

ABSTRACT. Most commonly, studies of geodesy on Riemann surfaces proceed on those surfaces without ramification. While it is true that every surface of finite type (stemming from a finitely generated Fuchsian group) has a finite-sheeted cover which is free of ramification, local geometric information is lost in this process.

We seek to analyze geodesics on ramified surfaces directly. After briefly reviewing (the by now well-understood) situation of ramification points of order 2, we turn to higher ramification. Examples are offered on the surface stemming from the squares of elements of the full modular group $\Gamma^2 \backslash \mathcal{H}$ of signature $(0; 3, 3, \infty)$. Already here we can see that a fundamental property of negatively curved surfaces fails: there are closed curves with apparently non-trivial homotopy, yet having no geodesic in their homotopy class. Next, we construct surfaces on which length L and number of self-intersections N of a closed bounded height geodesic are closely linked: There are constants a and b depending only the surface and said height such that $aL(\tau) < bN(\tau) < L(\tau)$. On such surfaces, long simple geodesic arcs cannot exist.