

ABSTRACT. We consider the question of the existence of a nontrivial continuous homomorphism from $(\beta\mathbb{N}, +)$ into $\mathbb{N}^* = \beta\mathbb{N} \setminus \mathbb{N}$. This problem is known to be equivalent to the existence of distinct p and q in \mathbb{N}^* satisfying the equations $p + p = q = q + q = q + p = p + q$. We obtain certain restrictions on possible values of p and q in these equations and show that the existence of such p and q implies the existence of $p, q,$ and r satisfying the equations above and the additional equations $r = r + r, p = p + r = r + p,$ and $q = q + r = r + q$. We show that the existence of solutions to these equations implies the existence of triples of subsets of \mathbb{N} satisfying an unusual Ramsey Theoretic property. In particular, they imply the existence of a subset A with the property that whenever it is finitely colored, there is a sequence in the *complement* of A , all of whose sums two or more terms at a time are monochrome. Finally we show that there do exist sets satisfying finite approximations to this latter property.