ABSTRACT. Let $p_n/q_n = (p_n/q_n)(x)$ denote the n^{th} simple continued fraction convergent to an arbitrary irrational number $x \in (0,1)$. Define the sequence of approximation constants $\theta_n(x) := q_n^2|x - p_n/q_n|$. It was conjectured by Lenstra that for almost all $x \in (0,1)$,

$$\lim_{n\to\infty} \frac{1}{n}|\{j : 1 \le j \le n \text{ and } \theta_j(x) \le z\}| = F(z)$$

where $F(z) := z/\log 2$ if $0 \le z \le 1/2$, and $\frac{1}{\log 2}(1 - z + \log(2z))$ if $1/2 \le z \le 1$. This was proved in [BJW83] and extended in [Nai98] to the same conclusion for $\theta_{k_j}(x)$ where k_j is a sequence of positive integers satisfying a certain technical condition related to ergodic theory. Our main result is that this condition can be dispensed with; we only need that k_j be strictly increasing.

[BJW83]W. Bosma, H. Jager, and F. Wiedijk, *Some metrical observations on the approximation by continued fractions*, Indag. Math. **45** (1983), 281–299, MR 85f:11059, Zbl 519.10043.

[Nai98]R. Nair, *On metric diophantine approximation theory and subsequence ergodic theory*, New York Journal of Mathematics **3A** (1998), 117–124, MR 99b:11088, Zbl 894.11032.