) : i = 1, \dots, m\}, \quad \tau^*(t) = \max\{\tau_i(t) : i = 1, \dots, m\},$$
$$\sigma_*(t) = \min\{\sigma_i(t) : i = 1, \dots, k\}, \quad \sigma^*(t) = \max\{\sigma_i(t) : i = 1, \dots, k\}.$$

Theorem 2.5. *Let $\tau^*(t) \le t$ for $t \in R_+$ and for large t* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \ge \sigma^*(t)$ *and*

$$\int_t^{+\infty} \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_t^{+\infty} \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds; \tag{2.6}$$

2) $\tau_*(t) \ge \sigma_*(t)$ *and*

$$\int_t^{+\infty} \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_t^{+\infty} \frac{1}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds; \tag{2.7}$$

3) $\tau_*(t) \le \sigma^*(t)$ *and*

$$\int_t^{+\infty} \frac{\tau_*(s)}{\sigma^*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_t^{+\infty} \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds; \tag{2.8}$$

4) $\tau_*(t) \le \sigma_*(t)$ *and*

$$\int_t^{+\infty} \frac{\tau_*(s)}{\sigma_*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_t^{+\infty} \frac{1}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds; \tag{2.9}$$

5) $\tau^*(t) \ge \sigma^*(t)$ *and*

$$\int_t^{+\infty} \frac{1}{\tau^*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_t^{+\infty} \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds; \tag{2.10}$$

6) $\tau^*(t) \ge \sigma_*(t)$ *and*

$$\int_t^{+\infty} \frac{1}{\tau^*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_t^{+\infty} \frac{1}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds; \tag{2.11}$$

7) $\tau^*(t) \le \sigma^*(t)$ *and*

$$\int_t^{+\infty} \frac{1}{\sigma^*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_t^{+\infty} \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds; \tag{2.12}$$

8) $\tau^*(t) \le \sigma_*(t)$ *and*

$$\int\limits_t^{+\infty} \frac{1}{\sigma_*(s)} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \int\limits_t^{+\infty} \frac{1}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds. \tag{2.13}$$

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.6. *Let $\tau_*(t) \ge t$ for $t \in R_+$ and for large t* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \ge \sigma^*(t)$ *and*
(i)

$$\int\limits_t^{+\infty} s^{n-2} \sum_{i=1}^m p_i(s)\,ds \ge \int\limits_t^{+\infty} s^{n-2} \sum_{i=1}^k q_i(s)\,ds$$

if n is even,
(ii) (2.3), (2,6) *hold and*

$$\int\limits_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \int\limits_t^{+\infty} s^{n-3} \sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is odd;

2) $\tau_*(t) \ge \sigma_*(t)$ *and*
(i)

$$\int\limits_t^{+\infty} s^{n-2} \sum_{i=1}^m p_i(s)\,ds \ge \int\limits_t^{+\infty} \frac{s^{n-2}}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is even,
(ii) (2.3), (2,7) *hold and*

$$\int\limits_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \int\limits_t^{+\infty} \frac{s^{n-3}}{\sigma^*(s)} \sum_{i=1}^k q_i(s)\sigma_i^2(s)\,ds$$

if n is odd;

3) $\tau_*(t) \le \sigma^*(t)$ *and*
(i)

$$\int\limits_t^{+\infty} s^{n-2} \frac{\tau_*(s)}{\sigma^*(s)} \sum_{i=1}^m p_i(s)\,ds \ge \int\limits_t^{+\infty} s^{n-2} \sum_{i=1}^k q_i(s)\,ds$$

if n is even,
(ii) (2.3), (2,8) *hold and*

$$\int\limits_t^{+\infty} s^{n-3} \frac{\tau_*(s)}{\sigma^*(s)} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \int\limits_t^{+\infty} s^{n-3} \sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is odd;

4) $\tau_*(t) \le \sigma_*(t)$ *and*

(i)

$$\int\limits_t^{+\infty} s^{n-2}\,\frac{\tau_*(s)}{\sigma_*(s)}\,\sum_{i=1}^m p_i(s)\,ds \geq \int\limits_t^{+\infty} \frac{s^{n-2}}{\sigma_*(s)}\,\sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is even,

(ii) (2.3), (2,9) *hold and*

$$\int\limits_t^{+\infty} s^{n-3}\,\frac{\tau_*(s)}{\sigma_*(s)}\,\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \int\limits_t^{+\infty} \frac{s^{n-3}}{\sigma_*(s)}\,\sum_{i=1}^k q_i(s)\sigma_i^2(s)\,ds$$

if n is odd;

5) $\tau^*(t) \geq \sigma^*(t)$ *and*

(i)

$$\int\limits_t^{+\infty} \frac{s^{n-2}}{\tau^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \int\limits_t^{+\infty} s^{n-2}\sum_{i=1}^k q_i(s)\,ds$$

if n is even,

(ii) (2.3), (2.10) *hold and*

$$\int\limits_t^{+\infty} \frac{s^{n-2}}{\tau^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \int\limits_t^{+\infty} s^{n-3}\sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is odd;

6) $\tau^*(t) \geq \sigma_*(t)$ *and*

(i)

$$\int\limits_t^{+\infty} \frac{s^{n-2}}{\tau^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \int\limits_t^{+\infty} \frac{s^{n-2}}{\sigma_*(s)}\,\sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is even,

(ii) (2.3), (2.11) *hold and*

$$\int\limits_t^{+\infty} \frac{s^{n-3}}{\tau^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \int\limits_t^{+\infty} \frac{s^{n-3}}{\sigma_*(s)}\,\sum_{i=1}^k q_i(s)\sigma_i^2(s)\,ds$$

if n is odd;

7) $\tau^*(t) \leq \sigma^*(t)$ *and*

(i)

$$\int\limits_t^{+\infty} \frac{s^{n-2}}{\sigma^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \int\limits_t^{+\infty} s^{n-2}\sum_{i=1}^k q_i(s)\,ds$$

if n is even,

(ii) (2.3), (2.12) *hold and*

$$\int\limits_t^{+\infty} \frac{s^{n-3}}{\sigma^*(s)}\,\sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \int\limits_t^{+\infty} s^{n-3}\sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is odd;

8) $\tau^*(t) \le \sigma_*(t)$ *and*

(i)

$$\int_t^{+\infty} \frac{s^{n-2}}{\sigma_*(s)} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \int_t^{+\infty} \frac{s^{n-2}}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i(s)\,ds$$

if n is even,

(ii) (2.3), (2.13) *hold and*

$$\int_t^{+\infty} \frac{s^{n-3}}{\sigma_*(s)} \sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \ge \int_t^{+\infty} \frac{s^{n-3}}{\sigma_*(s)} \sum_{i=1}^k q_i(s)\sigma_i^2(s)\,ds$$

if n is odd.

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.7. *Let* $\tau^*(t) \le t$ *for* $t \in R_+$ *and for any sufficiently large* t_0 *there exists* $t_1 = t_1(t_0) \ge t_0$ *such that* **one of** *the following* **eight** *conditions be fulfilled*

1) $\tau_*(t) \le \sigma^*(t)$ *and*

$$\int_{t_0}^t s \sum_{i=1}^m p_i(s)\tau_*(s)\tau_i^{n-2}(s)\,ds \ge \int_{t_0}^t s\sigma^*(s) \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds; \tag{2.14}$$

2) $\tau_*(t) \le \sigma_*(t)$ *and*

$$\int_{t_0}^t s\tau_*(s) \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_{t_0}^t s \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.15}$$

3) $\tau_*(t) \ge \sigma^*(t)$ *and*

$$\int_{t_0}^t s\sigma^*(s) \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_{t_0}^t s\sigma^*(s) \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.16}$$

4) $\tau_*(t) \ge \sigma_*(t)$ *and*

$$\int_{t_0}^t s\sigma_*(s) \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \ge \int_{t_0}^t s \sum_{i=1}^k q_i(s)\sigma_i^{n-1}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.17}$$

5) $\tau^*(t) \le \sigma^*(t)$ *and*

$$\int_{t_0}^t s \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_{t_0}^t s\sigma^*(s) \sum_{i=1}^k q_i(s)\sigma_i^{n-2}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.18}$$

6) $\tau^*(t) \le \sigma_*(t)$ *and*

$$\int_{t_0}^{t} s \sum_{i=1}^{m} p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_{t_0}^{t} s \sum_{i=1}^{k} q_i(s)\sigma_i^{n-1}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.19}$$

7) $\tau^*(t) \ge \sigma^*(t)$ *and*

$$\int_{t_0}^{t} s\,\frac{\sigma^*(s)}{\tau^*(s)} \sum_{i=1}^{m} p_i(s)\tau_i^{n-1}\,ds \ge \int_{t_0}^{t} s\sigma^*(s) \sum_{i=1}^{k} q_i(s)\sigma_i^{n-2}(s)\,ds \quad \text{for } t \ge t_1; \tag{2.20}$$

8) $\tau^*(t) \ge \sigma_*(t)$ *and*

$$\int_{t_0}^{t} s\,\frac{\sigma_*(s)}{\tau^*(s)} \sum_{i=1}^{m} p_i(s)\tau_i^{n-1}(s)\,ds \ge \int_{t_0}^{t} s \sum_{i=1}^{k} q_i(s)\sigma_i^{n-1}(s)\,ds \quad \text{for } t \ge t_1. \tag{2.21}$$

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

Theorem 2.8. *Let $\tau_*(t) \ge t$ for $t \in R_+$ and for any sufficiently large $t_0 \in R_+$ there exists $t_1 = t_1(t_0) \ge t_0$ such that* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \le \sigma^*(t)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1}\tau_*(s) \sum_{i=1}^{m} p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-1}\sigma^*(s) \sum_{i=1}^{k} q_i(s)\,ds \quad \text{for } t \ge t_1$$

if n is even,
(ii) (2.3), (2.14) *hold and*

$$\int_{t_0}^{t} s^{n-2}\tau_*(s) \sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \ge \int_{t_0}^{t} s^{n-2}\sigma^*(s) \sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad \text{for } t \ge t_1$$

if n is odd;

2) $\tau_*(t) \le \sigma_*(t)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1}\tau_*(s) \sum_{i=1}^{m} p_i(s)\,ds \ge \int_{t_0}^{t} s^{n-1} \sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad \text{for } t \ge t_1$$

if n is even,
(ii) (2.3), (2.15) *hold and*

$$\int_{t_0}^{t} s^{n-2}\tau_*(s) \sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \ge \int_{t_0}^{t} s^{n-2} \sum_{i=1}^{k} q_i(s)\sigma_i^2(s)\,ds \quad \text{for } t \ge t_1$$

if n is odd;

3) $\tau_*(t) \ge \sigma^*(t)$ *and*

(i)

$$\int\limits_{t_0}^{t} s^{n-1}\sigma^*(s)\sum_{i=1}^{m} p_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-1}\sum_{i=1}^{k} q_i(s)\sigma^*(s)\,ds \quad for \;\; t \geq t_1$$

if n is even,

(ii) (2.3), (2.16) *hold and*

$$\int\limits_{t_0}^{t} s^{n-2}\sigma^*(s)\sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-2}\sigma^*(s)\sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad for \;\; t \geq t_1$$

if n is odd;

4) $\tau_*(t) \geq \sigma_*(t)$ *and*

(i)

$$\int\limits_{t_0}^{t} s^{n-1}\sigma_*(s)\sum_{i=1}^{m} p_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-1}\sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad for \;\; t \geq t_1$$

if n is even,

(ii) (2.3), (2.17) *hold and*

$$\int\limits_{t_0}^{t} s^{n-2}\sigma_*(s)\sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-2}\sum_{i=1}^{k} q_i(s)\sigma_i^2(s)\,ds \quad for \;\; t \geq t_1$$

if n is odd;

5) $\tau^*(t) \leq \sigma^*(t)$ *and*

(i)

$$\int\limits_{t_0}^{t} s^{n-1}\sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-1}\sigma^*(s)\sum_{i=1}^{k} q_i(s)\,ds \quad for \;\; t \geq t_1$$

if n is even,

(ii) (2.3), (2.18) *hold and*

$$\int\limits_{t_0}^{t} s^{n-2}\sum_{i=1}^{m} p_i(s)\tau_i^2(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-2}\sigma^*(s)\sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad for \;\; t \geq t_1$$

if n is odd;

6) $\tau^*(t) \leq \sigma_*(t)$ *and*

(i)

$$\int\limits_{t_0}^{t} s^{n-1}\sum_{i=1}^{m} p_i(s)\tau_i(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-1}\sum_{i=1}^{k} q_i(s)\sigma_i(s)\,ds \quad for \;\; t \geq t_1$$

if n is even,

(ii) (2.3), (2.19) *hold and*

$$\int\limits_{t_0}^{t} s^{n-2}\sum_{i=1}^{m} p_i(s)\tau_i^2(s)\,ds \geq \int\limits_{t_0}^{t} s^{n-2}\sum_{i=1}^{k} q_i(s)\sigma_i^2(s)\,ds \quad for \;\; t \geq t_1$$

if n is odd;

7) $\tau^*(t) \geq \sigma^*(t)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1} \sum_{i=1}^{m} p_i(s) \, \frac{\sigma^*(s)}{\tau^*(s)} \, \tau_i(s) \, ds \geq \int_{t_0}^{t} s^{n-1} \sigma^*(s) \sum_{i=1}^{k} q_i(s) \, ds \quad for \ \ t \geq t_1$$

if n is even,
(ii) (2.3), (2.20) *hold and*

$$\int_{t_0}^{t} s^{n-2} \, \frac{\sigma^*(s)}{\tau^*(s)} \, \sum_{i=1}^{m} p_i(s) \tau_i^2(s) \, ds \geq \int_{t_0}^{t} s^{n-2} \sigma^*(s) \sum_{i=1}^{k} q_i(s) \sigma_i(s) \, ds \quad for \ \ t \geq t_1$$

if n is odd;

8) $\tau^*(t) \geq \sigma_*(t)$ *and*
(i)

$$\int_{t_0}^{t} s^{n-1} \sum_{i=1}^{m} p_i(s) \, \frac{\sigma_*(s)}{\tau^*(s)} \, \tau_i(s) \, ds \geq \int_{t_0}^{t} s^{n-1} \sum_{i=1}^{k} q_i(s) \sigma_i(s) \, ds \quad for \ \ t \geq t_1$$

if n is even,
(ii) (2.3), (2.21) *hold and*

$$\int_{t_0}^{t} s^{n-2} \, \frac{\sigma_*(s)}{\tau^*(s)} \, \sum_{i=1}^{m} p_i(s) \tau_i^2(s) \, ds \geq \int_{t_0}^{t} s^{n-2} \sum_{i=1}^{k} q_i(s) \sigma_i^2(s) \, ds \quad for \ \ t \geq t_1$$

if n is odd.

Let, moreover, the equation (1.2) *have Property A. Then the equation* (1.1) *also has Property A.*

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