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## Euclid The Creation of Mathematics

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The aim of this book is to present Euclid's *Elements* to general mathematical readers, including students. The author summarizes all of its 13 books in order of appearance of item or group of items (usually propositions, but also definitions and postulates), and in 16 interlaced chapters he relates the material to other developments in mathematics, usually more modern. A good bibliography is furnished (in various formatting conventions), though several items are not cited; the index is very sparse on topics.

### I

The author stresses that the identity of Euclid is unknown, and that he (or they?) was more often editor than inventor. Various other figures are discussed or quoted, especially Theaetetus (the author uses the Latin versions of Greek names), Theodorus, Plato, Archimedes and Proclus.

The subtitle of the book is non-standard. "Creation" can properly refer only to the systematic organisation of theories, where "Euclid" indeed seems to have been a pioneer; but the claim that "Mathematics was invented in Greek's classical period" (p. 2, *sic*) is an absurd dismissal of at least the Egyptians, Babylonian and Chinese. Again, "The Romans ran their Imperium without any mathematics" (p. 1) ignores at least their numeral system and profession of *Agrimensores*.

The book contains many excellent diagrams, prepared by the author; but he does not discuss the history or examine the role of the originals. These aspects, especially the semiotic side, have been interestingly handled in the recent book Netz 1999, along with aspects of proof. The author is rather light on proof, but he describes and even evaluates some. In particular, that of Pythagoras' theorem (I. 47) is "better than that any other proof" (p. 44); so no competitor is discussed, not even that of the generalised theorem about similar rectangles on the sides of the right-

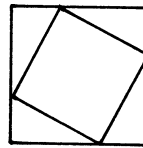


Fig. 1

angled triangle (VI. 31), which has a simpler proof using similar triangles (the author modernises it on pp. 154–155). In addition, that based upon Figure 1 is surely far more elegant and can be effected more quickly, within Euclidean means.

The author also reviews Euclidean arithmetic, as presented formally in Books 7–9; the account of the "near miss" over unique factorisation is good (pp. 178–182). He might have stressed more explicitly that the only numbers in the book are the integers from 2 onwards (as multiples of the unit 1) and the (Egyptian?) unit fractions from  $1/2$  onwards, and combinations thereof. Maybe "Euclid has preserved a historically earlier stage of mathematical theories" (p. 257) in his treatment; or perhaps it was placed here, when numbers had been used freely in the earlier Books, as preparation for the "higher" mathematics of the monumental Book 10 on incommensurable geometrical magnitudes. While not offering a theory of irrational numbers, the attendant Euclidean algorithm in that Book resembles the generation of continued fractions; they are mentioned sometimes but deserve more airing (compare Fowler 1999, Chapters 5 and 9).

### II

These last remarks exemplify the relationship between arithmetic and geometry in the *Elements*, which is handled disappointingly. In arithmetic Euclid multiplied numbers as usual, so that  $7^2 = 49$ : he even represented such numbers as plane figures (VII. Definitions 15–16), a main origin of our use "squared" to describe such numbers. But this move was unfortunate, since his geometry was *not* arithmeticised: he treated lines but not lengths, regions instead of areas, solids rather than volumes, angles but not degrees (Fowler 1999, especially Chapters 2–4). The *Elements* is concerned with comparing magnitudes but not at all with measuring them; for example, nothing in it relates directly to  $\pi$  (so that the author's pp. 74–76 are rather irrelevant). In the same approach he never multiplied together any kinds of geometrical magnitude, and so formed, for example, the square *on* a line, not *of* that line (I. 46).

In his summaries of the geometrical Books the author correctly writes, say, " $\square AB$ " for the square on line  $AB$ , with a similar notation for the rectangle "contained by" two adjacent perpendicular lines (but no notation for circles or spheres on their diameters). However, in contradiction of these notations he also states that "By 'the triangle'

Euclid means, as usual, its area" (p. 136; see also pp. 36–42, 145–146, 272), which is quite mistaken for all the geometry; and he also routinely writes expressions such as " $b^2$ " and " $ab$ " in his modernising passages, which forces false analogies with common algebra.

A central feature of the *Elements* is the theory of ratios of two magnitudes (of any kind, including numbers). The author mentions ratios regularly, but says too little about the theory of proportions between pairs of them: "proportion" is not even in his index. Euclid defined the condition of the ratio  $a : b$  being "in the same ratio" (V. Definition 5) as  $c : d$ , and consistently used similar phrases in his exegesis; but the author writes "=" instead (first on p. 7). The point is not trivial, for Euclid wrote of equality in many other contexts, such as "the three interior angles of a triangle are equal to two right angles" (I. 32). Although he also used "greater than" and "less than" between ratios, magnitudes and numbers, his avoidance of equality between ratios shows that he saw them as a new primitive, neither arithmetical nor geometric. However, the attendant theory hardly "much resembles the modern algebraic theories of groups, rings, fields, and similar algebraic structures" (p. 153). Similarly, the important technique of compounding ratios is not to be identified with multiplication in arithmetic or in any of these algebras.

The consequences of the geometry not being arithmetical are considerable, as I have shown in a paper cited in the bibliography but not discussed (Grattan-Guinness 1996). The relationship between geometry, algebra and arithmetic in Euclid differs fundamentally from that for an arithmetised geometry. If one wants to algebraise Euclid's geometry at all, then a Grassmannian type of algebra is needed. The matter is not only of historical interest; it also opens up various educational possibilities, especially the (merits of) teaching geometry and its applications *without* the hegemonies of arithmetic and common algebra.

### III

The interlacing chapters are strongest on physical manifestations: diagrams on coins (p. 50); the mysterious dodecahedral and icosahedral artefacts (pp. 300–303), showing the Romans at mathematics again; and architecture, where Fibonacci numbers intriguingly appear in the theatre in Epidaurus (pp. 239–240). But the author's own *Castrum Euclidis* which "show[s] the general architecture of the *Elements*" figuratively in a ground plan of a castle (pp. 314–315), makes a rather incoherent end to the book, for only a few features of the site are explained.

The other related themes come from mathematics itself: for example, solving quadratic equations, the theory of parallels, and symmetries. The comparison with Hilbert's foundations of geometry of 1899 is unhappy, since his famous remark about points, lines and planes being replaceable by tables, knives and beer-mugs was not a triviality about using other words (p. 50) but concerned *theories*, especially the model-theoretic possibility that an axiom system might admit interpretations other than the one intended. It is no accident that the theory of categoricity grew directly (and immediately) out of his work (Scanlan 1991).

### IV

While the author successfully conveys the main features and many details of the *Elements*, the relationship between its geometry and its arithmetic, and the role of algebra, is often misconceived, and the connections with other theories are misleading; Euclid is thereby distanced from the reader. The book contains both accurate summaries of the content of the *Elements* and key distortions of its intent.

### References

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