

Rotman, Joseph:

Journey into Mathematics An Introduction to Proofs

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Michael de Villiers, Durban (South Africa)

One of the main purposes of this book is to help the (novice) reader at the undergraduate level how to read and write proofs. The author points out at the beginning that a major function of proof is that of explanation; i.e. explaining why a result is true. On the other hand, he also uses some very good examples early on to illustrate the limitations of inductive reasoning. For example, he gives a spectacular example involving a special case of Pell's equation for which the first n for which it is false has 1115 digits! The author subsequently focuses on the verification function of proof by introducing mathematical induction as one method for checking the validity of a mathematical statement for *all* n .

The book continues with a discussion of binomial coefficients, polygonal areas, the irrationality of square root 2, the Pythagorean theorem, Pythagorean triples and the Diophantine method of finding Pythagorean triples. The author presents all these topics by interweaving historical information which makes it very entertaining; in fact, this is a strategy used effectively throughout the book. When some more difficult proofs are encountered on p. 15, the author eloquently encourages the reader as follows:

“Mathematics is not the realm of a few ‘magicians’; you are not expected to compete with Archimedes, Gauss, Hilbert, and Poincaré. Each of us is inventive to some degree, and the more one learns, the more proficient one becomes. In music, we can listen and thrill to the beauty of Bach, Mozart, and Beethoven. Even though we cannot compose the sonatas for unaccompanied violin, Don Giovanni, or the late quartets, we can still sing.”

Next the author passes on to disks, proving the area and circumference formulas, due to Eudoxus and Archimedes respectively. The limitations in these early notions of approximation are pointed out, and the reader is made to see how the modern view of convergence remedies these classical defects. The reader is then led through a discussion of the quadratic formula, complex numbers, the cubic formulas of Cardano and Vieta respectively, and the quartic formula. The text ends with proofs of the irrationality of e and π , which is essential reading for every high school mathematics teacher. Thus, geometry, algebra, number theory, and analysis are all intertwined in this entertaining journey.

A major feature of the book which makes it particularly relevant to novices, is the way in which the historical background has been interwoven into the discussion. It shows how techniques and understanding developed over time, with several people building on the work of their predecessors. It also seems to illustrate that some mathematical ideas and results have a Platonic, ideal existence, not de-

pendent on the subjective opinions of people considering them. For example, it comes as a surprise to many people to learn that in 1897 the State of Indiana passed a law which attempted to legislate the value of π to be 4.

Several exercises are given in the book where the reader can apply and extend the ideas discussed in a chapter. This clearly forms an integral part of the book, and the author correctly points out that merely reading about mathematics is no substitute for doing mathematics. The content ranges from the elementary to the fairly advanced, making it an excellent text for an undergraduate course. The author though might have considered a different ordering of the topics, starting instead with Pythagorean triples, areas and trigonometry before moving to mathematical induction which a lot of students seem to find difficult. However, the chapters do not follow a strict linear order, with some chapters being relatively independent from those immediately preceding it, so that any instructor could select and order the chapters in a variety of ways. In conclusion, this book is a valuable contribution to the teaching of proof at the undergraduate level, and strongly recommended.

Author

Villiers, Michael D. de, Prof., University of Durban-Westville,
Department of Applied Curriculum Studies, Private Bag
X54001, Durban 4000, South Africa.
E-mail: mdevilli@pixie.udw.ac.za