Developing Mathematical Thinking Socio-Culturally

Nunes, T.; Bryant, P. (Eds.):

Learning and Teaching Mathematics
An International Perspective

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During the early decades of this century, as mathematics education was seeking to establish its place in the university as a field of scholarly endeavor, it relied heavily on the young "master science" of the school-psychology – to provide theory, problems, and methodology. Connectionism, for example, yielded studies of arithmetic learning in which experimental designs were used to assess the effects of drill; gestalt psychology yielded studies of mathematical problem solving that relied on introspective accounts of thinking. Through numerous investigations of mathematical learning, transfer of training became as much of an issue for mathematics educators as it was for educational psychologists. By mid-century, developments in cognitive psychology as well as in the genetic epistemology of Jean Piaget were helping to expand and enrich the realm of research in mathematics education. During the 1970s and 1980s, however, many mathematics education researchers abandoned psychology and turned to other disciplines – notably, anthropology, sociology, linguistics, philosophy, and history – to orient their work. They were dissatisfied with what they saw as psychology's narrow interpretation of mathematics and its excessive focus on the individual learner.

Meanwhile, a number of psychologists began to consider mathematics from a broader perspective, looking at how it is learned and used not only in the schoolroom but also in the marketplace and home. They expanded their view of school mathematics beyond elementary computations with whole numbers and the solution of textbook word problems to consider how people cope with complex mathematical tasks both in and outside school. *Learning and Teaching Mathematics: An International Perspective* presents a compilation of recent work reflecting this expanded perspective on mathematics, its learning, and its use. The editors, Terezinha Nunes and Peter Bryant, are two eminent psychologists whose own work has helped strengthen the ties between psychology and mathematics education. The 26 authors are drawn from the ranks not only of developmental psychologists and social psychologists but also of researchers in mathematics education who have continued to adopt and adapt psychological constructs and methods.

The book is divided into four parts. The first part provides theoretical perspectives on mathematics and intelligence. The second contains articles that examine the development of understanding in different realms of mathematics from nursery school through secondary school. In the third part, the scene shifts to social and cultural influences on the learning of mathematics, with several of the chapters looking at mathematics learned outside school and its relation to school mathematics. The chapters in the final part enter the classroom, considering how knowledge is constructed there. In all, the book contains 15 chapters. Most of them attempt a review and synthesis of the literature on some topic, but a few report original research.

The authors come from 10 countries. The countries represented by the authors are France, the United Kingdom, Belgium, Brazil, the United States, Canada, Japan, Australia, Switzerland, and the Netherlands. That listing is based on the authors' current affiliations; China, Korea, Colombia, and Portugal would apparently also be included if countries of origin were counted. (Six of the U.S. authors are missing from the contributors' list given in the book, all of them presumably current or former students of the senior U.S. authors of their chapters. It is odd not to have their contributions acknowledged, particularly since two of them were the first two authors on one chapter. In the U.S., apparently, not all authors are created equal.) Although many countries in which research on the learning and teaching of mathematics is well established are missing from the list, the claim for an international perspective made by the subtitle is reasonably accurate.

The main title, however, is somewhat misleading. The book deals more with mathematical understanding and reasoning than with learning, and it deals more with the organization and engineering of instruction than with teaching. A more accurate, if less appealing, title would have been something like "The Socio-Cultural Development of Mathematical Intelligence". Also somewhat misleading is a claim on the back cover that "one theme, which will be new to most readers, is the importance of the 'didactic contract' between teacher and pupils". The didactic contract is a theme of one chapter, a thoughtful account of the development of their own research on social interactions and mathematics learning by Maria Luisa Schubauer-Leoni and Anne-Nelly Perret-Clermont in Part III. It is also touched on but not elaborated in the book's final chapter, by Régine Douady, on didactical engineering. But the didactic contract plays essentially no role elsewhere in the book.

Doudy's chapter illustrates and amplifies one of the true themes of the book, the contrast between mathematics as tool and mathematics as object, or in Anna Sfard's terms, as cited by Carolyn Kieran in her chapter, the process and object conceptions of mathematics. The tool-object contrast is also explicated in the first chapter, by Gérard Vergnaud, on the nature of mathematical concepts. Vergnaud introduces his theory of conceptual fields, in which the concepts of scheme, operational invariant, concepts-in-action, and theorems-in-action play a central role. Taken together, the chapters by Douady and Vernaud provide one of the clearest, most articulate introductions available in English to the central ideas of French research in the didactics of mathematics.

In their introduction to Part I, the editors characterize mathematical knowledge as a form of intelligent behavior that involves reasoning, problem solving, and understanding; the processes of perception and memory are seen as belonging to a different realm of behavior. “The view of mathematical knowledge as intelligence excludes from the set of observations to be explained, for example, comparisons of numerosity that can be accomplished perceptually or the mechanic memorisation of addition and multiplication tables through repetition” (p. 2). This characterization of knowledge negates the common distinction, promoted by educators such as Ralph Tyler and Benjamin Bloom, between knowledge and “higher order” thinking. For Bryant and Nunes, knowledge *is* higher order thinking. Their exclusive focus on mathematical reasoning and problem solving, however, although a welcome correction to experimental psychology’s historically narrow view of mathematical behavior, risks neglect of the role that perception and memory play in building mathematical knowledge, however defined. Moreover, using the terms *mathematical knowledge*, *mathematical reasoning*, and *mathematical intelligence* interchangeably does not represent a step forward in our thinking about these constructs.

In addition to Vergnaud’s chapter, Part I contains a chapter by Nunes entitled “Systems of Signs and Mathematical Reasoning” in which she develops a theory of mathematical “knowledge” built on the idea of mediated action. For Nunes, thinking is carried out through representations and not directly through actions on situations or objects. How mathematical thinking relies on specific forms of representation is a theme that runs through the volume, although explicit attention to systems of signs, or even signs themselves, is rare. Like Vergnaud, Nunes stresses the importance of invariants, but unlike Vernaud, she also emphasizes that such invariants are constructed by the problem solvers and depend on how the problem has been represented.

Many chapters in the book deal with some aspect of mathematical understanding and reasoning in a social context. An example of a chapter that succeeds admirably in synthesizing the relevant literature is Kieran’s chapter in Part II on the learning of algebra and functions. She examines the distinction between these two domains of mathematics, considers the literature relating to each separately, and argues, using the process-object model, for a more integrated approach. Like most of the other chapters in Part II, the emphasis is more on the mathematical ideas and how students think about them than on how students come to know them. Another example of an notable synthesis, this one in Part III, is Giyoo Hatano’s “Learning Arithmetic With an Abacus”. This chapter sits somewhat uneasily with the others in that it deals with the overlearning of a form of calculation largely excluded from mathematical knowledge as the editors have characterized it. Nonetheless, it is a lovely essay that does, as the editors say, demonstrate “the power of the cultural tool, the abacus, in shaping the mental processes used during calculation” (p. 161).

An example of a report of original research that does not attempt a larger synthesis is the chapter in Part III by Guida de Abreu, Alan Bishop, and Geraldo Pompeu, Jr., entitled “What Children and Teachers Count as Mathematics”. The chapter reports on a study by Abreu demonstrating that third graders and sixth graders in Madeira had learned that “the school culture does not acknowledge ... out-of-school practices as proper mathematics” (p. 252). It also reports on Pompeu’s successful implementation of an “ethnomathematical” approach that allowed teachers in Brazilian schools to capitalize on rather than deplore the contrasts between school and out-of-school mathematics. The authors wonder why out-of school mathematics is disfavored, but in Abreu’s study the students were asked to solve problems as they would if asked by a teacher, so it is hardly surprising that they used school-taught approaches. That quibble aside, the chapter raises important issues about how schools validate certain forms of knowledge and discredit others.

The theory of “realistic mathematics education” developed at the Freudenthal Institute in the Netherlands is highlighted in Part IV. The late Leen Streefland illustrates, through selected tasks and children’s responses to them, a new course on fractions that has been developed at the Institute in light of the theory. Streefland suggests that the chapter provides a framework for other courses that might elaborate the approach in different ways. Evidence is not provided in support of the approach; rather, the chapter attempts to provoke questions and raise hypotheses. It does not elaborate the theory. That is done by Koeno Gravemeijer in a chapter that comes across more as advocacy than as analysis. Gravemeijer dismisses the use of material objects and representations (so-called manipulatives) to help students acquire abstract mathematical knowledge, contending flatly that “research has shown that this approach does not work” (p. 318). Instead, one needs a “bottom-up” approach in which the students take the initiative in developing models themselves rather than having the teacher or the text do it for them. Fortunately, such an approach is at hand in the form of realistic mathematics education, so the errors of Montessori, Dienes, Bruner, and others can now be corrected. Unfortunately, Gravemeijer’s view of instruction clashes with the tool-object contrast mentioned above: “From a constructivist point of view, a drawback [of connecting students’ informal knowledge with a mathematical system such as the system of decimal fractions] is that prefabricated knowledge is taken as an immediate goal for instruction. That is, for instance, expressed by the use of ‘tools’ as a metaphor” (p. 319). The clash seems not merely semantic. Tools for doing mathematics are socially constructed inside and outside the classroom. As Vergnaud says, “In mathematics, concepts usually emerge as tools” (p. 27). To the extent that Gravemeijer’s realistic mathematics education cannot handle the tool-object dialectic in mathematics learning, it seems curiously unsuited to a volume that promises, on the back cover, “for the first time in English a comprehensive description of teaching methods based on the idea of social construction”.

As the remarks above suggest, the chapters in this book, as in most edited books in the field, do not fit together as smoothly as one might wish, despite the editors’ efforts to find common threads linking them. The book also shows some signs of the haste that seems to go into almost all scholarly book production these days. For example, page 195 ends in the middle of a sentence, and a new section begins at the top of 196. The subject index is incomplete and occasionally inaccurate (try looking up “didactic contract”). Nonetheless, the book provides a valuable survey of recent work that takes mathematical thinking as socially constructed and culturally embedded. It belongs in the library of every mathematics educator who wants to know where the field of mathematics education stands vis-à-vis psychology as the century ends.

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