Expanding Context and Domain: A Cross-Curricular Activity in Mathematics and Physics

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Abstract: This article is based on my 15 years of experience as a teacher of mathematics and physics in the Danish Gymnasium (high school), and it gives an example of an interdisciplinary course between mathematics and physics. The course is centered around the concept of exponential functions. The starting point is that concepts are rooted in practice and gain their meaning through application, and the concept of a function is regarded as a tool for modelling real-world situations. It is the intention to teach a course that emphasizes factors that promote transfer of the concept and use of the various representations of the concept, to make it more practical and meaningful for the students. It is concluded that a coordinated cross-curricular activity between mathematics and physics, by offering a great variety of domain relations and context settings, has a great potential for creating a learning environment where the students, through applicational and modelling activities, are engaged actively in constructing and using knowledge.

Kurzreferat: *Erweiterung von Kontext und Bereich: eine fächerübergreifende Aktivität in Mathematik und Physik.* Dieser Beitrag, der auf 15 Jahren Erfahrung als Mathematik- und Physiklehrer an dänischen Gymnasien basiert, stellt ein Beispiel eines fächerübergreifenden Kurses in Mathematik und Physik (Radioaktivität) vor. Im Mittelpunkt des Kurses steht der Begriff der Exponentialfunktion. Es wird davon ausgegangen, daß Begriffe ihren Ursprung in der Praxis haben und ihre Bedeutung durch Anwendung gewinnen. Der Funktionsbegriff wird als Werkzeug zur Modellierung von Situationen der realen Welt angesehen. Zweck des Kurses ist es, Faktoren hervorzuheben, die einen Transfer des Begriffes und die Anwendung der verschiedenen Begriffsdarstellungen fördern, um den Begriff für die Schüler handhabbarer und sinnvoller zu machen. Der Autor kommt zu dem Schluß, daß eine koordinierte fächerübergreifende Aktivität zwischen Mathematik und Physik, die eine Vielfalt von Beziehungen zwischen verschiedenen Bereichen und von Kontexten bietet, ein großes Potential zur Schaffung einer Lernumgebung hat, in der die Schüler durch Anwendungs- und Modellbildungsaktivitäten aktiv Wissen aufbauen und anwenden.

ZDM-Classification: M10, M50

1. Cross-curricular activities in the Danish Gymnasium

In the Danish school system the elementary school includes grades 1 to 9. After 9 or 10 years of elementary school instruction, the students can visit three types of 3 years upper secondary schools, namely Technical School, Commercial School and Gymnasium. The percentage attending the different schools changed significantly over the past two decades with a strong increase of the number of students opting for Gymnasium. On average more than 30% of the students go to the Gymnasium now. When entering Gymnasium, students have to opt for a linguistic branch or a science branch.

In the Gymnasium, structure, organisation and tradition play a dominant role for the possibility of cross-curricular activities. During their three years of instruction the students work on about 15 different subjects ranging from physics and mathematics to music, arts, and sports. Each subject has its own syllabus. Since the 1989 reform of the Gymnasium, several subjects, for example mathematics and physics, are taught on different levels, and some of the levels are optional for the students. This structure greatly hinders the possibilities of carrying out cross-curricular activities. But there has been some valuable pragmatic cooperation between for example mathematics and other subjects.

A possible goal of a cross-curricular activity between mathematics and other subjects could be to improve students' conceptual knowledge by using applications to take advantage of their potential, in a cooperation between different subjects and domains. By a cross-curricular activity I here mean a course where concepts, principles, techniques, or reasoning methods from different subjects are restructured from an educational perspective with a view to organizing an integrated course. The process of integration is preceded by an analysis of the content structure of the subject matter within each of the participating subjects with the question of elementarizing the fundamental concepts and principles as the focal point. This process of differentiation within each of the subjects is then followed by an integration based on didactical principles to present the integrated topic as something accessible to the intended learners.

In describing the possibilities of cross-curricular activities it is convenient to distinguish between three ways of organizing cross-curricular activities:

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a) a cross-curricular activity within one subject with a perspective towards other subjects, where selected elements from the other subjects are adopted in the instruction, for example to provide the instruction in mathematics with suitable sources for application.
b) a cross-curricular activity where coordinated instructional sequences of specific themes are arranged locally between two or more other subjects, for example to coordinate differential calculus in mathematics and kinematics in physics.
c) an activity that goes beyond the subjects, for example in a project work about the topic of pollution, where the participating subjects become indistinguishable.

But as mentioned above, the structure of the Gymnasium dictates the extent to which teachers can utilize cross-curricular activities, so the first approach is carried out regularly, the second rather sporadic, and the third only on special occasions. Christiansen (1998) describes an example of a cross-curricular activty within mathematics in the Danish Gymnasium. I this article I will describe an example of a coordinated cross-curricular activity between mathematics and physics.

2. Interdisciplinary activities between mathematics and physics

The fact that mathematics occurs in applied areas in many other subjects, for example in physics, biology, economics and commerce has made mathematics a suitable subject for cross-curricular activities. In the Danish Gymnasium there was a long period, i.e. from the sixties to the beginning of the eighties, with close relations between mathematics and physics instruction. Especially in grades 11 and 12, coordinated cross-curricular activity between mathematics and physics provided numerous opportunities for teaching applied calculus. However, since the reform of 1989 these relations have become weakened or disappeared in many places. This is partly due to the opening of mathematics instruction to other applicational subjects than physics, but also due to the above mentioned structure of the Gymnasium. In mathematics as well as in physics, there are two teaching levels, so from grade 11 not all students in a classroom follow the same courses in mathematics and physics. This only leaves space for interdisciplinary courses between mathematics and physics at the first year of Gymnasium instruction. Of course another reason lies in the fact that the number of teachers who teach both mathematics and physics is decreasing.

Blum and Niss (1989) emphasize the importance of maintaining the close instructional contact between mathematics and physics at school level:

"It may be said, somewhat paradoxically, perhaps, that the more mathematics is being applied to areas and subjects outside physics, the more important it is to have access to representative cases from physics to shed light on possibilities, conditions, difficulties and pitfalls of application and modelling in fields with smaller degrees of well-established mathematisation". (Blum & Niss, 1989, p. 17)

This prompts me to focus on the fact that in the teaching of both mathematics and physics, and other subjects as well, the modelling techniques that emphasize problem-solving processing and embedding this instruction in an existing subject matter domain plays an increasing role. It is a well-established view among the teachers of mathematics in the Danish Gymnasium that the incorporation of modelling aspects is well suited to assist students in acquiring mathematical concepts by providing the motivation and relevance of mathematical instruction. Furthermore, it also contributes to training students to perform mathematical techniques in different contexts, within and outside mathematics. Also in the guidelines from the Ministry of Education, it is stated that the "model aspect"

"... must give students knowledge of the construction of mathematical models as representations of reality and impressions of the possibilities and limitations of applying mathematical models, and also equip them to accomplish a modelling process in simple situations". (Translated from Bekendtgørelse, 1997)

So during their three years of mathematics instruction, the students are required to work with mathematical models. The applicational aspect of mathematics has entered the written examination, though only to a modest extent and in the form where the task for students seems to be to perform a standard mathematical exercise disguised in applicational clothes.

3. The concept of function in a cross-curricular activity

In mathematics education of the Danish Gymnasium the concept of function is placed in the centre of attention. It has a central and organizing role around which many other important mathematical ideas resolve. In the majority of textbooks used for instruction, a function is defined by two sets A and B with a rule which assigns exactly one member of B to each member of A. The textbooks are still the primary organizer of the mathematics instruction in the Gymnasium. This gives a restricted view of functions by the dominance of an algebraic symbolism, where the function is exclusively presented in the $f(x)$ form.

It is rare that the students are given the opportunity to acquire a perception of a function as an appropriate tool to organize the physical world. Of course many teachers are aware of this formalistic approach, and there are teachers with a pragmatic view of the function concept as a tool to model real-world situations by forming a mathematical representation of the quantitative and qualitative relationships in that situation. The view that functions should first appear as models of relationships and that the textbook definition of a function is too formal and abstract for the student, is supported by Vinner (1991) and Sierpinska (1994). In addition the formal approach gives the students the impression, that a function has to be described by a formula, and that the only possible actions on them are algebraic. Although the function is described as a unifying concept, functions appear and behave in different ways. There are different representations, and according to Schwarz & Dreyfus (1995) and Confrey & Doerr (1996) research has shown that the rich diversity of multiple representations leads to a more robust and flexible understanding of functions. The most common representations of functions are graphical, algebraic and tabular representations. To my view the verbal representation of the function should also be among the common representations, to

let the students develop the ability to reason qualitatively about the concept. Actually concepts are rooted in practice and gain meaning when they are subjected to negotiation, interpretation and application.

I will here describe an example of a cross-curricular activity between mathematics and physics, which presents a view of the function concept as a tool to model real-world situations.

4. An example: Exponential growth in 10th grade

The example is centered around a class of 28 15-16 year-old students. The course was organized about introducing the concept of exponential functions in mathematics and radioactivity in physics to the students. At this time, the students had been introduced to the function concept and they were familiar with proportionality and linear functions. The teachers of both subjects participated in the course.

In the first lesson, after an orientation by the teacher of physics, where a number of everyday life situations in which the use of ionizing radiation might be an issue was discussed, and the basic information and skills about the nature, effects and sources of X-ray and radioactivity were introduced. The random nature of the process of radioactive decay was introduced by an analogy: The students were asked to consider a situation where a large group of students are in a room: Each student represents a radioactive nucleus, and they all throw dice, saying "one to five I stay, six I leave the room", and the process is iterated.

In the following lesson the students worked with ionizing radiation. The starting point were two well-known experiments from physics: Measuring with a monitor the decay of a radioactive source and absorption of gamma rays in metal. Students worked in groups of five, and their first task was to collect the experimental data. Each group had to carry out four experiments: absorption of gamma-rays in lead and in aluminium plates of different thickness and the decay of two different radioactive sources. The data was put on tabular form, and the students now had to discuss the relations between the variables of their experiments. They plotted the points in a normal system of co-ordinates, and many of the students seemed to be in favour of assuming linearity, and the main topic of the group discussions was whether or not it was reasonable to assume this. However nearly all groups ended up with four curved graphs, and in the following open class discussion the groups presented their work, and they had to give a verbal description and interpretation of their graphs. Also models for the two processess were discussed. There was agreement on a model for the decay processes, where the change in the number of nuclei in a short time interval is proportional to the number of nuclei present, and a model for the absorption process, where the change of intensity in a narrow metal plate is proportional to the intensity. By discussing the similarities between the two models, it was now possible to formulate "the exponential model": The relative change in the dependent variable equals a constant value for a fixed increase in the independent variable.

Now the context and domain was changed. The groups had to work with deposit on a bank account with a fixed rate of interest and with the reduction of the value of a car by a fixed percentage every year. Most of the groups used their calculators to set up a tabular representation, and then plotted the graphs. But several of the students were familiar with calculations with percentage, so they were able to set up the formula for extrapolation of the account, $k_n = k_0(1+r)^n$, and correspondingly for the car value, $k_n = k_0(1-r)^n$, where n stands for the setting period, r for the percentage divided by 100, k_0 for the start deposit account/value, and k_n for the deposit/value after n settings. When discussing the results of the group work, focus was on the exponent in the expressions for k_n. Until this moment the students were only familiar with powers of integer values, and it caused problems when the students for example wanted to calculate the deposit on the account after 2 years and 5 months. The students had to realize that their original conception was inadequate, and the job of the teachers was to help the students to expand the concept of power to a situation with a real valued exponent.

Later on the students worked out standard problems to be familiar with the new concepts. Then the teachers again turned the students' attention to the expressions for k_n, and the relationship between the two variables n and k_n pointed towards the concept of function. The students now were able to write down the well-known algebraic formula for the exponential functions: $f(x) = ba^x$.

Several of the students had already noticed the similarities between the graphs from their experimental work and the graphs of k_n versus n, and the students were now asked to substantiate their statement. By interpreting their results in group discussion the students made backward references to the original "exponential model", and exposure to the new concept of exponential functions. In this work the students were also introduced to the semi logarithmic paper. Focus was not on the construction of the logarithmic axes, but on the different aspects of a graph, for example, individual points versus more global features. Finally in the class with the teacher mainly functioning as a chalkholder, it was concluded that for an exponential function the relative change of the dependent variable equals a constant value for a fixed increase of the independent variable. Now the students could classify the processes of absorption and decay, and write formulae for the intensity I, and the number of nuclei N : $I = I_0a^x$ and $N = N_0a^t$, where x stands for thickness of the metalplates, t for time, and I_0 and N_0 for the starting values. By looking on the semilogarithmic plots, concepts like half life and half length were introduced.

When discussing the formulae for the intensity and the number of nuclei, some students asked the question: So the numbers we observed at the display window when we were measuring the decay were the number of radioactive nuclei? After discussing this question the students agreed that it is an impossible task to count the numbers of nuclei. But by making reference to the experimental equipment, especially the monitor and the fact that the students had knowledge about charged particles, the focus was now turned on the number of nuclids decaying in some fixed interval of time, and the concept of activity. There was

general agreement about proportionality between activity and the number of nuclei. After the formulation of the appropriate algebraic formula for the activity as a function of time, some of the students turned attention to the constant determined by the conditions of the situation when the value of time is zero. The effect of "starting the watch at another time" was discussed. So the introduction of the concept of activity offered an opportunity to study two processes on the exponential functions: multiplying by a constant and translation of the independent variable.

In the following class lessons the results were summarized and concepts as dose and absorbed dose were introduced. In the discussion the students revealed a wide knowledge and interest about radiation effects. Especially the Tjernobyl nuclear power plant accident turned up several times and the following question put into the discussion: What is the effect of taking iodine pills after a nuclear power plant accident? This was the starting point for focusing on a radioactive decay chain consisting of a radioactive nucleus decaying to a radioactive nucleus (iodine), which again decays to a stable nucleus. The teachers refered to the analogy from the start of the course: The students leaving the room go to another room and throw dice, saying: "One to four, I stay. Five or six, I leave", and the process here too is iterated. It was now decided to round off the course with modelling the decay-chain.

In the groups the students using the concepts previously discussed pursued a number of approaches to determine more facts about the decay-chain. The students had to revise the construction of the model for the decay process, and nearly all the groups were able to describe the problem. But especially the first step in the decay-chain caused problems. It was difficult for the students to cope with a situation, where a number of nuclei had to be added, caused by the decay of the first member of the chain, and at the same time were decreased by an exponential factor. The students were very uncomfortable with this situation, so when one of the groups by revising the original model of the decay process was able to reformulate the problem to a set of difference equations, it was decided that the whole class should go on with this idea. The system of equations could be programmed directly with a spreadsheet, so by the end of the course all groups were able to present a graphical representation of the problem, showing the exponential decrease in the number of the first group of nuclei and after about a week a maximum of the number of nuclei from the second group (iodine).

When the course was completed, each of the groups was asked to deliver a written report about their work in the course. The students were requested to evaluate and criticize their models in the report. Here the students first of all pointed towards the unrealistic situation of considering the rate of interest as a constant for a longer period. But several groups also mentioned that radioactive pollution from a nuclear power is a rather complex problem to model as you have to include factors like wind force, wind direction, distance, etc.

5. Formation of the concept of exponential functions

The goal of the course was to give the students understanding and insight into the concept of exponential functions through experience gained from their own experimental activities. The students were dealing with a collection of objects belonging to the areas studied, and then, by the process of mathematization, these were transfered into mathematical objects and relations. This mathematization is an essential part of the curriculum, argues Freudenthal (1973). Through the processes of application of the exponential model, the students gradually developed the concept of exponential functions. In the example presented, the students were forced to explore the experimental situations and to develop a model resulting in the required concept. This process of conceptual mathematization will, according to de Lange (1996), help the students to better acquire and understand mathematical concepts. Based on the students' source knowledge gained through actions by reflection and generalizing, the students will develop a more complete concept on which the more formal mathematical notion can be based.

A current view is based on the idea that any development of a mathematical concept is a process that starts with an action on objects. Recent focus has been on the process of acquisition of a mathematical concept, and there are several theories (Dubinsky 1991, Sfard 1991, Schwarz and Dreyfus 1995) assuming that the acquisition of knowledge starts with actions, some of which become processes and then later are conceived as objects, which on a higher level are starting points of new actions. The theories of concept formation by Dubinsky and Sfard all contain an aspect of decomposing the subject matter into a learning sequence with several levels of understanding. To construct the concept, the students have to pass through these levels.

According to the theory of Sfard, a mathematical concept, e.g. a function, has an inherent process-object duality, and the concept can therefore be conceived in two fundamentally different ways: operationally as processes and structurally as objects. This model of concept formation implies that a mathematical concept like exponential functions should only be regarded as fully developed when it can be conceived both operationally and structurally. Sfard emphasizes that the operational view of a concept in terms of a process to be carried out seems to precede the structural view using objects and formal definition, both in the cognitive and historical development of the concept. Here, one should only add that after all the historical starting point of many mathematical concepts is more or less a practical problem. The formation of the concepts is described by a hierarchical scheme composed of three stages: interiorization, condensation and reification.

At the stage of interiorization the students get acquainted with the processes which will give rise to the concept. With the physical experimentation as a starting point in the example, the students are engaged in gathering data and posing conjectures about the relevant variables and their relations.

In the phase of condensation, the learner becomes capable of seeing the process as a whole, and capable of al-

ternating between different representations of the concept. In the example, the instruction of the students is based on a systematic use of several representations of the function concept, and the process of switching representations is stressed from the beginning in order to let the students grasp the view that verbal, graphical, algebraic and tabular representations of the function are distinct representations of a single object.

Reification of a given process occurs when the process solidifies into an object, and simultaneously with the interiorization of a higher-level process. Reification is the final stage in the acquisition of the concept, and it brings relational understanding of a concept. It is argued by Sfard that reifying is a difficult process, and that great effort is required to achieve it. In the example, the emphasis is on the representations giving the students the time demanded to get through the phase of condensation. But also the first operations on the object take place, i.e. translation of the independent variable and multiplying with a constant value. So some of the activities in the condensation phase are already directed towards reification. And in the final part of the course, the students perform more complex operations on the exponential functions, when forming their models of the radioactive decay chain.

The question arises, how can the teacher be sure that the students have reified the concept of exponential function? It is of course difficult, if not impossible, to observe the actual transition from one stage to the next, so the teacher has to focus on which stage the students are in. In that connection I want to stress that qualtitative reasoning is an important qualification. Students regularily solve a problem posed in explicitly quantitative terms, but they are seldom asked for a qualitative analysis of the same problem. To strengthen the qualitative aspect in the example, the verbal representation of the function was placed among the common representation, and the students were discussing with other students and with the teachers, and in written reports, motivated, and forced, to interpret and seek meaning throughout the course. In addition qualitative reasoning of the students allows the teacher to probe for students' conceptual understanding, and allows students to work on tasks that require them to explore their reasoning. This will offer the teacher a splendid possibility to identify the students' stage of concept formation.

6. Competences of transfer and broadening

In the learning activities of the example, the students constructed the concept in an applied environment with the perception of a function as an appropriate tool for organizing the physical world. This is in agreement with the function model proposed by O'Callaghan (1998). Rooted in a problem-solving enviroment, the model consists of four component competencies: modelling, interpreting, translating, and reifying. But also the competences of transfer and broadening should be mentioned. Transfer refers to the situation where the concept is transferred to other context settings, and broadening to process of expanding the domain of concept validity.

It is a well-known fact that the context in which knowledge develops influences the extent to which the knowledge can be applied in other contexts. Greeno (1992) draws attention to the fact that what he calls inert knowledge cannot be applied in contexts other than the formal context in which it was learned. In the example, the concept was first introduced in a simple modelling context, followed by a more formal mathematical context, and then in a more complex modelling context, to let the students acquire a repertoire of significant conceptual and procedural knowledge, but also an ability to transfer their knowledge from the specific contexts in which it is presented to a new and apparently different setting.

By the learning activities of the example, the students were led to construct the concept of exponential functions by a process, where the individual student synthesizes different aspects of the concept in different domains. Dreyfus (1991) emphasizes the process of synthesis and the need for activities designed to lead the students to synthesize different aspects of a concept, or different concepts within a domain or even within different domains. By introducing the exponential functions as a model of absorption of gamma-rays, a model of calculation of interest etc., the domains of validity for the concept are gradually broadened. This perspective is emphasized by Niss (1997, chapter 2, p. 8) in

> "The First Main Finding of the Didactics of Mathematics: When a pupil or student engages in learning mathematics, the specific nature, content, range, and flavour of a mathematical notion or concept that he or she is acquiring or building up are greatly influenced, if not determined, by the domains in which that notion or concept is anchored and imbedded for that particular person".

So the more extended the contexts and the richer the set of domain relations, the more comprehensive and multifaced will the concept be. Here we have the power of interdisciplinary activities. They offer a rich set of domains, and a wide range of different context settings.

7. Creation of knowledge in the classroom

The view of learning presented in this example is to consider learning as an active process of acquiring knowledge, where the students experience the process of constructing knowledge not only by reading about results from other people, but first of all by being actively engaged in knowledge construction themselves. It is, on one hand, an active, strictly personal process of mastering the subject matter, but also an active social process experienced during discussions with other students and teachers.

Molander (1993) introduces the concepts of technical knowledge and directive knowledge as analytical tools for understanding knowledge. Types of technical knowledge are instrumental knowledge and mastery of technique. It is characteristic for these types of knowledge that they do not themselves lead to direct action. It is the type of knowledge brought into play when the students are solving standard exercises, and perceive the quantitative aspect of the solution as being primary. Focus is not on analysis and understanding of the question posed, but on the answer. It is an educational idea of acceptance of answers without questions, which is often present in the problems of the written national examination held at the end of grade 12. Here we have only to cite Cobb (1988, p. 4):

"... students who have constructed instrumental beliefs about mathematics (...) anticipate that future classroom mathematical experiences will 'fit' these beliefs. They intend to rely on an authority as a source of knowledge, they expect to solve tasks by employing procedures that have been explicitly taught, they expect to identify superficial cues when they read problem statements, and so forth. Alternative ways of operating do not occur to them".

Opposite to the technical knowledge is the idea of a directed knowledge-forming process taking place in a dynamic question-and-answer frame of increasing skill, insight and enlightment. In my example, the concept of exponential functions is formed in an interdisciplinary environment directed towards applications of the concept in a broad range of circumstances, and in a variety of different situations. It is an integrated modelling process that emphasizes questions as well as answers, and the construction of the appropriate concepts and laws as they are needed. The goal is to encourage students to seek answers by trying to make sense out of the questions through reflection and through interactions with the teacher and other students. This will offer students the opportunity to make conjectures, to interpret and to integrate existing knowledge.

Following the view that acquiring knowledge must be a dynamic, directive and orientated process, it is the task of the teacher to create a learning enviroment which provides the students with space for inquiry and interpretation in situations that are meaningful for the students. In their model of constructive learning, Glyn & Duit (1995) argue that learning meaningfully calls for five conditions to be present: (a) existing knowledge is activated, (b) existing knowledge is related to educational experiences, (c) intrinsic motivation is developed, (d) new knowledge is constructed, and (e) new knowledge is applied, evaluated, and revised.

8. Conclusion

This example shows that cross-curricular activities between mathematics and other subjects need not necessarily be centered around the use of mathematics as a tool for calculations. By describing an interdisciplinary activity between mathematics and physics, I want to point out that cross-curricular activities have a large potential for creating a learning environment in which the students, through applicational and modelling activities, are engaged actively in constructing and using knowledge. Regarding the concept of function as a tool for modelling real-world situations, the interdisciplinary activity offers a variety of different contexts and a rich set of domains to let the students investigate a broad range of circumstances to properly refine models and develop the concept. Using this framework, conceptual development is promoted by placing emphasis on the students' discussions of the questions. And the role of the teacher is to help the students to pay attention to those ideas that will help them reach a satisfactory conclusion.

In my opinion, an important goal of teaching mathematics, and other subjects too, is to create meaningful situations, where the students through exploratory activities develop conceptual knowledge. Cross-curricular activities offer such situations, so the curriculum must create the space that is needed to encourage cross-curricular activities. Also, in the Danish Gymnasium it is a well-known fact that instruction is greatly influenced by assessment methods. Strangely enough, a test of the students' ability to perform active modelling processes and to transfer skills learned with one set of problems to a different set of problems is only very seldom on the agenda of assessment in the Gymnasium. Of course it is important to focus on assessment methods to take into account that the students now solve many problems using technological support, but why not bring up for debate how the evaluation procedures could be broadened so that the students are also given exam credit for reasoning? In my view students' written reports and students' portfolios constitute a reasonable category of tasks and activities to be subjected to an assessment – i.e. the students are given credit for inquiry, reasoning and interpretation.

Finally, I want briefly to touch on the following problem: Even teachers who have been exposed to the ideas of cross-curricular activities have seldom had the opportunity to carry out such activities. The structural organization of the Danish Gymnasium calls for creativity of the teachers to organize a cross-curricular activity between two, or more, subjects. Nor is there any time in the hectic school day to design such activities. If teachers are expected to apply their ideas into practice, they need to be knowledgeable about educationally relevant theory, and to have the time to design interdisciplinary instruction.

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