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PROPERTY A OF HIGH ORDER LINEAR DIFFERENTIAL EQUATIONS WITH SEVERAL DEVIATIONS

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1. Introduction

Consider the equation

$$u^{(n)}(t)+\sum_{i=1}^{m}p_i(t)u(\tau_i(t))=0, \tag{1.1}$$

where $n\geq 2$, $n,m\in N$, $p_i\in L_{loc}(R_+;R_+)$, $\tau_i\in C(R_+;R_+)$ and $\lim\limits_{t\to\infty}\tau_i(t)=+\infty$ $(i=1,\dots,m)$.

Oscillatory properties of higher order ordinary differential equations are studied well enough (see e.g. [1,2]) Analogous questions for the equation (1.1) are studied in [3,4] (see also references therein). Using comparison theorems presented in [5] and earlier results, we can derive effective criteria for the equation (1.1) have Property A (for definition of Property A see [5]). In the case $m=1$ some of the results of the present paper are given in [6].

Let $l;j\in N$; $\alpha_i;c_i\in(0,+\infty)$ $(i=1,\dots,j)$. Below we will make use of the following notation

$$M_{l,j}=\inf\left\{\frac{\sum_{i=1}^{j}c_i\alpha_i^{\lambda}}{\prod_{i=0}^{n-1}|\lambda-i|}:\lambda\in(l-1,l)\right\}. \tag{1.2$_{l,j}$}$$

2. Effective criteria for Property A

Theorem 2.1. *Let $m=k$, $\tau_i(t)\leq t$ for $t\in R_+$, $\alpha_i;c_i\in(0,+\infty)$ $(i=1,\dots,m)$, and* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t)\geq\alpha_i t$ $(i=1,\dots,m)$,

$$\liminf_{t\to+\infty} t\int_t^{+\infty}\tau_i^{n-2}(s)p_i(s)\,ds\geq c_i\alpha_i^{n-2}\quad(i=1,\dots,m) \tag{2.1}$$

and $M_{n-1,m}>1$*;*

2) $\tau_i(t)\leq\alpha_i t$ $(i=1,\dots,m)$,

$$\liminf_{t\to+\infty} t\int_t^{+\infty}s^{-1}\tau_i^{n-1}(s)p_i(s)\,ds\geq c_i\alpha_i^{n-1}\quad(i=1,\dots,m) \tag{2.2}$$

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and $M_{n-1,m} > 1$, *where* $M_{n-1,m}$ *is defined by* $(1.2_{n-1,m})$.

Then the equation (1.1) *has Property A.*

Theorem 2.2. *Let* $k = m$, $\tau_i(t) \geq t$ *for* $t \in R_+$, $\alpha_i; c_i \in (0, +\infty)$ $(i = 1, \dots, m)$ **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \geq \alpha_i t$ $(i = 1, \dots, m)$,

(i)

$$\liminf_{t \to +\infty} t \int_t^{+\infty} s^{n-2} p_i(s)\, ds \geq c_i \quad (i = 1, \dots, m)$$

and $M_{1,m} > 1$ *if* n *is even,*

(ii) (2.1) *holds,*

$$\liminf_{t \to +\infty} t \int_t^{+\infty} s^{n-3} \tau_i(s) p_i(s)\, ds \geq c_i \alpha_i^{n-1} \quad (i = 1, \dots, m),$$

$$\int_t^{+\infty} t^{n-1} \sum_{i=1}^{m} p_i(t)\, dt = +\infty, \tag{2.3}$$

$M_{2,m} > 1$ *and* $M_{n-1,m} > 1$ *if* n *is odd;*

2) $\tau_i(t) \leq \alpha_i t$ $(i = 1, \dots, m)$,

(i) (2.3) *holds,*

$$\liminf_{t \to +\infty} t \int_t^{+\infty} s^{n-3} \tau_i(s) p_i(s)\, ds \geq c_i \alpha_i \quad (i = 1, \dots, m)$$

and $M_{i,m} > 1$ *if* n *is even,*

(ii) (2.2), (2.3) *hold,*

$$\liminf_{t \to +\infty} t \int_t^{+\infty} s^{n-4} \tau_i^2(s) p_i(s)\, ds \geq c_i \alpha_i^{n-1} \quad (i = 1, \dots, m),$$

$M_{2,m} > 1$ *and* $M_{n-1,m} > 1$ *if* n *is odd, where* $M_{n-1,m}$ *and* $M_{2,m}$ *are definmed by* $(1.2_{2,m})$ *and* $(1.2_{n-1,m})$.

Then the equation (1.1) *has Property A.*

Theorem 2.3. *Let* $m = 1$, $\alpha \in (0, 1)$, $\tau_i(t) \leq t$ *for* $t \in R_+$ *and* **one of** *the following* **two** *conditions be fulfilled:*

1) $\liminf_{t \to +\infty} \dfrac{\tau_1(t)}{t^\alpha} > 0$ *and* $\liminf_{t \to +\infty} t^\alpha \displaystyle\int_t^{+\infty} \tau_1^{n-2}(s) p_1(s)\, ds > 0$;

2) $\liminf_{t \to +\infty} \dfrac{\tau_1(t)}{t^\alpha} < +\infty$ *and* $\liminf_{t \to +\infty} t^\alpha \displaystyle\int_t^{+\infty} s^{-\alpha} \tau_1^{n-1}(s) p_1(s)\, ds > 0$.

Then the equation (1.1) *has Property A.*

Theorem 2.4. *Let $k = m$, $\tau_i(t) \le t$ for $t \in R_+$, $\alpha_i; c_i \in (0, +\infty)$ $(i = 1, \dots, m)$ and* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \ge \alpha_i t$ $(i = 1, \dots, m)$,

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s^2 \tau_i^{n-2}(s) p_i(s)\, ds \ge c_i \alpha_i^{n-2} \quad (i = 1, \dots, m)$$

and $M_{n-1,m} > 1$;

2) $\tau_i(t) \le \alpha_i t$ $(i = 1, \dots, m)$,

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s \tau_i^{n-1}(s) p_i(s)\, ds \ge c_i \alpha_i^{n-1} \quad (i = 1, \dots, m) \tag{2.4}$$

and $M_{n-1,m} > 1$, *where* $M_{n-1,m}$ *is defined by* $(2.2_{n-1,m})$.

Then the equation (1.1) *has Property A*

Theorem 2.5. *Let $k = m$, $\tau_i(t) \ge t$ for $t \in R_+$, $\alpha_i; c_i \in (0, +\infty)$ $(i = 1, \dots, m)$* **one of** *the following* **two** *conditions be fulfilled:*

1) $\tau_i(t) \ge \alpha_i t$ $(i = 1, \dots, m)$,

(i) (2.3) *holds,*

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s^{n-1} \tau_i(s) p_i(s)\, ds \ge c_i \alpha_i \quad (i = 1, \dots, m)$$

and $M_{1\,m} > 1$ *if* n *is even,*

(ii) (2.3), (2.4) *hold,*

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s^{n-2} \tau_i^2(s) p_i(s)\, ds \ge c_i \alpha_i^2 \quad (i = 1, \dots, m),$$

$M_{2,m} > 1$ *and* $M_{n-1,m} > 1$ *if* n *is odd;*

2) $\tau_i(t) \le \alpha_i t$ $(i = 1, \dots, m)$,

(i) (2.1) *holds,*

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s^{n-1} \tau_i(s) p_i(s)\, ds > c_i \alpha_i \quad (i = 1, \dots, m)$$

and $M_{1,m} > 1$ *if* n *is even,*

(ii) (2.3), (2.4) *hold,*

$$\liminf_{t\to+\infty} t^{-1} \int_0^t s^{n-2} \tau_i^2(s) p_i(s)\, ds \ge c_i \alpha_i^2 \quad (i = 1, \dots, m),$$

$M_{2,m} > 1$ *and* $M_{n-1,m} > 1$, *for* $l = 2, n-1$ *if* n *is odd, where* $M_{2,m}$ *and* $M_{n-1,m} > 1$ *are defined by* $(1.2_{2,m})$ $((1.2_{n-1,m})$.

Then the equation (1.1) *has Property A.*

Below we will make use of the following notation

$$\tau_*(t) = \min\{\tau_i(t) : i = 1, \dots, m\}, \quad \tau^*(t) = \max\{\tau_i(t) : i = 1, \dots, m\}$$

Let $\alpha_i \in (0, +\infty)$ $(i = 1, \dots, k)$, $\alpha_* = \min\{\alpha_i : i = 1, \dots, \}$, $\alpha^* = \max\{\alpha_i : i = 1, \dots, k\}$.

Theorem 2.6. *Let $\tau^*(t) \le t$ for $t \in R_+$, $\alpha_i; c_i \in (0, +\infty)$ $(i = 1, \dots, k)$ and* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \ge \alpha^* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} \sum_{i=1}^m p_i(s) \tau_i^{n-2}(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.5}$$

and $M_{n-1,k} > 1$*;*

2) $\tau_*(t) \ge \alpha_* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} \sum_{i=1}^m p_i(s) \tau_i^{n-2}(s)\, ds \ge \frac{1}{\alpha_*} \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.6}$$

and $M_{n-1,k} > 1$*;*

3) $\tau_*(t) \le \alpha^* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} \frac{\tau_*(s)}{s} \sum_{i=1}^m p_i(s) \tau_i^{n-2}(s)\, ds \ge \alpha^* \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.7}$$

and $M_{n-1,k} > 1$*;*

4) $\tau_*(t) \le \alpha_* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} s^{-1} \tau_*(s) \sum_{i=1}^m p_i(s) \tau_i^{n-2}(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.8}$$

and $M_{n-1,k} > 1$*;*

5) $\tau^*(t) \ge \alpha^* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} \frac{1}{\tau^*(s)} \sum_{i=1}^m p_i(s) \tau_i^{n-1}(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.9}$$

and $M_{n-1,k} > 1$*;*

6) $\tau^*(t) \ge \alpha_* t$,

$$\liminf_{t \to +\infty} t \int_t^{+\infty} \frac{1}{\tau_*(s)} \sum_{i=1}^m p_i(s) \tau_i^{n-1}(s)\, ds \ge \frac{1}{\alpha_*} \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.10}$$

and $M_{n-1,k} > 1$*;*

7) $\tau^*(t) \le \alpha^* t$,

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{-1} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.11}$$

and $M_{n-1,k} > 1$;

8) $\tau^*(t) \le \alpha_* t$,

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{-1} \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \ge \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.12}$$

and $M_{n-1,k} > 1$.

Then the equation (1.1) *has Property A, where the constant* $M_{n-1,k}$ *is defined by* $(1.2_{n-1,k})$.

Theorem 2.7. *Let* $\tau_*(t) \ge t$ *for* $t \in R_+$, $\alpha_i; c_i \in (0,+\infty)$ $(i = 1,\dots,k)$ *and* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \ge \alpha^* t$ *and*
(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-2} \sum_{i=1}^m p_i(s)\,ds \ge \sum_{i=1}^k c_i$$

if n *is even,*
(ii) (2.3), (2.5) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \sum_{i=1}^k c_i \alpha_i$$

if n *is odd;*

2) $\tau_*(t) \ge \alpha_* t$ *and*
(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-2} \sum_{i=1}^m p_i(s)\,ds \ge \frac{1}{\alpha_*} \sum_{i=1}^k c_i \alpha_i$$

if n *is even,*
(ii) (2.3), (2.6) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \ge \sum_{i=1}^k c_i \alpha_i$$

if n *is odd;*

3) $\tau_*(t) \le \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} s^{n-3}\tau_*(s)\sum_{i=1}^m p_i(s)\,ds \geq \alpha_* \sum_{i=1}^k c_i\alpha_i$$

if n *is even,*

(ii) (2.3), (2.7) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} s^{n-4}\tau_*(s)\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n *is odd;*

4) $\tau_*(t) \leq \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} s^{n-3}\tau_*(s)\sum_{i=1}^m p_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n *is even,*

(ii) (2.3), (2.8) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} s^{n-4}\tau_*(s)\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i^2$$

if n *is odd;*

5) $\tau^*(t) \geq \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} fracs^{n-2}\tau_*(s)\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i$$

if n *is even,*

(ii) (2.3), (2.9) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} \frac{s^{n-3}}{\tau^*(s)}\sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n *is odd;*

6) $\tau^*(t) \geq \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int\limits_t^{+\infty} \frac{s^{n-2}}{\tau^*(s)}\sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \frac{1}{\alpha_*}\sum_{i=1}^k c_i\alpha_i$$

if n *is even,*

(ii) (2.3), (2.10) *hold*, $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} \frac{s^{n-3}}{\tau^*(s)} \sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \frac{1}{\alpha_*} \sum_{i=1}^k c_i\alpha_i^2$$

if n is odd;

7) $\tau^*(t) \leq \alpha^* t$ *and*
(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \alpha^* \sum_{i=1}^k c_i$$

if n is even,
(ii) (2.3), (2.11) *hold*, $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n is odd;

8) $\tau^*(t) \leq \alpha_* t$ *and*
(i) $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-3} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n is even,
(ii) (2.3), (2.12) *hold*, $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} t \int_t^{+\infty} s^{n-4} \sum_{i=1}^m p_i(s)\tau_i^2(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i^2$$

if n is odd, where $M_{1,k}$, $M_{2,k}$ *and* $M_{n-1,k}$ *are defined by* $(1.2_{1,k})$, $(1.2_{2,k})$ *and* $(1.2_{n-1,k})$.

Then the equation (1.1) *has Property A, where the constants* $M_{l,k}$ *are defined by* (1.2_{lk}).

Theorem 2.8. *Let* $\tau^*(t) \leq t$ *for* $t \in R_+$, $\alpha_i; c_i \in (0,+\infty)$ $(i = 1,\dots,k)$ *and* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \leq \alpha^* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s \sum_{i=1}^m p_i(s)\tau_*(s)\tau_i^{n-2}(s)\,ds \geq \alpha^* \sum_{i=1}^k c_i\alpha_i^{n-2} \tag{2.13}$$

and $M_{n-1,k} > 1$*;*

2) $\tau_*(t) \leq \alpha_* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s\tau_*(s) \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i^{n-1} \tag{2.14}$$

and $M_{n-1,k} > 1$;

3) $\tau_*(t) \geq \alpha^* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^2 \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \geq \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.15}$$

and $M_{n-1,k} > 1$;

4) $\tau_*(t) \geq \alpha_* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^2 \sum_{i=1}^m p_i(s)\tau_i^{n-2}(s)\,ds \geq \frac{1}{\alpha_*} \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.16}$$

and $M_{n-1,k} > 1$;

5) $\tau^*(t) \leq \alpha^* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \geq \alpha^* \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.17}$$

and $M_{n-1,k} > 1$;

6) $\tau^*(t) \leq \alpha_* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s \sum_{i=1}^m p_i(s)\tau_i^{n-1}(s)\,ds \geq \sum_{i=1}^k c_i \alpha_i \tag{2.18}$$

and $M_{n-1,k} > 1$;

7) $\tau^*(t) \geq \alpha^* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^2 \sum_{i=1}^m p_i(s) \frac{\tau_i^{n-1}}{\tau^*(s)}\,ds \geq \sum_{i=1}^k c_i \alpha_i^{n-2} \tag{2.19}$$

and $M_{n-1,k} > 1$;

8) $\tau^*(t) \leq \alpha_* t$,

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^2 \sum_{i=1}^m p_i(s) \frac{\tau_i^{n-1}(s)}{\tau^*(s)}\,ds \geq \sum_{i=1}^k c_i \alpha_i^{n-1} \tag{2.20}$$

and $M_{n-1,k} > 1$.

Then the equation (1.1) *has Property A, where the constant* $M_{n-1,k}$ *is defined by* $(1.2_{n-1,k})$.

Theorem 2.9. *Let* $\tau_*(t) \geq t$ *for* $t \in R_+$, $\alpha_i; c_i \in (0,+\infty)$ $(i = 1,\dots,k)$ *and* **one of** *the following* **eight** *conditions be fulfilled:*

1) $\tau_*(t) \leq \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-1}\tau_*(s) \sum_{i=1}^m p_i(s)\,ds \geq \alpha^* \sum_{i=1}^k c_i$$

if n *is even,*

(ii) (2.3), (2.13) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-2}\tau_*(s) \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \alpha^* \sum_{i=1}^k c_i\alpha_i$$

if n *is odd;*

2) $\tau_*(t) \leq \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-1} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i$$

if n *is even,*

(ii) (2.3), (2.14) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-2} \sum_{i=1}^m p_i(s)\tau_*(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i^2$$

if n *is odd;*

3) $\tau_*(t) \geq \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n} \sum_{i=1}^m p_i(s)\,ds \geq \sum_{i=1}^k c_i$$

if n *is even,*

(ii) (2.3), (2.15) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-1} \sum_{i=1}^m p_i(s)\tau_i(s)\,ds \geq \sum_{i=1}^k c_i\alpha_i$$

if n *is odd;*

4) $\tau_*(t) \geq \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n} \sum_{i=1}^m p_i(s)\,ds \geq \frac{1}{\alpha_*} \sum_{i=1}^k c_i\alpha_i$$

if n *is even,*

(ii) (2.3), (2.16) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n-1} \sum_{i=1}^m p_i(s)\tau_i(s)\, ds \ge \frac{1}{\alpha_*} \sum_{i=1}^k c_i \alpha_i^2$$

if n is odd;

5) $\tau^*(t) \le \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n-1} \sum_{i=1}^m p_i(s)\tau_i(s)\, ds \ge \alpha^* \sum_{i=1}^k c_i$$

if n is even,

(ii) (2.3), (2.17) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n-2} \sum_{i=1}^m p_i(s)\tau_i^2(s)\, ds \ge \alpha^* \sum_{i=1}^k c_i \alpha_i$$

if n is odd;

6) $\tau^*(t) \le \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n-1} \sum_{i=1}^m p_i(s)\tau_i(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i$$

if n is even,

(ii) (2.3), (2.18) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n-2} \sum_{i=1}^m p_i(s)\tau_i^2(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i$$

if n is odd;

7) $\tau^*(t) \ge \alpha^* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t s^{n} \sum_{i=1}^m p_i(s) \frac{\tau_i(s)}{\tau^*(s)}\, ds \ge \sum_{i=1}^k c_i$$

if n is even,

(ii) (2.3), (2.19) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int\limits_0^t \frac{s^{n-1}}{\tau^*(s)} \sum_{i=1}^m p_i(s)\tau_i^2(s)\, ds \ge \sum_{i=1}^k c_i \alpha_i$$

if n is odd;

8) $\tau^*(t) \ge \alpha_* t$ *and*

(i) $M_{1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^n \sum_{i=1}^m p_i(s) \frac{\tau_i(s)}{\tau^*(s)}\, ds \geq \sum_{i=1}^k c_i \alpha_i$$

if n is even,

(ii) (2.3), (2.20) *hold,* $M_{2,k} > 1$, $M_{n-1,k} > 1$ *and*

$$\liminf_{t\to+\infty} \frac{1}{t} \int_0^t s^{n-1} \sum_{i=1}^m p_i(s) \frac{\tau_i^2(s)}{\tau^*(s)}\, ds \geq \sum_{i=1}^k c_i \alpha_i^2$$

if n is odd, where $M_{1,k}$, $M_{2,k}$ *and* $M_{n-1,k}$ *are defined by* $(1.2_{1,k})$, $(1.2_{2,k})$ *and* $(1.2_{n-1,k})$.

Then the equation (1.1) *has Property A, where the constant* M_{lk} *is defined by* (1.2_{lk}).

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