Studies of Argumentation in Primary Mathematics Education[1]

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Abstract: The participation of students in processes of collective argumentation is seen as a fundamental condition for enabling mathematics learning in classroom settings. Using data from a finished research project on argumentation in primary mathematics classrooms it will be shown, that in elementary education these processes are of a narrative character.

Kurzreferat: *Untersuchungen zur Argumentation im Mathematikunterricht der Grundschule.* Die Teilnahme von Schülern an Prozessen kollektiver Argumentation ist eine wesentliche Voraussetzung für das Lernen von Mathematik im Unterricht. Mithilfe von Daten eines abgeschlossenen Forschungsprojekts zur Argumentation im Mathematikunterricht der Grundschule wird gezeigt, dass solche Prozesse im Primarbereich von narrativem Charakter sind.

ZDM-Classification: C52

1. Introduction

The central research purpose of this paper is to examine the relationship between the participation of students in classroom interaction and individual mathematical learning. The theoretical orientation is drawn from ethnomethodology (Garfinkel 1967), symbolic interactionism (Blumer 1969) and cultural psychology (Bruner 1996).

In the following, elements of an interactional theory concerning content related learning in classroom-situations are discussed in greater detail. First, I clarify the concept of the "social constitution of learning" and the role the concept of "culture" plays for this matter. Second, regarding the empirical material of interaction processes in primary education, I outline the narrative character of the reconstructed classroom culture. Here I contend that the classroom culture is characterized through processes of common storytelling. This is seen as a feature of rational acting in primary classroom interaction, and is based on reflexive argumentation. Finally the empirical results which have been presented in these chapters will be brought together to create elements of a theory of interactions of mathematical learning. Practical implications will be described.

2. The social constitution of learning

Learning is seen here as a social process that takes place in the interaction between human beings. Students act in ways that seem *sensible* and *tenable* to them. In order to do this, they *interprete* their classroom situations: They reflect, set up and review hypotheses, and make rational decisions; common features are accomplished that temporarily enable them to participate. In such a classroom situation students develop their mathematical understanding not only to be mutually regarded as responsible and capable but in order to participate in the joint creation of the interactions. Thus, in such a situation their minds are challenged, which they employ and develop simultaneously.

In order to conceptualize the process of the individual learning while participating in such situations of social interaction Bruner 1983 introduces the term "format" as a "standardized, initially microcosmic interaction pattern between an adult and an infant that contains demarcated roles that eventually become reversible" (p. 120f).

Bruner's work on this concept is related to the field of early child language acquisition. Later and with regard to a larger domain of knowledge acquisition he employs the concept of "plot", with which he articulates learning through participation in formats in a more general and more advanced way as learning through participation in processes of story-telling. Bruner 1990 states:

> "When we enter human life, it is as if we walk on stage into a play whose enactment is already in progress – a play whose somewhat open plot determines what parts we may play and toward what denouements we may be heading. Others on the stage already have a sense of what the play is about, enough to make negotiation with a newcomer possible" (p. 34).

Thus, learning is not simply the appropriation of culture, it rather occurs during the co-creation of interaction. Especially with regard to primary education it is often stated that basic cultural contents such as reading or arithmetic are taught and acquired. From the perspective of cultural psychology, this seems to be an insufficient point of view: Children do not only learn the subject matters of culture, instead, through their contributions in reading or calculating they also create "a" or "the" culture. If we integrate these two aspects, we arrive at what could be described as "classroom culture". *It is a culture of subject matter and a culture of learning.*

3. The narrativity of classroom-culture

It is well-known, that in primary education children like to tell and listen to stories. The argument presented here goes beyond this empirical evidence: I claim that children learn by participating in the telling of these stories. In addition, they learn the content of different school subjects. When classroom culture of primary schools is characterized by narrativity, the social constitution of classroom-learning can be described in models of participation in situations of story-telling. This is also relevant for mathematics classes, and the analysis of processes of interaction concerning this subject matter can demonstrate the importance of this thesis in general.

In the following, I begin by presenting the characteristics of such narrations in the observed mathematics classes. My aim is to clarify that *the narrative classroom culture of primary education is based on rationality, and the social constitution on classroom-learning is the participation in the interactional accomplishment of argumentative, narratively structured sequences of actions.*

This thesis does not imply that in classroom-situations "stories" are told endlessly and that beyond educational goals in native language classes, classroom-education intends to teach children the telling of stories. Rather, the concept of "narrative" is used here to describe a specific

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phenomenon of everyday classroom conversation. It is not meant in the sense of narrative found in literary science.

Four characteristics for narrative accomplishments can be identified (Bruner 1990, p. 50):

- "sequentiality",
- "factual indifference between the real and the imaginary",
- "unique way of managing departures from the canonical", and
- "dramatic quality".

Here, the first and third point are of special interest. The supposed narrativity of classroom culture is seen in the patterned sequentiality of classroom interaction, and the specifity of an event, such as the elaborated solving process for a new mathematical task, is presented in relation to the canonical management of such events or problems.

In the following, evidence is provided for this theoretical approach by reconstructing in detail several classroom episodes.

3.1 Introductory example – The narrative plot

In this section, I illustrate the narrative character of classroom culture with empirical data from a second grade mathematics class. The students are supposed to present their results to word problems in front of the class after having solved previously in groups of two. In a certain sense this resembles a typical situation of narrating: one reports, what one did and experienced. But it is not only the single reporting child who generates a story. Rather, in the interaction with the teacher a new story accomplishes, which tells, how one should solve problems of the given kind.

One demand which results in the presentation of the children is to rearrange all that happened in the team work according to the solution of the problem in a comprehensive order. Two second graders, Sibylle and Stefano, for example, are coping with this narrative problem of "sequentiality" in the following way. (On the worksheet a drawing which shows 5 plants in a flowerbox is attached to the text.)

78 Sibylle (reads) The gardener puts the plants into the flower-box for Mrs. Müller. He charges her four marks for one plant.

79 Stefano 4 times 5 (.) marks equals twenty marks.

They are reading aloud the text of the task (Sibylle), give a number-problem, and then proceed to the result (Stefano). This issue can be differentiated more precisely in an episode from the same lesson in which two children present a wrong answer:

126 Lisa (reads) Elke has 35 marbles. Four friends of hers are playing with her. Every child
127 gets the same amount of marbles.
128 Marcel 35 (..) divided by 4 equals five.
129 (2 sec: Marcel and Lisa are on their way back to their seats)
130 teacher So what did you figure out now' (2sec)
131 Marcel Actually she has figured it out. (pointing at Lisa)
132 Lisa You'
133 teacher I don't know, four friends are playing with her. Every child gets the same
134 amount of marbles
135 Marcel 5 marbles
136 teacher 5'
137 Marcel 5 divided (..) from those 5 (.) so' the four children get five marbles.

Again, one can recognize in this episode the sequentiality of a narrative presentation as mentioned above. However, this time the delivered story is told incorrectly; both the approach to the problem and the result of the calculation are wrong. The teacher asks for an interpretation of the result (what) $< 130 >$. She is not asking about the process of the children's prior cooperative groupwork (how did you solve it?). This one can understand as a lesson: in front of the class it is not appropriate to investigate the genesis of the solution of these two children. It seems more appropriate to stress the missing credibility or verisimilitude of the whole story. As Bruner 1990 puts it:

"The function of the story is to find an intentional state that mitigates or at least makes comprehensible a deviation from a canonical pattern. It is this achievement that gives a story verisimilitude" (p. 49f.).

Together, the participants of such a narratively structured process of interaction must fulfill the demand and expectation, to conclude from the details of a given story onto its concatenation and the continuity of the whole. Frequently, from the perspective of a theory of narrativity one uses for this relationship the concept of "plot". Bruner 1998 defines it as "how and in what order the reader becomes aware of what happened" (p. 19). Bruner refers to the "reader" because the context of this quotation refers to written stories. One can easily substitute for the reader a listener according to a verbally presented story or by a participant according to a process on interaction in which a story emerges. Important in this quotation is that a plot characterizes the sequence of action in its totality. A plot describes something that is already fixed such as the plot of a movie. But an unfolding plot connotes something fragile, not yet entirely executed, still changeable. Both aspects are essential and the tension between these two dimensions of this concept is crucial for its adaptation for classroom interaction and its function for learning.

Elements of such a plot in the class discussion previously illustrated become more conceivable in the following example with Stefanie. (On the problemsheet one can see a container with 12 bottles.)

65 Stefanie (Reads) Susi carries that to the kindergarten. She can bear two bottles
66 at one time. Calculation (.) 2 times 6 equals
67 12. Answer She must go six times

First, Stefanie reports the text of the task $< 65,\ 66 >$, then she presents her calculation, and finally she formulates a sentence as answer. This is the rough draft of the plot of the working-process on a word problem. As one can reconstruct with respect to the whole classroom report, the plot consists of the following parts:

– reading the text
– possibly, rephrasing and emphasizing the relevant bits of the text
– developing a solving approach which is restricted to these bits,
– conducting the calculations
– formulating sentences as an answer which contains the results of the calculation.

In the episodes presented, the plot is only partially realized. The children not only follow the single steps of the presented solution to do mathematics, they also comprehend these steps as parts of a more global scheme or plot for the solving word-problems. This scheme is "wrapped" in the story of pairwork reports. In this lesson it is neither explicated and/or commented as a recommended working-scheme, nor is it argumentatively brought into prominence by contrasting it, for example, with different possibly inappropriate or wrong solving-procedures. Thus this scheme is not only a plan for the presentation of an already solved problem but also the plot for future solving-activities of such tasks.

3.2 The academic task structure (ATS)

The chosen material of this project contains whole class-discussion and group-work as well. For a deeper clarification of the narrative character of processes of interactive mathematical problem-solving in elementary classes empirical episodes from the group-work data are presented. Here generally two or three children work together. Their successful cooperation demands two different achievements. They have to clarify (a) what shall be done at what time and (b) who shall do it at what time. The structure of actions and interactions with regard to the first issue can be called "academic task structure" (Erickson 1982). It is based on the understanding of the situation of the problem as shared among the students. It is not identical with logical considerations about a sequence of problem-solving steps according to the subject matter of mathematics (see Vollmer & Krummheuer 1997). A second issue can be described in reference to Erickson as "social participation structure". Both structures are mutually dependent (see Erickson 1982, p. 156 and Vollmer & Krummheuer 1997).

"The academic task structure (ATS) can be thought of as a patterned set of constraints provided by the logic of sequencing in the subject-matter content of the lesson. The social participation pattern (SPS) can be thought of as a patterned set of constraints of the allocation of interactional rights and obligations of various members of the interacting group" (Erickson 1982, p. 154).

One episode might help to identify typical features of ATS:

What Number?

Here, members of a sports club sit together. The numbers on their shirts create a sequence of numbers. Which number should the boy to the very right have at the front of his T-shirt?

Once again? What number?

And which number should be on his shirt?

In the "shirt-task" the boys third-graders Daniel, Slawa and Stanislaw are confronted with the presentation of numbers at the back of T-shirts which represent the first parts of a number sequence. Their task is to determine the fifth element of this sequence which is:

$$\{3 - 8 - 15 - 24 - ?\}$$

for the first part of the task.

For this problem, Slawa can quickly give a solution:

47	Slawa	*(pointing at the picture)* Here comes five, here comes seven'..
49	Slawa	here comes *(.)* nine'
52	Slawa	He gets an eleven-
53	Daniel	Why eleven'
54	Stanislaw	Why'
55	Slawa	Well eleven. Look', *(precariously whispering)* how much plus three,
56		look', at this number. five-
57<	Daniel	Well', from three to eight are five.
58<	Slawa	*(directed to Daniel and still pointing at the picture)* here comes already
59		seven', seven-
60	Daniel	seven-
61	Slawa	nine' *(.)* eleven.
62	Stanislaw	*(inarticulate)* well yeah.
63	Slawa	Eleven plus twenty-four. add it here. then one gets *(figures about 2 sec)* thirty-five.

From a mathematical stance, one can view in Slawa's solution the thematization of the general concept of the sequence of differences and the first four numbers of a specific sequence of differences {5 - 7 - 9 - 11}. The boy cannot name them. He does not define them explicitly and in a certain way he is not talking *about* them, but *through* them. His two classmates cannot follow him. Slawa is obliged to explain; generally, he reacts in the way just described: He names the four elements of the sequence of difference. One short scene of this episode might demonstrate this:

77<	Slawa	This are five. here *(points at paper)* then seven', here comes nine
78<	Daniel	five *(mumbles inarticulately)* from eight to fifteen are seven'
79	Slawa	add always two to it.
82<	Slawa	Thus here comes eleven', Daniel *(points at number sequence)* here
83<	Daniel	seven'
		yes from, yes
84<	Slawa	comes eleven to that number
85<	Daniel	from fifteen to twenty-four are nine.
86	Slawa	Thus here you get thirty-five. *(inarticulate)* thirty-five.
87	Stanislaw	Whoop-
88	Daniel	Yes, nine'

One recognizes how Daniel in <78, 84, and 88> agrees to the numbers 5, 7, and 9 as the difference between the given elements of the initial number sequence. He and Stanislaw as well do not conceptualize the numeration of these numbers as the elements of a number sequence which emerges by finding the differences. Even Slawa's meta-comment about the rule for this sequence of differences in $< 79 >$ does not help. Slawa's finding of the solution, his presentation and his justifications are narratively oriented. In order to understand his solution one must, firstly, recognize the phenomenon of a sequence of difference and, secondly, the defining characterization $x_{n+1} = x_n + 2$ while repeating the numbers 5, 7, 9, 11. Those who cannot infer this argument from the numeration of the numbers, do not understand the sense of the story.

In summary, the four following conclusions can be drawn from the interpretation of this and the other episodes form the mentioned project:

1) The mathematical concepts which are necessary for understanding the ATS are not introduced explicitly. In a narrative way, they are rather pointed at implicitly. Not all students are able to recognize the ATS by this way of presentation. The plausibility of this solving process might be inscrutable for them.
2) For the accomplishment of the different steps of the ATS the boys need certain mathematical competencies such as addition and subtraction of positive integers.
3) Only few or no meta-commentaries about the functionality of the ATS or certain steps of the ATS are given. This characterizes narratively organized interaction.
4) The presentation of the solution steps proceeds mainly by verbalization. The boys do not use alternative presentations such as visualization or embodiments. This is an additional characteristic for narratively organized processes of interaction in mathematical group work.

These four issues describe aspects of the ATS and its narrative generation in group work. In general these are

> "(a) the logic of subject matter sequencing; (b) the information content of the various sequential steps; (c) the 'meta-content' cues toward steps and strategies for completing the task; and (d) the physical materials through which tasks and tasks components are manifested and with which tasks are accomplished" (Erickson 1982, p. 154).

These four issues are applicable to the first example as well.

4. Reflexive argumentation in narrative classroom interaction

4.1 Substantial argumentation

The mentioned four issues demonstrate in such solving-processes that on the surface of their communication the participants only talk about calculation steps and not much else. This widely known phenomenon of students mere vocalization about the steps in calculation is considered within narratively structured processes of interaction which contain an argumentative aspect under the surface. Typically these processes are characterized through (a) their specific sequentiality of the actions and their dominant verbal presentation, and (b) the missing explication of the plot of the story and the implicit assumption that the participants and other listeners are able to follow this plot by themselves. With regard to the four components of ATS in a narratively structured realization of a problem-solving process the components 2 and 4 are explicitly dealt with, whereas the components 1 and 3 are rather taken as implicitly effectual. Thus, this kind of classroom interaction fosters a rationality of task-oriented acting that reveals argumentation about the "rightness" of a solution to the students as much as they are able to infer the expectation for argumentation from the plot of the evolving story themselves. Further differentiation of this thesis is provided by a discussion of the concept of argumentation as used in this article.

We should notice that both the concept of argument and that of argumentation do not need to be exclusively connected with formal logic, as we know it from mathematical proofs or as the subject matter of logic. There are more human activities and human efforts which are argumentative but not in a strictly logical sense. As Toulmin 1969 points out, if these formally logical conclusions would be the only legitimate form of argumentation at all, then the domain of rational communication would be considerably restricted and argumentation would be rather irrelevant as a possible way of a rational communication (p. 40f., p. 123ff.; see also Krummheuer 1995, p. 23ff.).

A logically correct deduction, for example, contains in its conclusion nothing that is not already a potential part of the premises. It explicates aspects of the meaning of the premises by means of deduction. These kinds of argumentations are "analytic" (Toulmin 1969, p. 113). In contrast, "substantial" arguments (p. 113) expand the meaning of such propositions insofar as they soundly relate a specific case to them by actualization, modification and/or application. Thus, substantial argumentations are informative in the sense that the meaning of the premises increases or changes by the application of a new case to it, whereas analytic ones are tautological, that is a latent aspect of the premises is elaborated visibly (p. 125). Usually, these kinds of substantial argumentations do not have the logical stringency of formal deductions, which is not to be taken as a weakness, but rather as a sign that fields of problems exist which are not accessible to formal logic. Substantial argumentation has a right to exist in itself. By substantial argumentation a statement or decision is gradually supported (see also Perelman & Olbrechts-Tyteca 1969, p. 1 and Krummheuer 1995, p. 236 and 1997, p. 29ff.).

This distinction helps to clarify the conceptual framework chosen for the analysis of argumentation in mathematics classroom situations in primary education. It is the substantial argumentation that is seen here as more adequate for the reconstruction. With regard to folk psychological assumptions of learning the impact of argumentation can be reconstructed in a more appropriate manner when the analysis is based on the concept of substantial argument (for further clarification see Krummheuer 1995, p. 236ff.).

4.2 Ethnomethodology and rationality

The previous discussion suggests that the rationality of everyday activities, such as those in the observed classes,

should not be measured by the standards of a scientific model of rationality which in mathematics is oriented toward deductive logic.

Specifically, ethnomethodology emphasizes that the participants in the affairs of everyday life constitute their model of rationality interactively, which one might call the "inner logic" emerging in these situations. One of the aims of ethnomethodology is to explore such interactive constitutions of rational acting (Garfinkel 1972). Thus, from the stance of ethnomethodology the rationality of everyday actions is interactively generated while acting in a social setting; it is not an invariant part of such actions.

The participants of everyday encounters are continuously concerned with showing and clarifying the rationality of their actions for themselves and for the others as well (see Lehmann 1988, p. 167ff.). The participants use so-called "accounting practices", which are techniques and methods that help to demonstrate the rationality of the action while acting. In the process of accomplishing an action the participant is already trying to make his actions accountable. This concept of "account" is a key concept in Garfinkel's work (see e. g. Attewell 1974, p. 183).

The concept has an application that is broader than that of argumentation. Rational processing envelops all types of experience. The unifying character of all accounts is that the denoted action is to be made understandable, and that the proposed claims are intersubjectively acceptable (see also Voigt 1995). In order to do so, one endows it with the "status of an intersubjective object" (Leiter 1980, p. 162). With regard to the concept of substantial argumentation, this transfer changes, through the interactive establishment of such an argumentation, the collectively accepted basis as well as the collectively shared means for this transfer.

For the ethnomethodological approach, it is essential that the accomplishment of an action and the demonstration of its accountability are not separate activities. Garfinkel understands it as the central issue of his studies that "the activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings 'accountable'" (Garfinkel 1967, p. 1). Thus, in the process of social interaction participants make their actions understandable and accountable as well, a fact which is usually termed "reflexivity" (Mehan & Wood 1975, p. 137ff.; Lehmann 1988, p. 174; Voigt 1995 and Yackel 1995). This reflexivity does not necessarily mean, as the spelling of "accountable" suggests, that the rationality of one's actions is explicated entirely or sufficiently while accomplishing these acts. Rather, in the episodes presented here, reflexivity constantly emerges from the specific sequentiality of children's solving steps which reflexively expresses the accountability of both the single calculation steps and their entire solving approach.

5. Summary: Arguing and learning in a narrative classroom culture

Parts of the classroom culture in primary mathematics education can be reconstructed as narratively structured ones. The following describes how the content related learning of mathematics is situated in such kinds of interaction.

The basic assumption is that the students do learn mathematics content during these interaction processes. They form the classroom-processes in a specific way, because they assume, that by this learning is rendered feasible and that the students carry along the cognitive supply (equipment) so that such narrative processes can work as social constituents for content related learning.

In order to reconstruct this folk psychological assumption I use the perspective of interactionism (c.f., Bauersfeld et al. 1985). This perspective contains as basic assumptions that participants make sense of a situation by means of their individual interpretation. The related inner-psychic process is called the "definition of the situation". The learning theoretical consideration being presented in the following applies to this inner-psychic disposition and examines the social conditions of the possibility to affect (influence) these processes of the definition of the situation.

A second basic assumption of this theoretical approach is that the definition of the situation evolves from cooperative interactive processes for example through resolution of contradictions. At the level of interaction, the processes of "negotiation of meaning" about different interpretations of the situation result in ideas which are taken as shared. In a successful case a "working consensus" is accomplished.

Principally it might happen, that in such a working consens interpretations of the situation are created, which are both taken as shared and go systematically beyond the definition of the situation for at least some of the participants. Frequently in mutually complementing interactional moves a tightly woven sequence of utterance emerges which (a) could not be created solely by single participants, because they could not have produced the contributions of the other participants based of their idiosyncratic definitions of the situation; and (b) leads to the negotiation of meaning, a process which has to be ascribed to the "dynamics" of the interaction process.

Such interactive accomplishments can be transient and the participants might judge them as less important. For the case, however, that these common constitutions of meaning are appearing with a certain regularity and credibility in the sense of a format or a plot, one can assume, that the influence of these social events become more influential for the content related internal processings of the participants: The momentary interactional moves and those to be expected for the next moment as well can be anticipated by such individuals, if they are able to comprehend *the rationality of the whole plot.*

The rationality of the whole process expresses itself through the conduct of an ATS which is already known or which is created novelly. The ATS relates to a sequence of working-steps seen by the proceeding persons as appropriate to expect a plausible result. In the presented episodes this happens in the form of a narrative. The working steps according to the sequence of actions of an ATS are told as far as the participants are able to proceed with these steps. Typically, in a narratively structured ATS the inner logic of the whole endeavor is not expressed explicitly. It is left to the participants to infer the inner logic of the plot

for themselves.

On this base, I elaborate a model of participation in narratively structured classroom-processes as follows: The children are participating in an ATS-governed interaction and they conduct those steps of the interaction that they realize. Through several repetitions of such sequences patterned according to a specific plot, a student might be able to conduct all steps of the ATS by him/herself. This gain of autonomy might be an indicator of successful learning. But it also can evoke an illusion of learning if children do not recognize the similarity of the argumentation presented in the plot of each problem-solving situation, and if they do not construct on this basis a convincing argument for the correctness of all solutions (for more details see e. g. the concept of topos in Krummheuer 1995).

In such classroom situations, the properties of content related learning might not occur consistently or in a complete way. The solution process was the core event. In order to increase the likelihood of successful learning the teacher could initiate repeatedly stories of a similarly structured plot. But possibly only the solving of a certain class of problems defined by its ATS become routinized. Practically the same problem with changing numbers could be solved autonomously as long as the student is able to conduct the single solving-steps. This is the case of the illusion of an increasing autonomy. The essential learning increment, which plays at the level of the plot, must not occur necessarily: learning happens indirectly (Bauersfeld 1995).

This kind of narrative classroom culture is characterized by stability in everyday primary school teaching and learning situations and there are many students who daily proceed successfully in their content related learning development by participating in this classroom culture. In regard to those students who do not proceed successfully, one issue needs to be addressed: It is the fact that writing and application of other illustrating tools are missing. The observed interaction processes are solely based on oral exchange. Bruner 1996 speaks with regard to this point of the necessity of an externalization tenet. "Externalization, in a word, rescues cognitive activity from implicitness, making it more public, negotiable, and 'solidary'. At the same time, it makes it more accessible to subsequent reflection and meta-cognition" (Bruner 1996, p. 24f.).

The starting point of my argument is that in the project episodes preferentially verbal productions can be observed. Generally, the quick evaporation and the situative uniqueness of verbal accomplishments impedes the reflection on such interactive procedures – at least for some, the so-called "weak" students. Complementing such reflections with a written presentation of the result and especially of the work process seems helpful. Bruner 1996 refers to the concept of the œuvre of the French psychologist Ignace Meyerson. Œuvre does not mean a somehow standardized scientific presentation. Rather, it means that the children find a productive form of written presentation of their thoughts by themselves. Œuvres, produced in such a way, facilitate easier listening and possible repetitions, if necessary.

"Creative and productive writing" in such a sense is not only a category of native language classes, but in general a platform for reflection on classroom related processes of symbolization. It is not the question if the children should write down something that is correct in the sense of the subject matter, but rather that the children are to find means for presenting their thoughts which lasts over a longer span of time.

Such classroom cultures provide all participants with well-founded possibilities to negotiate meaning productively and to produce shared meaning. The specific problem might be to identify forms of externalization which enable all students (not only the teacher or researcher) to pursue a specific problem-solving process. Regarding arithmetic, we can refer here to standardized iconic ways of presentation. However, they need to be assessed and enhanced for this special use of providing reflection for the students.

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Vorschau auf Analysethemen der nächsten Hefte

Für die Analysen der Jahrgänge 32 (2000) bis 33 (2001) sind folgende Themen geplant:
- Computergestütztes Lösen offener Probleme im Mathematikunterricht
- Mathematik an Hochschulen lehren und lernen
- Analysis an Hochschulen
- Mathematik in der Ingenieurausbildung
- Theoretische Betrachtungen zu Schulbuchanalysen.

Vorschläge für Beiträge zu o.g. Themen erbitten wir an die Schriftleitung.

Outlook on Future Topics

The following subjects are intended for the analysis sections of Vol. 32 (2000) to Vol. 33 (2001):
- Computer-aided solution of open problems in mathematics teaching
- Teaching and learning mathematics at university level
- Calculus at universities
- Mathematics and engineering education
- Concepts and issues in textbook analyses.

Suggestions for contributions to these subjects are welcome and should be addressed to the editor.