From Formal Standards to Everyday Practice of Mathematics Learning
Illustrations from the TIMSS Case Study Project in Japan

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Abstract: This paper uses the example of six Japanese teachers and their mathematics lessons to illustrate how clear, high standards for mathematics instruction are combined with teachers' holistic concern for students. We draw upon data from the Third International Math and Science Study Case Study Project in Japan that was designed to elucidate the context behind the high achievement of Japanese students. Using everyday examples of classroom practice, we illustrate both flexibility in teachers' approach to teaching and adherence to Monbusho's (Ministry of Education, Science, Sports, and Culture) *Course of Study.* Our purpose is to emphasize how flexibility and attention to individual needs by Japanese teachers combine with quality mathematics instruction based on the detailed Japanese curricula.

Six teachers' characteristics and lessons (two teachers at each educational level – elementary, junior high, and high school) are described in order to show the variety of teachers who exist in Japan. These teachers use their understanding of the *Course of Study* and are supported by their school environment to enhance their students' conceptual understanding of the fundamentals of mathematics. Characteristics of their teaching include: 1) involving the whole class in learning, 2) using extremely focused curriculum guidelines that expect mastery of concepts at each grade level, 3) thoroughly covering mathematics units in an organized and in-depth manner, 4) leading classes as facilitators or guides more often than as lecturers, and 5) focusing on problem solving with the primary goal of developing students' ability to reason, especially to reason inductively. The examples in this paper show how these methods develop in individual classrooms.

Kurzreferat: *Von formalen Standards zur täglichen Praxis des Mathematiklernens. Anschauliche Beispiele des TIMSS-Fallstudien-Projekts in Japan.* Der Beitrag zeigt anhand von sechs japanischen Lehrern und ihrem Mathematikunterricht wie klare, hohe Standards für den Mathematikunterricht mit einer ganzheitlichen Sichtweise im Hinblick auf die Bedürfnisse der Schüler kombiniert werden. Grundlage sind Daten des TIMSS-Fallstudien-Projekts in Japan, mit dessen Hilfe Ursachen für die hohe Leistung japanischer Schüler herausgefunden werden sollten. Alltägliche Unterrichtsbeispiele zeigen sowohl die Flexibilität der Lehrer in der Auswahl von Unterrichtsmethoden sowie ihre enge Anlehnung an curriculare Richtlinien des Kultusministeriums. Insbesondere soll damit das Zusammenspiel von Flexibilität und Beachtung individueller Bedürfnisse durch japanische Lehrer und gutem, auf den detaillierten japanischen Curricula basierendem Mathematikunterricht herausgestellt werden.

Merkmale und Unterrichtsstunden von sechs Lehrern (je zwei Lehrer pro Schulstufe – Primarbereich, Sekundarstufe I und II) werden beschreiben, um die Verschiedenartigkeit japanischer Lehrer zu demonstrieren. Diese Lehrer nutzen ihr Verständnis der curricularen Richtlinien, unterstützt durch die Schulumgebung, um das begriffliche Verstehen mathematischer Grundlagen durch die Schüler zu fördern. Ihr Unterricht ist u.a. charakterisiert durch: 1) Einbeziehen der ganzen Klassse beim Lernen, 2) Anwendung extrem detaillierter curricularer Richtlinien, die die Beherrschung von Begriffen auf jeder Schulstufe verlangen, 3) sorgfältig organisierte und in die tiefe gehende Behandlung von Mathematikeinheiten, 4) Leitung der Klasse eher als Berater statt als Vortragender, und 5) Schwerpunktlegung auf das Problemlösen mit dem Hauptziel, die Denkfähigkeit der Schüler, insbesondere zum induktiven Denken, zu entwickeln. Die Beispiele dieses Beitrags zeigen die Entwicklung solcher Methoden in den einzelnen Unterrichtsstunden.

ZDM-Classification: D10, D40

Japanese students continue to astound researchers and educators alike with their consistently high scores on various international assessments of mathematics achievement. Although ousted from their previous number one ranking by Singapore and Hong Kong in the Third International Mathematics and Science Study (TIMSS), Japan still presents a formidable challenge to the other industrialized nations of the world in terms of students' performance on mathematics achievement tests. Among the 41 countries participating in the Population 2 (eighth grade) assessment of the TIMSS, Japan was ranked first in geometry; second in number sense, algebra, and measurement; third in data analysis and probability; and fourth in proportionality (Peak, 1996). Japanese fourth graders achieved at equally high levels, placing third among a total of 17 nations.

So, what explains these consistently high scores? From genetics to cultural beliefs, explanations concerning Asian mathematics superiority have been numerous and varied. However, converging evidence seems to suggest that, in part, these cross-national differences can be attributed to differences in schooling and the quality of mathematics instruction. Gilford cited a need for small, in-depth studies that "permit cross-cultural comparisons of a myriad of causal variables not recognized in large-scale surveys" (1993, p. 22). LeTendre specifically addresses studies on Japan when he suggests that more comparative qualitative work may help "inhibit the recurrent use of stereotypes and reasoning based on stereotypes" that has burdened much of the debate about Japanese educational practices (LeTendre 1999).

The Case Study Project was an attempt to compare innumerable contextual variables across three levels of schooling in each of three countries. As such, it provides an initial foray into sorting out which differences in context may contribute to Japanese mathematics success. We suggest that clear national guidelines and supportive school environments encourage mathematics teachers to focus on teaching for conceptual understanding of fundamentals, while allowing for flexibility in approach.

The Case Study Project was included as part of TIMSS to examine daily practices within schools. It included classroom observations and interviews with teachers, students, parents, administrators, and government officials and complements and complicates the information gathered through questionnaires, test scores, and other parts of the TIMSS study. Case studies were conducted in three countries: The US, Japan, and Germany. Three locations were visited in each country: one primary site in a large city in a central region and two secondary sites located in smaller cities in different regions of each country. Three schools at each level of schooling were chosen in consultation with local education authorities in the main location and one school at each level was chosen in each secondary location. Schools at the primary site were chosen to represent successful, average, and less successful schools in terms of scores on achievement tests and high school and college entrance examinations. Interviews and observations were extensive; for example, over 255 hours of math and science classroom observations were conducted in the three countries. In Japan, over 494 hours of interviews were conducted with 247 people.

In each country, there were four major topics of investigation – National Standards, Individual Differences, Adolescents' Lives, and Teachers' Lives – each studied by one researcher who spent two to three months at the major research site. In addition, other researchers conducted supplementary observations and interviews. All interviews were tape-recorded, transcribed, and translated. Extensive notes taken of observations and conversations were also entered into a shared computer data base. Carol Kinney conducted the two month investigation into Teachers' Lives at the primary site in Japan, and the examples in this paper are drawn largely from her work.

One goal of the Case Study Project was to link everyday mathematics learning to general standards and test scores. This paper clarifies how *Monbusho* (Ministry of Education, Science, Sports, and Culture) guidelines help shape mathematics learning in Japan while also pointing out the anomalies and contradictions that exist in everyday life in Japanese schools. Using examples of teachers observed during the Case Study Project, this paper seeks to provide a more detailed account of a few Japanese mathematics classrooms and the daily activities of six teachers in Japan. These individuals continually interact with the structure of education in Japan. Examining how each lesson adheres to as well as diverges from a model lesson illustrates how the structure of education shapes individuals' behavior and how individuals in turn shape that structure. This paper provides contextual information that emphasizes how a commitment to fostering conceptual understanding by mathematics teachers in Japan is situated within what teachers described as their primary concern for the overall needs of students.

The first section reviews research related to various characteristics of Japanese mathematics education. The remaining sections describe the daily activities of six Japanese teachers, two from each level of schooling.

1. Characteristics of Japanese mathematics education

1.1 The curriculum

The Japanese educational system centers on the activities of Monbusho. One of the primary responsibilities of Monbusho is the establishment of national curriculum guidelines for all subjects at all educational levels. These guidelines, also known as the *Course of Study,* provide specific information to teachers and textbook developers concerning course content and sequence, as well as time allotment for each subject area. The information provided in the mathematics portion of the Course of Study forms the basis of the Japanese mathematics curriculum. All instructional materials, such as student textbooks and workbooks, along with teacher manuals, are carefully reviewed at Monbusho for adherence to these standards.

The Course of Study is widely distributed in most major bookstores across Japan in the form of three booklets, one for each level of schooling. Although in outline form, the information provided in the Course of Study is fairly detailed. For the area of mathematics, specific concepts that must be mastered at each grade level are listed according to the four main mathematical strands (numbers and operations, quantity and measurement, geometrical figures, quantitative relations). For example, in terms of numbers and operations, fourth graders are expected to

a) perfect their place-value concept by learning about the one hundred millionth place *(oku)* and the trillionth place *(cho)* (the Japanese number system is based on units of 10,000, not 1,000),
b) develop their understanding of number estimation,
c) become proficient in the multiplication of integers,
d) extend their understanding as well as skill in executing division problems,
e) develop an understanding of decimals as well as fractions,
f) refine their understanding of the four basic operations of arithmetic and,
g) become able to perform addition and subtraction problems on an abacus (Monbusho, 1989).

In the second section of this paper, Mr. Nomura's review lesson on mixed operation equations and Ms. Tanaka'a lesson on long division demonstrate how teachers pace their classes to master these concepts.

A defining characteristic of the curriculum is its emphasis on *conceptual* understanding of fundamental mathematical concepts. For example, the fourth grade math guidelines in the *Course of Study for Elementary Schools* (Monbusho 1989) do not merely state that students must learn how to divide by the end of the school year. As our lessons show, students in the fourth grade are expected to extend their understanding of the meaning of division as well as increase their skill in solving more complicated division problems, such as larger digit problems with remainders. Specifically, the guidelines state that students must learn how to perform division of two-digit integers and understand the following relationship: (Dividend) = (Divisor) × (Quotient) + (Remainder).

A closer examination of the mathematics section of the Course of Study reveals several other interesting facets of the Japanese math curriculum. First of all, instead of recycling the same concepts year after year, the curriculum is extremely focused and expects mastery of concepts at each grade level. As a result, mathematical concepts are organized within each grade level and across grades, such that new concepts build upon previously mastered material. Let us look at the elementary school guidelines for quantity and measurement as an example. The first grade objectives are remarkably simple. Rather than introduce specific terms and concepts, the only requirement put forth in the Course of Study is the development of a *general* sense of quantity and measurement. This is accomplished primarily by focusing on the direct comparison of objects and using real-life objects as units of measurement. Standardized units of length, such as meters, centimeters, and millimeters along with those units associated with volume (e.g., liters, deciliters) are introduced in the second grade. The third grade objectives focus mainly on the concept of weight, and extend students' understanding of length by acquainting them with larger standardized units, such as the kilometer. In the fourth grade, students become acquainted with the concept of area and are introduced to terms and units related to the area of squares and rectangles such as squared centimeter and squared meter.

The Japanese mathematics curriculum also strongly espouses the problem-solving approach to learning. According to Monbusho, the primary goal of mathematics education is not skill development but rather "the development of students' ability to reason" (Monbusho 1998B, p. 11). This stress on problem-solving and mathematical reasoning skills is not only emphasized throughout the Course of Study but is evident in student textbooks as well. In a study comparing three seventh grade Japanese student textbooks to four parallel American student textbooks, Mayer, Sims, and Tajika (1995) found that over 80% of space in Japanese textbooks and only 36% of space in US textbooks was devoted to worked out examples and explanations of problem-solving procedures. In fact, the majority of space in American textbooks was composed of unsolved exercises and attractive but nevertheless irrelevant illustrations. Moreover, the study found all three Japanese books (and only one American textbook) provided multiple representations of problems through the use of words, symbols, and pictures. Additionally, the authors found material in Japanese textbooks to be arranged in an inductive fashion, with problems first being introduced through students' real-life experiences and ending with formal mathematical statements of the solution rule.

One should note that Monbusho is currently undergoing efforts to revise the Course of Study, primarily as a result of the change toward the five-day school week system. At present, students in Japan still attend school in the mornings of the first and third Saturdays of every month. However, starting in the year 2003, students will no longer be required to attend school on Saturdays, thus reducing the number of school days (Monbusho 1998A). In order to accommodate this change, the New Course of Study has reduced the number of school hours devoted to each subject. For example, the new standards now require elementary school teachers to teach anywhere from 114 to 150 hours of mathematics, depending on the grade level, as opposed to the previous allotment of 136 to 175 hours (Monbusho 1998B). Additionally, some changes have been made to the actual subject content. According to a recent report published by Monbusho, "the teaching contents will be reduced to ones necessary for daily life and contents that are, at present, considered difficult for students to master will be moved to later grades" (Monbusho 1998B, p. 11). For example, concepts such as congruence and symmetry of geometrical figures or ratios and proportional expressions will now be introduced in junior high school, rather than in elementary school. Regardless of these changes to the content the general philosophy of the curriculum remains intact: the new standards equally emphasize mastery of fundamental mathematical concepts and problem-solving.

1.2 Mathematics instruction

The extant literature on Japanese mathematics instruction, although concentrated at the elementary level, suggests that for the most part, the practices of Japanese teachers complement the goals of the mathematics curriculum. For example, to compensate for the hierarchical nature of the curriculum and its expectation of mastery of concepts, instruction is thorough, in-depth, and organized. In fact, Japanese teachers are only expected to teach one (at the most two) topics each month.

The beliefs and attitudes of Japanese teachers also seem to be in line with the primary objectives of the curriculum. The results of the TIMSS Videotape Study serve as a case in point. As part of the study, which was conducted in eighth grade mathematics classrooms in Japan, Germany, and the United States, teachers were asked to state what they wanted students to learn from the lessons they videotaped. While German and American teachers emphasized skills, Japanese teachers overwhelmingly stressed thinking and understanding (Peak 1996).

The curriculum's strong emphasis on the development of mathematical reasoning skills is also reflected in the organization of most classrooms in Japan. In contrast to the trend in the US toward reductions in class size and an increasing focus on small group collaborative learning, an overwhelming majority of class time in Japanese schools is still spent in whole class instruction. In fact, a large-scale comparative observational study conducted by Stevenson and his colleagues found 95% of elementary level Japanese mathematics lessons to be organized in this fashion (Stevenson & Lee, 1995). In this method of teaching, the teacher's role is not that of lecturer, but rather more of a guide or facilitator of discussion. In this approach, students are actively encouraged to provide comments, explanations, and even evaluations of other students' responses. In fact, it is the students' contributions that make up the bulk of the Japanese mathematics lesson. In this manner, students are expected to learn from each other and observe the many possible ways of solving one mathematics problem, thus strengthening and honing their mathematical problem-solving ability.

While students' responses are critical to the success of the whole-class instructional method, it still, nevertheless, takes a skilled teacher to conduct these lessons. Indeed, Japanese teachers have been found to be quite adept at presenting coherent lessons and engaging students in discussions that require higher-order reasoning skills (Peak, 1996; Perry, Vanderstoep, & Yu 1993). Teachers do this, in large part, by posing "hatsumon", or thought-provoking questions throughout their lessons. These questions generally do not have one correct answer. Rather, they serve to engage students in the mathematics lesson and prompt students to explore or think of various solution strategies.

Additionally, other studies examining Japanese mathematics instruction have found that Japanese teachers are more likely to subscribe to a constructivist view of learning, use manipulatives and other real-world objects with greater frequency, and encourage students to construct multiple representations of problems (Stevenson & Stigler 1992).

1.3 Learning to teach mathematics

All Japanese teachers study both their specialty subject and pedagogy in college, although elementary school teachers must take relatively more pedagogy courses while junior and senior high school teachers study more in their subject. For example, a junior high school teacher of mathematics must take courses in algebra, geometry, analytical geometry, probability and statistics theory, and computers. Elementary school teachers are required to have taken a minimum of two college courses in arithmetic. Although universities design their own teacher training courses, Monbusho certifies courses and provides oversight. Teachers study the Monbusho Course of Study for teacher employment examinations and enter their profession with a firm grasp of content guidelines and goals (Ministry of Education, Science, and Culture 1995).

Teachers work together to increase their expertise in mathematics teaching. In all schools teachers organize themselves into subject-based and grade-based groups. These groups work together to develop the best methods of teaching a subject or grade and share their information with other teachers. Teachers who are more experienced work together to develop ideas for teaching, and then meet with all teachers to discuss implementation of the teaching methods. More informally, when there are questions about teaching, experienced teachers are identified and sought out for ideas. At the city or prefecture level there are also subject-based study groups where mathematics teaching skills are discussed. Prefecture-wide or city-wide study groups disseminate findings to individual schools, through handbooks, formal and informal discussions, teaching demonstrations. In addition, as Lewis and Tsuchida (1998) describe thoroughly, research lessons are regularly scheduled to allow teachers to hone their teaching skills. Finally, teacher manuals focus on in-depth explanation of the content of lessons; this enables all teachers to fully grasp the fundamentals of mathematics for all lessons (see Lee and Zusho 1998).

2. Teaching arithmetic in elementary school: Two teachers' approaches

The Monbusho approved curriculum and texts shape how mathematics teaching and learning takes place in Japan, and several researchers have documented recurring strategies for teaching, but understanding how individual teachers use and modify the curriculum is also important. In the rest of this paper we provide examples of single lessons and descriptions of six teachers to help illuminate the daily implementation of the curriculum. How do elementary school teachers approach arithmetic teaching? Two teachers, Mr. Nomura and Ms. Tanaka (all names of people and locations are pseudonyms in this paper for teachers and schools observed during the TIMSS Case Study Project) illustrate some of the variety found among elementary school teachers. Both adhere to Monbusho goals and content recommendations while carefully considering the needs of their classrooms.

2.1 Mr. Nomura: Teaching individuals and promoting student discussion

> "You can see within the group of children that some think in certain ways. And then others think, 'Oh, that's a different way of thinking'. Discussion is really good for learning. They can consider doing problems in different ways. Today we had 56×25. If they do it as 14×4 and then multiply by 25, 4×25 is 100, right? And then they can do 14×100 .... One student says, '14×4 is 56', but another says, 'no no, 7×8 is easier', so 8×25 also works. It's good when students discuss things, but difficult to teach this way".
>
> Mr. Nomura – Matsu Elementary

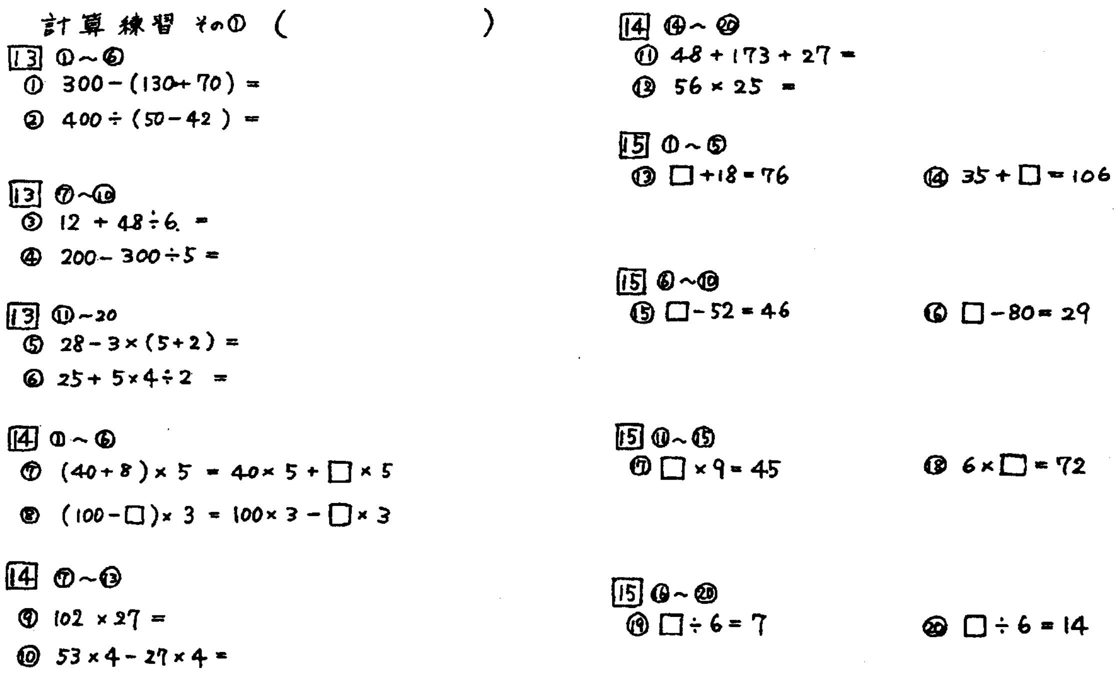
Mr. Nomura teaches fourth grade at Matsu Elementary school in Naka City. Matsu is located in a mixed income area and has a middle income student population; it was the middle range elementary school sampled for the TIMSS Case Study Project. There are three fourth grade classes at Matsu; Mr. Nomura is the head teacher for the fourth grade. As in most schools, teachers were assigned to their grades so that there was a more experienced teacher as the head teacher, a teacher with some experience, and a novice teacher. These three teachers consulted each other frequently on the days they were observed.

Mr. Nomura is married with two teenage children and his wife is a full-time housewife. Mr. Nomura became an elementary school teacher after first getting a masters degree in weather science and then working in a private high school as a geography teacher. During his university years, he worked in a *juku* or private preparatory school, where he became known as "Math Nomura" because, "I was known for being able to teach anything to any junior high school student!" He also developed his own mathematics study manual which became known locally as the "Nomura method". He shares his somewhat unusual background and expertise with the teachers at Matsu and by meeting in study groups a few nights a week with other teachers. He and four other teachers in a study group have published a fourth grade science teachers' manual for use in Naka City schools. The manual describes details of teaching, how to do experiments, how not to make a mess, and other minute details for each lesson. The study group periodically adds new ideas and make changes to the manuals. They are writing a handbook for the fifth and sixth grade science curricula, too. He told the interviewer, "we made it as material for teachers who are teaching from the text for the first time. If they teach as this recommends, they won't make mistakes". The goal of their manual, as for all teachers' manuals, is to help teachers fully understand the conceptual basis and teaching strategies for lessons.

2.1.1 Review of equations and calculations: The order of operations and the use of parentheses

Mr. Nomura's class was a review of the unit he was just ending on "equations and calculations". He used a handwritten worksheet (see Figure 1) to check each student's mastery of what he described as a difficult unit in their textbook. Mr. Nomura told the observer after class, "the addition gets difficult, so some get stuck on that. In today's class I made comments like, 'be careful at this point'". His comments during the class focused on alerting students to pitfalls when approaching equations; the purpose of the unit had been to deepen the students' understanding of using parentheses in equations and understanding how to calculate equations involving all four mathematical operations.

Mr. Nomura rang the bell and wrote problems on the board that many students had missed. He explained that although it was not really wrong to set $48 + 173 + 27$ equal to $(173 + 27) + 48$ (problem #11 in Figure 1), it was altogether wrong in the following example when there are



Note: The large numbers in boxes refer to the unit in students' workbooks and the circled numbers to practice problems in the workbooks. Mr. Nomura's problems are numbered 1–20 and are circled.

Fig. 1: Mr. Nomura's handout on mixed operation problems with and without parentheses

A look into texts and teacher's manuals for this lesson provides insight into how Mr. Nomura's review class is related to the overall goal of enhancing conceptual understanding of the fundamentals of arithmetic. The unit he was reviewing was, according to a teachers' manual, an extension of previous units that covered 1) equation writing using boxes for unknowns and 2) using parentheses with the associative principle for addition and multiplication. The manual further reminds teachers where students have previously been exposed to problems that involve all four operations. An overall stated goal was to have children understand the usefulness of equations. Specifically, the goal was to extend their understanding of the use of parentheses and to learn how to read and write mixed operations problems. In the manual, the order of operations and the use of parentheses were described as specific skills to be practiced (Shinpan Sansuu 1993, p. 180).

Mr. Nomura began class by ringing a little bell and saying "All right, arithmetic". Unlike in most observed classrooms, he had students' desks arranged in three rows of half circles "so the children can see each others' face while they are talking". Students either worked on the worksheet they had received the previous class (see Figure 1) or rushed up to his desk to have their work checked and receive a sticker. After fifteen minutes mixed operations: $15 + 4 \times 3 = (15 + 4) \times 3$. Even in this review class he encouraged students to think inductively; he gave a concrete example to make a point about the ordering of mixed operations.

Students worked independently while Mr. Nomura periodically called attention to both mistakes and correct solutions he found on their worksheets. For example, he pointed out that some students had set up the following equation: $53 \times 4 - 27 \times 4 = (53 - 27) \times 4$ (problem #10 in Figure 1). He asked how students got to this equality, an example of a hatsumon, and prodded students to induce the general principle (of the distributive property) from specific strategies. Students worked independently but sometimes shared work. Mr. Nomura asked two students who were finished to help struggling students nearby. He ended class by working his way slowly to the back of the room to check on each student.

Mr. Nomura's leadership as a teacher and his eagerness to try Monbusho's new directives are important to understanding how ideas evolve around lesson development. Monbusho's current recommendations are to individualize instruction and allow students to work at their own pace, but this requires deviation from norms of whole class teaching at times. When the interviewer commented on how students individually approached his desk and the room became noisy, he responded, “the purpose is to teach the children the lesson, the purpose isn’t to keep them quiet”, Mr. Nomura demonstrated his willingness to try new approaches and the freedom individual teachers have to run their classrooms as they see fit.

Mr. Nomura emphasized to the interviewer and his co-teachers that addition is not just a question of skill – it can be studied more effectively through discussions of mistakes and different solution strategies. Mr. Nomura tried to reach every child. As in other schools, teachers at Matsu mentioned that they also tutor struggling students in small groups before or after school or during lunch.

2.2 Ms. Tanaka: Teaching the standard curriculum with a concern for individuals

> “(In this school) There are also issues about academic ability, but if I show affection and try to teach well, they respond. I can’t just tell students to learn. I have to put enthusiasm into it and really get them to master things”. –
> Ms. Tanaka, Hasu Elementary

Ms. Tanaka teaches at Hasu Elementary school, located in a working class neighborhood near the harbor – the lower achieving elementary school in the TIMSS Case Study Sample. She has been teaching for nineteen years, and currently teaches fourth grade during her ninth year at Hasu. As she explained in her interview, Ms. Tanaka has remained single partly because of her dedication to her work. Like Mr. Nomura, she spends long hours at the school. At Hasu, more students reportedly struggled with learning than at the other two observed schools. In Ms. Tanaka’s home room there were three or four students who were disruptive and two or three others who struggled with their lessons.

2.2.1 Arithmetic: Practicing long division with more digits and remainders

According to the Course of Study, students are first introduced to division in third grade when they learn about the conceptual basis of division and how to calculate simple (i.e. one-digit) division problems. In the fourth grade, students are expected to extend their understanding of division and sharpen their computational skills (Monbusho 1989). The following fourth grade lesson was a continuation of the larger unit on division. The unit was organized so as to gradually introduce division with more digits, first through problems without remainders and then through problems with remainders. This division unit concluded the introduction to the four mathematical operations involving integers.

As Ms. Tanaka and the observer walked upstairs to her home room, she explained that she scheduled arithmetic for the first period when the children are most wide awake. After home room period she began by putting a long division problem on the board: $52 \div 26$. She called upon students to do the steps of the problem and then worked on three more problems as a whole class. This was a review of examples from the last class period. After asking students to open their textbooks, Ms. Tanaka called upon a boy to read the top of page 50 from the text, and helped him with words he could not easily read (see Figure 2).

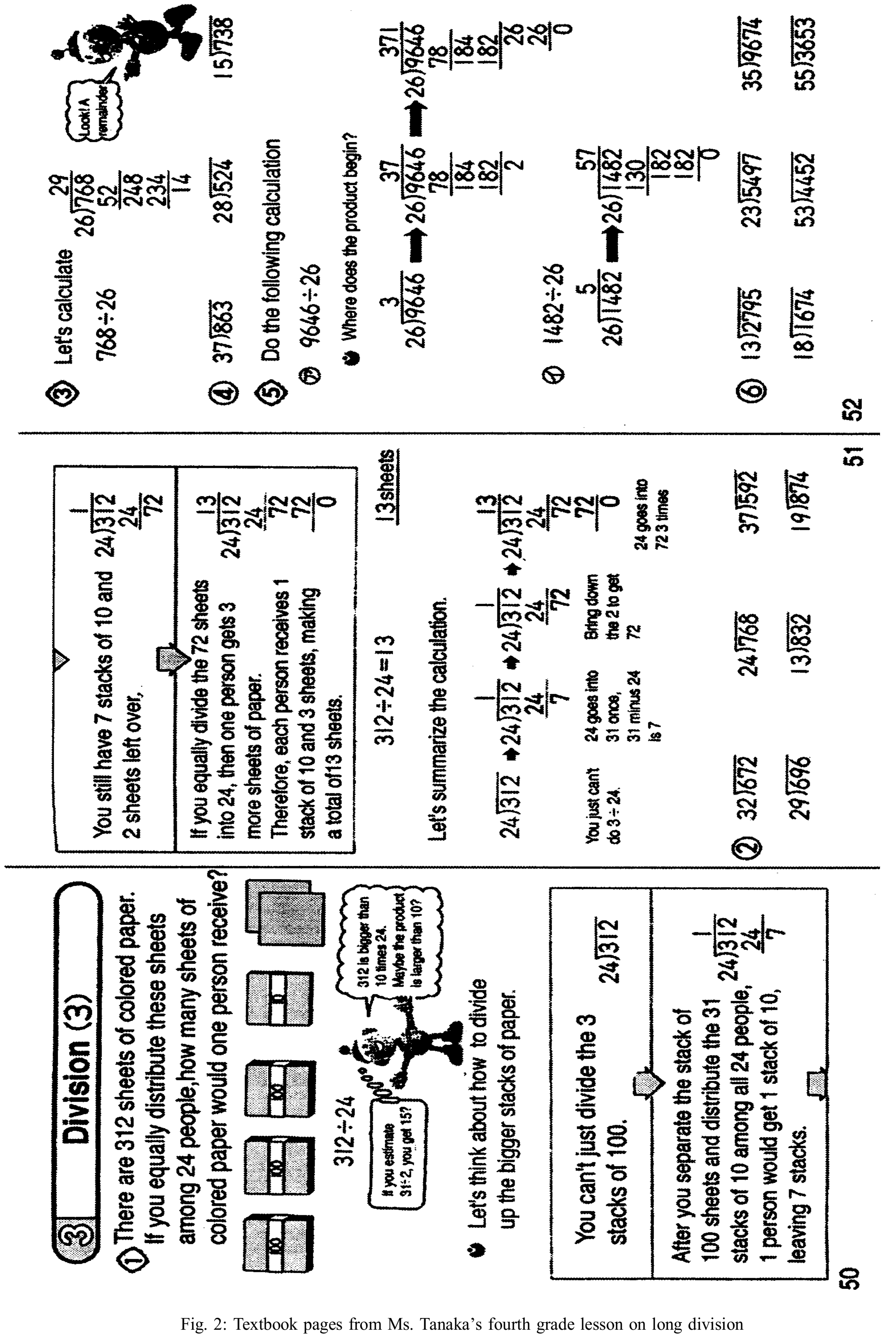
Ms. Tanaka drew a box of sticks on the board and labeled it “100”. She asked the class how many boxes of 10 could be made. Then she changed it to “300” and asked how many boxes of ten. She linked this to their class by asking if there were 38 students, how many would not get a box of 10. She had set up the lesson by providing pictorial, symbolic, and textual representations of similar problems, basing her example on the textbook.

Ms. Tanaka spent about twenty minutes reviewing division with no remainders by working problems together and then individually; for example, she worked the first problem, $672 \div 32$, together with students and then had students try the next problem at their seats: $768 \div 24$ (see Figure 2). She called upon students to explain their work to the class.

Next, she reviewed remainders by first putting $903 \div 43$ on the board and working through it together. She covered numbers as they worked together and had students call out the progressive steps of the solution in unison. Having practiced together, she proceeded to $863 \div 37$, (#4 on p. 52 in Figure 2), which has a remainder. The students began calling out answers and initially called out that 37 would go into 863 24 times. When this did not work, they tried 23, and Ms. Tanaka then explained that while 3 is the largest number that would work, there was a remainder of 12, and reminded them how to write the answer as “24 remainder 12”. She assigned the next two problems in section four in the text for seatwork. After about three minutes of working quietly, students began to talk with each other. Ms. Tanaka put one of the assigned problems on the board: $524 \div 28$, and asked if anyone could solve it. Half the class raised their hands and one boy was called upon to come to the front and present his solution. He explained it just like Ms. Tanaka had been doing, “there is one 28 in 52, so it is 1, then 52 minus 28 is 24 and you bring the 4 down ... 20 is smaller than 28 so the remainder is 20”. Ms. Tanaka reviewed the general rules about remainders and directed the class to the next page in their textbook, and the next challenges in section 5 on page 52: the division of larger numbers. Immediately she told students to gather their belongings and move to the science laboratory for the next period. Ms. Tanaka devoted the class to working examples and problem solving procedures as is typical in Japan, but the fluidity of the day was also apparent. In a later conversation, which reflected the views of other teachers observed, Ms. Tanaka explained that she watches children’s moods and behaviors and tries to time her teaching to keep their attention; she said that in this class she switched from individual work to whole–class instruction again because she perceived the children as restless.

Ms. Tanaka and her principal described Hasu Elementary as having many students from low income homes who seem to need extra attention. She said that especially at this school she sees:

> “really big differences in academic ability. Students who are good at studying and those who aren’t, and I don’t want those who can’t study well to have the sense that they are no good. I try to tell them to look for people’s good points. It’s what people can do and what they need to be able to build their lives that are most important”.



3 Division (3)

① There are 312 sheets of colored paper. If you equally distribute these sheets among 24 people, how many sheets of colored paper would one person receive?

100 100 100 10

312÷24

If you estimate 31÷2, you get 15?

312 is bigger than 10 times 24. Maybe the product is larger than 10?

● Let's think about how to divide up the bigger stacks of paper.

You can't just divide the 3 stacks of 100. 24)312

After you separate the stack of 100 sheets and distribute the 31 stacks of 10 among all 24 people, 1 person would get 1 stack of 10, leaving 7 stacks.

50

You still have 7 stacks of 10 and 2 sheets left over.

If you equally divide the 72 sheets into 24, then one person gets 3 more sheets of paper. Therefore, each person receives 1 stack of 10 and 3 sheets, making a total of 13 sheets.

312÷24=13 13 sheets

Let's summarize the calculation.

You just can't do 3÷24.

24 goes into 31 once, 31 minus 24 is 7

Bring down the 2 to get 72

24 goes into 72 3 times

② 32)672 24)768 37)592

29)696 13)832 19)874

51

③ Let's calculate 768÷26

Look! A remainder

④ 37)863 28)524 15)738

⑤ Do the following calculation

㋐ 9646÷26

● Where does the product begin?

㋑ 1482÷26

⑥ 13)2795 23)5497 35)9674

18)1674 53)4452 55)3653

52

Fig. 2: Textbook pages from Ms. Tanaka's fourth grade lesson on long division

Recognizing the range of ability, she works in her arithmetic classes to convey the material to every child, and, like most elementary school teachers, to alternate between individual work, group work, and working problems on the board for the whole class.

3. Junior high school: Teaching for understanding when examinations are looming

At the junior high school level, teachers teach one or two subjects. Mathematics teachers are usually well trained in the discipline. In smaller schools a non–math major may teach mathematics but will almost always be paired with an experienced math teacher as mentor. Ms. Ogawa and Ms. Yoshino were both mathematics majors at the prefecture's education university. Ms. Ogawa, at Midori Junior High School, the more average school in our sample, and Ms. Yoshino, a teacher at Chuo Junior High, the academically highest level junior high school in the case study sample, were both 37 years old and have two children. Both also moved to their current schools only two months before the observations. Despite their similarities, the two junior high schools were quite different, and the teachers' styles varied, too.

3.1 Ms. Ogawa's 8th grade class: Practicing the graphing of linear functions

> "(A mathematics teacher) knows the subject well. You must be able to explain things well and understand them and not get tired of the subject". – Ms. Ogawa, Midori Junior High

Ms. Ogawa mostly *team teaches* at Midori Junior High. Team teaching refers to at least three different types of teaching, mostly experimental at the time of the TIMSS Case Study. At Midori, Ms. Ogawa's other team member, Ms. Suzuki, provided extra help to students who are struggling or who have questions. Midori Junior High got a special grant last year for a "team teacher". In another example, at an elementary school, teachers taught three classes together in the gymnasium, setting up "stations" where each teacher took responsibility for teaching specific content. These are ways schools try to help all students keep up with the mathematics curriculum. Finally, although probably rare in Japan, as part of the Case Study Project in a secondary site, Naoko Moriyoshi observed "team teaching" in an elementary school where students from two home rooms were separated by ability and were taught arithmetic targeted at two learning levels. Other research suggests that this type of ability grouping is not what is usually meant by team-teaching in Japan.

Ms. Ogawa currently does not have time for many evening study groups or other professional development opportunities, but she feels lucky because, "my first school was very big, so there were many people to talk with". Midori has older teachers, in fact she is the youngest, so she has been able to learn from many kinds of teachers. During her second year of teaching she went to evening research groups and says she learned much from other teachers and still uses many of those ideas.

Ms. Ogawa's second year class, which is the equivalent of 8th grade in the United States, was practicing graphing linear functions. This was a review class of this material that began with an example from the textbook unit in a section called "Drawing Graphs of Linear Functions". Students had covered slope, linear functions, and graphing during the first year of junior high school. The Course of Study states that during the second year students should deepen their understanding and reasoning about slope and correspondence, understand the characteristics of linear functions, and expand their ability to solve these functions (Monbusho 1994, p. 39). Much of this class period was spent on practicing skills; about ten minutes into the lesson students began working on a handout that Ms. Ogawa had made (see Figure 3).

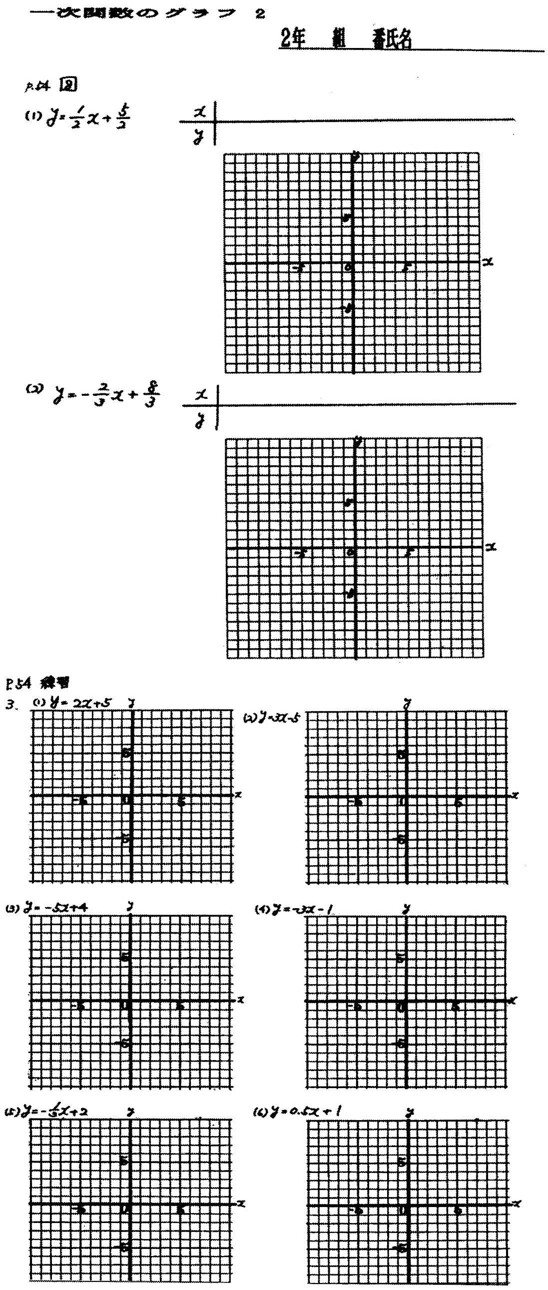


Fig. 3: Ms. Ogawa's handout on graphing linear functions

Fourth period was right before lunch on a hot day in early July. The girls had returned from physical education outdoors, and the boys came rushing back five minutes late, many with dripping wet hair from having run it under the faucets after their basketball game. Due to students arriving late and their boisterousness, class did not begin until about ten minutes after the chimes rang when Ms. Ogawa directed the class to their textbooks. Ms. Suzuki circulated to check that all students were looking at the right examples. Ms. Ogawa put the example $y = 2x+1$ on the board and began to show how to graph it. She elicited two possible values for x and y from the class: (1,5) and $(3, 11)$. They first graphed the equation by plotting these two sets of values and drawing a line through them. She worked several examples of similar equations on a large chalkboard printed with a graph that was propped against the front chalkboard, asking students to suggest possible values.

After this review she told students to turn to the next page and try the practice problems. She scolded, "just because the test is over, you shouldn't forget it!" She then exhorted them to memorize the formula "a equals y over x" (slope equals the change in y over the change in x) over summer vacation – they had been exposed to this formula in a previous class. She put a sample problem on the board that required students to first derive the slope from two sets of points and then plot the corresponding line. After practicing this alternative graphing method as a class, Ms. Ogawa assigned the handout for individual work at their seats. Both teachers circulated to check students' work.

After several minutes during which students worked on the handout, Ms. Ogawa asked for solutions to the problems on the handout, and wrote answers on the chalkboard, using a large triangle to graph the answers and continually asking students to comment on whether the solutions were correct and whether there were alternative methods. Several students had completed the handout, while others worked slowly on the first few problems, seemingly waiting for the explanations to be put on the board.

Ms. Ogawa did more of the writing on the chalkboard herself than in other classes observed, and said this was because she was speeding up the pace of classes before the examinations. Her class was typical of junior high mathematics classes with

1) a review with the whole class and individual practice of already learned material,
2) an introduction to a new form of the work and assigned problems that are discussed or checked during class, and
3) several exhortations to remember specific formulas and methods.

Less time was spent getting students to inductively come to the formulas or principles than in elementary school – partly because the formulas were previously introduced. During class, one student repeatedly complained of a stomach-ache, several students said they were hungry, and most students looked hot and tired. Ms. Ogawa may have been more directive in this class because of the heat, the time of the day, and the fact that the class started late following a physical education class which made this class seem less attentive than most to the observer.

To Ms. Ogawa, a good teacher is "someone who can provide an individual response to various kinds of people". Her ideal is to be a teacher who can "look at each child and treat them all equally while seeing their differences". But she also believes that the "system is hard on the students ... they have to study so much every day, poor things". The tension between her ideal and the preparation for high school entrance examinations was clear on a hot day when students seemed hot, tired, and hungry and she was trying to cover enough material.

3.2 Ms. Yoshino's third year class on applications of factoring

> "Every time (after every class) I wonder if it was OK, I'm always reflecting. Did that child understand? ... I feel that I want to make even one child who doesn't like math begin to like math". – Ms. Yoshino, Chuo Junior High

At the start of Ms. Yoshino's third year mathematics class at Chuo Junior High during first period one boy sheepishly came to the front to report that he left his school bag on the train. Most junior high schools in Japan are neighborhood schools where students commute by foot or bicycle; commuting by train indicates the way parents bend the rules – for example, by using a business or relative's address as a home address – to get their child into this well respected school. Although junior high schools in Japan are neighborhood schools that are not tracked by ability or other student characteristics, certain schools may gain a reputation for higher or lower achievement than others.

Third year junior high students learn how to factor and the Course of Study states they will learn how to use four formulas for factoring. For example, one formula is $(a + b)^2 = a^2 + 2ab + b^2$. In this class students were practicing how to set up equations to find the answers to word problems; the solutions always required factoring. Ms. Yoshino read aloud an example from the text to begin the lesson that continued a focus on the factoring (see Figure 4):

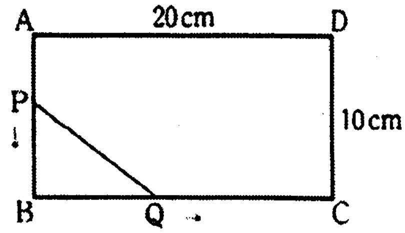
> "The circumference of $ABCD$ is made up of $AB = 10$ centimeters, $BC = 20$ centimeters. The point P is on line AB and can move one centimeter a second from A to B; point Q on line BC can move from B to C at two centimeters a second. If P and Q start at the same time, how many seconds later will the area of the triangle PBQ become 24 square centimeters?"

She carefully explained the problem twice. She then drew a picture of the problem on the board and asked them to draw what they thought would be useful in their notebooks. She carefully explained each aspect of the picture and told the class that they had to carefully understand the picture in order to begin the problem. She gave them a hint, "after departing it takes another x seconds", and asked how many of the students could understand the answer just from the picture. About half the students raised their hands. Again, she read the problem carefully while she walked around the room checking their drawings in their notebooks. A few students went directly to a solution without drawing a picture. Then she said, "OK, I'd like someone to put it on the board. Who has finished?" Four boys immediately raised their hands and she chose one of them. She asked the boy who put the problem on the board to explain it, but he said he could not do it. One of his friends volunteered and carefully explained the problem. The student concluded that it would be four seconds

Matsu Junior High

Ex. 3

For rectangle ABCD, AB=10 *cm* and BC=20 *c*
Point P is on line AB and moves from
A to B at a rate of 1 *cm/sec*.
Point Q is on line BC and moves from
B to C at a rate of 2 *cam/sec*.



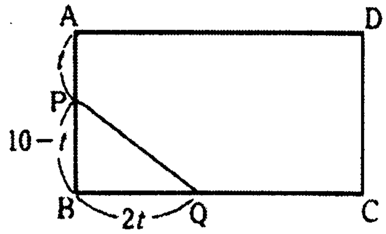
If P and Q start at the same time, how many sec
later will the area of triangle PBQ become 24 m^2?

If t = time area of triangle PBQ equals 24 cm^2 t

$$\frac{1}{2}(10 - t) \times 2t = 24$$

If you solve the above equation,

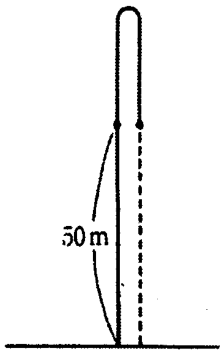
$$t^2 - 10t + 24 = 0$$
$$(t-4)(t-6) = 0$$
$$t = 4, 6$$



8 If you throw an object straight up in the air at a rate of 35 *m/sec*, a
height of the object t seconds later, then h can be expressed using
following relationship:

$$h = 35t - 5t^2$$

(1) How many seconds later will the height be 50 *m* ?
(2) How many seconds later will the object return to its original lo



9 For triangle ABC, angle C = 90 degrees,
AC = BC = 15 *cm*.

As displayed in the figure to the right,
point D is drawn on AB to form
rectangle DECF with an area of 56 cm^2.

What is the length of CE?

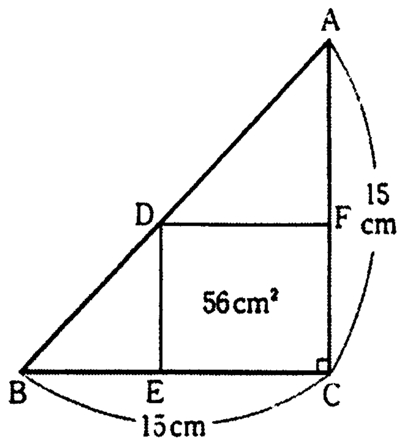


Fig. 4: Example problems from the textbook in Ms. Yoshimo's ninth grade class at Matsu Junior High

and six seconds later. Ms. Yoshino asked two boys in the back of the classroom if they understood. They nodded. She then asked the entire class. No student indicated any problems. This problem took about fifteen minutes.

Ms. Yoshino then assigned problems 8 and 9 (see Figure 4) to do in class while she put the problems on the board. Some students worked out the problems neatly in their notebooks, while others wrote out only the final part of the solution. After five minutes of seatwork, Ms. Yoshino suddenly threw her chalk straight up in the air and caught it. The students laughed. Ms. Yoshino pointed out that the chalk passed each height twice but only passed the top point once, as a real life illustration of problem 8. Ms. Yoshino drew a picture of the trajectory of the chalk. She explained that if you were lying down, 0 seconds would be at the bottom. She asked, "how many seconds would it take?" She encouraged the students to think hard.

Most students worked on problem number 8 (see Figure 4) but a few were done and worked ahead in their math workbook. Ms. Yoshino asked who had completed problem 9, and one student wrote it on the chalkboard. But there was no time, so Ms. Yoshino said, "I'll explain, watch up here and you can write it in your notebooks later", adding a few details to the figures on the board, and exhorting them to fully understand the diagrams. Although the class ended without a final summing up, Ms. Yoshino had presented a problem in words, pictures, and by a concrete illustration (throwing her chalk up), and had encouraged students to draw figures carefully and think about each step of the problem fully. As Figure 4 shows, the textbook devoted almost a page to explanations of one problem and procedures for solutions.

Ms. Yoshino and the other teachers at this school talked mostly about students' needs to the interviewer. Several teachers joined in at various times to talk about how privileged most students at Chuo Junior High School are, describing them as coming from well off families with fathers who are company presidents, school teachers, Buddhist priests, and other professionals. The school was described as being a "greenhouse" where relations between people were peaceful and calm. They compared this school to Midori Junior High which is a mixture of nice homes and lower income apartments.

Ms. Yoshino aims for whole class teaching that reaches each student:

> "If you try to help each individual learn within the whole class, you first must think about the topic and then ... usually I try to find out the different ways of thinking about something. Then I elicit different ways to arrive at a solution, and I put the children's way of thinking up on the TV screen right away (she uses a monitor that can project a page from a notebook onto an overhead TV). I show their work and while explaining it say, 'Oh, this child got this part right, and this boy got this right, and that way of thinking is nice', and that sort of thing".

However, the class described here was not like this. Ms. Yoshino explained, "afterall, there are also parts that must just be taught. Today it was the earlier part of a unit, so ...". Although Ms. Yoshino described her teaching as eliciting several solutions from students and having a discussion, the reality of the lesson described was that two students each fully presented one problem.

Ms. Yoshino also expressed concern with the way examinations structure mathematics learning,

> "Ideally, I would know what the students understand, and construct a class that addresses the points students want to know. That's the best kind of class. If it is what students want to know, they will be motivated. We could have classes that build upon each other, and if I could do that every time, they would gradually learn things and we would have a purpose for classes, and what I wanted to teach and what the students wanted to know would become the same. That would be ideal, but that depends on the time allowed. I have to teach too much content. There is only a year ... until examinations. If math were really lifelong learning ... they would study it for all of their lives. Instead, they must plod through tests, so there are parts that they want me to teach and I must teach it. There is pressure everyday".

Although the Monbusho Course of Study introduces only a few topics a month, Ms. Yoshino still wished for a more thorough, slower pace. At Chuo Junior High School where most students are pushed by their families to study hard, she could encourage an inductive approach to mathematics learning, and serve more as a facilitator of the lesson, rather than reiterating formulas learned previously and emphasizing memorization, as Ms. Tanaka did.

4. Two high school teachers: Mr. Endo and Mr. Koyama

> "This school is an academic preparatory school, so if you don't direct yourself toward that priority, you won't answer the students' needs ... The children at this school aren't the kind who study on their own, but if you guide them well they will do some of what you advise them to do. Teachers who advise effectively are good teachers who try to raise students' motivational level". – Mr. Endo, Meiji High

> "To be a good teacher, there is one thing: you get students to understand. I am always saying to students, it isn't just looking and listening, you won't remember that way ... if you just watch the mathematics that the teacher does and think you understand it, that's not enough. If you think you have understood it, try it on your own. I tell them to practice a lot". – Mr. Koyama, Naka Vocational

At the high school level teachers identify themselves by their discipline; mathematics teachers majored in mathematics, or a related field in colleges. Along with their deep interest in mathematics, teachers reported that concern for students' motivation toward mathematics learning was a priority. Mathematics teachers face the least competition to become teachers, because teacher employment tests require a high level knowledge of mathematics that only successful college math concentrators possess. Depending on the type of high school, math teachers may teach calculus and other mathematics aimed at difficult college entrance examinations, or they may teach remedial mathematics to students in less academically challenging schools. The TIMSS Case Study indicated that at both types of schools, math teachers said they try to convey their material in interesting and accessible manners.

Most mathematics classes in high school seemed to follow the format of presenting new material, requiring some individual seatwork on related problems (either in class or as work at home), having students put solutions on the board, and finally discussing how solutions were reached and eliciting ideas for better strategies. In the vocational school studied, early morning drill sessions were added as preparation for certification tests in specialty areas such as electrical work, and games and competitions around math problem solving took place within mathematics classrooms. In the academic track high schools, games in mathematics were observed in elective mathematics classes and some third year classes focused entirely on review for college examinations.

Mr. Endo is the head mathematics teacher at Meiji High School, the high achieving college preparatory school in our sample, although not the highest ranking school in the prefecture, and Mr. Koyama is the head teacher at Naka Vocational High School, a high school from which most students will not go on to further schooling. Both said they want to emphasize conceptual understanding of mathematics and try to connect students' learning to real life. This is sometimes difficult, because the looming college examinations or licensing examinations for trades keep students only focused on passing tests.

Mr. Endo, in his 40s, graduated from the prefecture's Education University in mechanical engineering. He has three children and his wife is also a high school teacher. He previously taught at a less academically oriented high school that was outside of Naka City, and, most recently, at a very high ranking high school.

Mr. Koyama is 59 years old, one year from retirement, and has been teaching at Naka Vocational High School for the past 24 years. He has five children and bicycles to school everyday. He, too, is the head of the mathematics department. He enjoys teaching:

> "Everyone says they get tired when they teach classes. When I teach classes, I don't get at all tired. Everyone just plays together with me, because my feeling is that I am going to have a good time. So I really don't get tired at all!"

Like most high school teachers, he teaches about sixteen hours of classes a week, plus supervising a home room class. He studied education in the liberal arts department of a neighboring prefecture's university, where he focused on mathematics.

In this section we include two descriptions of high school mathematics department meetings in order to emphasize how high school teachers tailor their teaching to students' needs and future plans. At Meiji High, teachers must balance teaching mathematics for college entrance examinations against their own goals of teaching the deeper concepts behind mathematics. At Naka Vocational High, the teachers often focus on the broader goal of preparing their students for life outside of school, with specific mathematics learning sometimes seeming to be less of a priority.

4.1. A mathematics department meeting: Preparing for entrance examinations

Mr. Endo led the June 1st mathematics department meeting in a discussion of the new mathematics curriculum for next year. One teacher pushed for adding a computer class for the third year students where they would teach programming. The other teachers expressed pessimism that a practical computing course could fit into the examination oriented mathematics curriculum at this school. Next they discussed the results from a recent mathematics test. Meiji's first year students ranked above the prefecture's average score, but had gone down from rank 13 last year to number 16 this year. Teachers discussed each of their home room's performances and some of the reasons for the lower overall scores. One teacher pointed out that it was mainly questions about vectors that gave the students trouble. He suggested they revisit the first year curriculum and teaching methods to try to improve on that area. Discussion continued, with three teachers emphasizing that the students are working hard and several nodding when one teacher commented that these students do not have much ability so it is hard for them.

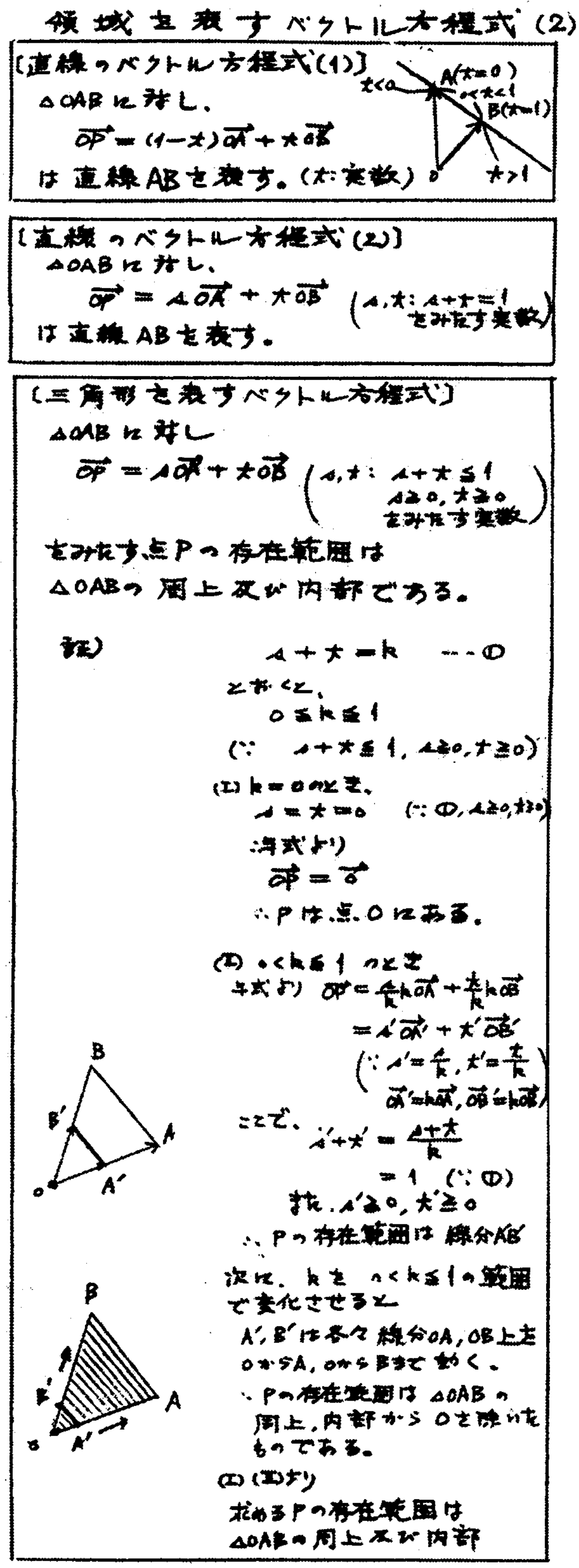
Teachers also discussed how to cut the number of mathematics credits to meet the new Monbusho requirements for next year when two Saturdays a month become vacation days. Should they really cut the number of minutes in class or just change the way the days are organized to keep the same amount of class time? The meeting illustrated some of the tension between trying to teach for conceptual understanding and relating mathematics to everyday life (e.g. through computers) and the practical need to prepare students to pass examinations. These are ongoing discussions in the weekly mathematics department meeting.

4.2. A Mathematics department meeting: Preventing drop outs and finding jobs

The mathematics department meeting at Naka Vocational focused on how to prevent students from dropping out. Mr. Koyama emphasized his concern about this and described some ways he encourages students he knows to stay in or return to school. He criticized the committee formed to address the issue of drop-outs for doing too much research and not taking enough action on the problem. After agreeing that they should convey their concerns to the committee, teachers reported on a model car race in which many students will participate and plans for a job fair and visits to factories where students can learn about jobs. One teacher ended the meeting by emphasizing the difficult job market this year and reporting that third year home room teachers will meet tomorrow to discuss how to help their students find jobs. In the weekly mathematics department meeting at Naka, teachers focused less on improving mathematics skills and more on the relationship between school and students' futures.

4.3 A Mathematics class at Meiji High School: Review for examinations

Mr. Endo taught this fifth period class after lunch for third year students in the science track at this school who are not taking a calculus examination for college entrance. The textbook was a compilation of examination questions from various college entrance examinations. Each question was followed by a footnote about when and where the question was used. The book reviews algebra, geometry, calculus, and other areas.



領域を表すベクトル方程式(2)

[直線のベクトル方程式(1)]
△OABに対し、
$\vec{OP}=(1-t)\vec{OA}+t\vec{OB}$
は直線ABを表す。(t:実数)

[直線のベクトル方程式(2)]
△OABに対し、
$\vec{OP}=s\vec{OA}+t\vec{OB}$ (s,t: s+t=1 をみたす実数)
は直線ABを表す。

[三角形を表すベクトル方程式]
△OABに対し
$\vec{OP}=s\vec{OA}+t\vec{OB}$ (s,t: $s+t\leqq 1$, $s\geqq 0$, $t\geqq 0$ をみたす実数)
をみたす点Pの存在範囲は△OABの周上及び内部である。

証) $s+t=k$ …① とおくと、
$0\leqq k\leqq 1$
($\because$ $s+t\leqq 1$, $s\geqq 0$, $t\geqq 0$)
(I) $k=0$のとき、
$s=t=0$ ($\because$ ①, $s\geqq 0$, $t\geqq 0$)
与式より
$\vec{OP}=\vec{0}$
∴Pは点Oにある。
(II) $0<k\leqq 1$のとき
与式より $\vec{OP}=\frac{s}{k}k\vec{OA}+\frac{t}{k}k\vec{OB}$
$=s'\vec{OA'}+t'\vec{OB'}$
($\because$ $s'=\frac{s}{k}$, $t'=\frac{t}{k}$, $\vec{OA'}=k\vec{OA}$, $\vec{OB'}=k\vec{OB}$)
ここで、$s'+t'=\frac{s+t}{k}$
$=1$ ($\because$ ①)
且つ $s'\geqq 0$, $t'\geqq 0$
∴Pの存在範囲は線分A'B'
次に、kを$0<k\leqq 1$の範囲で変化させると
A', B'は各々線分OA, OB上をOからA, OからBまで動く。
∴Pの存在範囲は△OABの周上,内部からOを除いたものである。
(I)(II)より
求めるPの存在範囲は
△OABの周上及び内部

Fig. 5: Mr. Endo's handout on vectors

Mr. Endo began with a ten minute review of the midterm examinations, paying attention to which questions gave most students problems and what types of mistakes students made. Mr. Endo said that if they got less than 80% of the problems correct, they should redo their problems and hand the test in again. He walked around to check homework. Quite a few students had not finished and he marked this in his grade book. During this time one girl put a long problem about vectors on the board; the emphasis on vectors grew out of the discussion at the departmental meeting. Mr. Endo reminded students that they all should be able to do the work in this class, and emphasized that entrance examinations were soon.

As in many high school mathematics classes, Mr. Endo corrected work students wrote on the chalkboard. He asked for explanations and encouraged students to push the student presenting to explain the logic more fully. Students sometimes volunteered that they didn't understand certain points. Mr. Endo pointed out two types of reasoning, and presented the alternative approach.

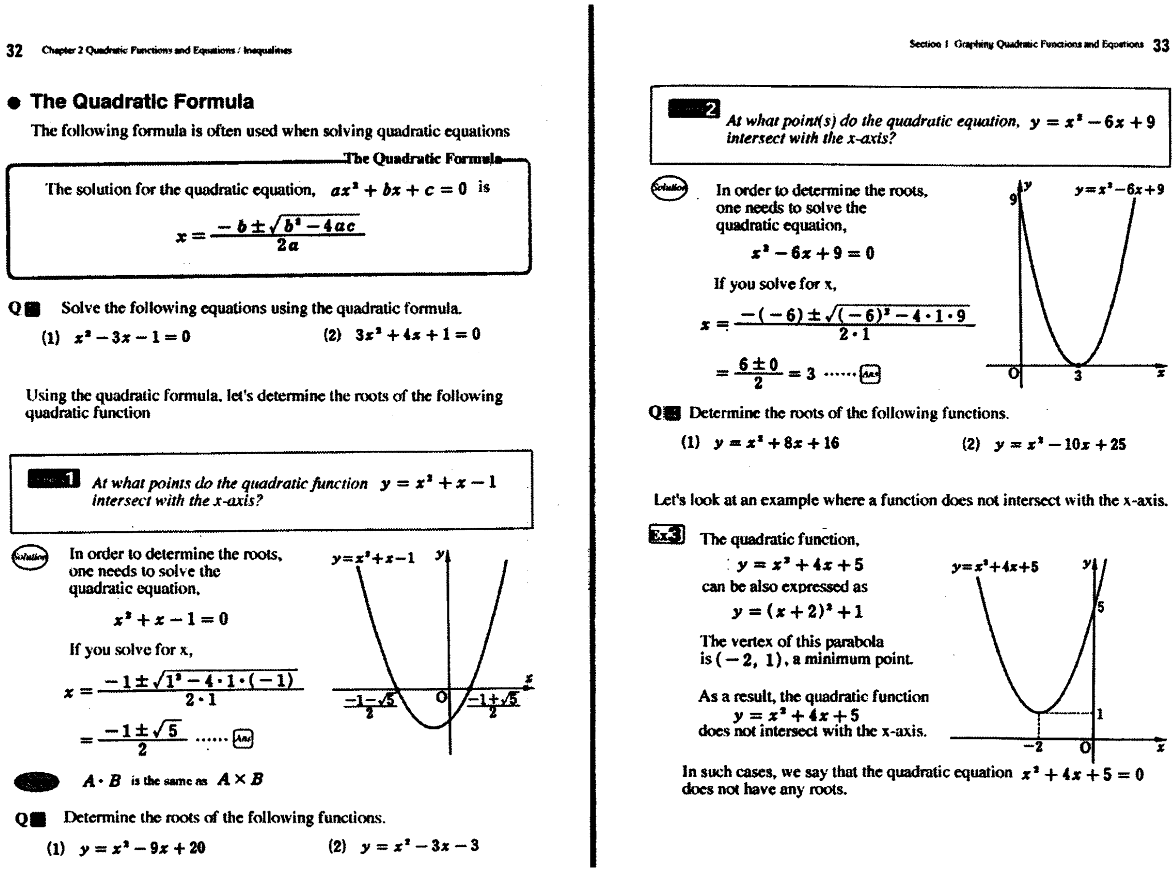
Next, Mr. Endo passed out a handout on vectors (see Figure 5) with examples and emphasized how complicated vectors are. Students worked on the handout while Mr. Endo circulated to answer questions. With only five minutes left in the class, Mr. Endo suddenly whistled to get their attention. Students smiled at this antic and he said, "let's do the previous problem a shorter way", again emphasizing versatility in approaching solutions. He wrote while a student corrected his mistakes. Class ended with one more problem quickly written on the chalkboard; Mr. Endo explained it while students copied into their notebooks.

The third year in academic preparatory high schools is often devoted to review for entrance examinations, as at Meiji High School. Mr. Endo emphasized understanding of solutions and searching for alternative methods, but the class was structured to practice problem solving and to prepare students for examinations. If students had not mastered fundamental mathematical concepts before this year, there was little time to approach learning in the time-intensive manner recommended in the Course of Study. Mr. Endo attempted to foster more conceptual learning through a brief lesson on vectors, but this lesson was not connected to an ongoing lesson.

4.4 Early morning practice and a lesson on the quadratic formula at Naka Vocational

Mr. Koyama's interaction with students began before home room period with a practice mathematics session. His first year home room, electrical mechanics concentrators, arrived early for practice with calculators. They practiced for speed and accuracy on a variety of algebra and trigonometry problems. Students seemed to enjoy themselves, and Mr. Koyama joked and pushed them to try harder. After taking care of home room tasks, Mr. Koyama started the first period mathematics class. Students at Naka Vocational studied lower level mathematics than at Meiji High School; this lesson was on the quadratic formula, similar to third year junior high classes.

Mr. Koyama reminded students they were to memorize the quadratic formula for today. He called on a student to repeat it and commented, "you probably even learned this in junior high". He then read again from the box at the top of page 32 (see Figure 6). Mr. Koyama joked with and scolded students to keep their attention. He spent five minutes reviewing factoring out parts by working problems in section 2 on page 32 together with the students. He asked them, "did you do the multiplication in your head?



32 Chapter 2 Quadratic Functions and Equations / Inequalities

● The Quadratic Formula

The following formula is often used when solving quadratic equations

The Quadratic Formula

The solution for the quadratic equation, $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q Solve the following equations using the quadratic formula.

(1) $x^2 - 3x - 1 = 0$ (2) $3x^2 + 4x + 1 = 0$

Using the quadratic formula, let's determine the roots of the following quadratic function

1 *At what points do the quadratic function* $y = x^2 + x - 1$ *intersect with the x-axis?*

Solution In order to determine the roots, one needs to solve the quadratic equation,

$x^2 + x - 1 = 0$

If you solve for x,

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$= \frac{-1 \pm \sqrt{5}}{2} \ \ldots\ldots \text{Ans}$$

$A \cdot B$ is the same as $A \times B$

Q Determine the roots of the following functions.

(1) $y = x^2 - 9x + 20$ (2) $y = x^2 - 3x - 3$

Section 1 Graphing Quadratic Functions and Equations 33

2 *At what point(s) do the quadratic equation,* $y = x^2 - 6x + 9$ *intersect with the x-axis?*

Solution In order to determine the roots, one needs to solve the quadratic equation,

$x^2 - 6x + 9 = 0$

If you solve for x,

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{6 \pm 0}{2} = 3 \ \ldots\ldots \text{Ans}$$

Q Determine the roots of the following functions.

(1) $y = x^2 + 8x + 16$ (2) $y = x^2 - 10x + 25$

Let's look at an example where a function does not intersect with the x-axis.

Ex 3 The quadratic function,

$y = x^2 + 4x + 5$

can be also expressed as

$y = (x + 2)^2 + 1$

The vertex of this parabola is $(-2, 1)$, a minimum point.

As a result, the quadratic function $y = x^2 + 4x + 5$ does not intersect with the x-axis.

In such cases, we say that the quadratic equation $x^2 + 4x + 5 = 0$ does not have any roots.

Fig. 6: Example textbook problem from Mr. Koyama's first year high school class

You should try that". Mr. Koyama explained that in simple examples the quickest way is to use the formula. He had one student do problem 2 in section one ($3x^2 + 4x + 1 = 0$) by factoring. Then he had another student use the formula and turned to the class to say, "see, it is the same answer". Then Mr. Koyama put a different problem on the board. A student joked, "Teacher, you already know the answer", and he responded, "you have to know the answer to know if it will work!" He then explained that learning about factoring requires practice at guessing quickly which numbers will work.

Saying that this had been a review of what they studied yesterday, he turned to new material. He drew the graph from example problem 1 in the textbook on the board, highlighting the two points where the curve intersects the line. He asked many rapid questions about this problem from the text, and called on students for quick answers. Some students collaborated with a nearby student before answering.

Next, he had several students put problems they had already prepared, 1 and 2 from section 3, on the chalkboard. Mr. Koyama walked around checking work and commented on the answers on the board, explaining how to find the most efficient solution and discussing strategies to use when answering questions on a test. He repeatedly checked to see if students understood the work their peers presented, and pointed out the different strategies students used to answer the same questions. Then he worked example problem 2 from page 33 on the board. In this problem, the number under the square root sign worked out to be 0. Mr. Koyama asked what the square root of 0 is, and then pointed out that they can then erase that section of the problem to shorten their work. During the last ten minutes of the class he had two students put problems 1 and 2 from section 4 on the board. Just before class ended, he checked the work on the board, pointing out that there is no answer if the number under the square root sign is negative.

Mr. Koyama used the textbook with a note to himself about the date each page was covered in years past. He explained that although he is required to make official teaching plans for the year, he does not use them much because he knows his plan for individual lessons from long experience.

5. Discussion

Through the examination of the varied teaching practices of six teachers, this paper shows how teachers aim to convey a conceptual understanding of the fundamentals of mathematics. We focus on five ways this understanding is achieved:

1) an emphasis on whole class teaching,
2) the use of extremely focused curriculum guidelines that aim for mastery of concepts at each grade level,
3) a curriculum that thoroughly covers mathematics units in an organized and in-depth manner,
4) teachers acting as facilitators or guides more often than as lecturers, and
5) a focus on problem solving with the primary goal of developing students' ability to reason, especially to reason inductively (see Table 1). Our purpose was to show how this can happen with flexibility and imagination.

Table 1: Ways conceptual understanding of fundamentals mathematics is conveyed in everyday practice

Ways Japanese classrooms aim for conceptual understanding of the fundamentals of mathematics	Examples of practice
1) Emphasis on involving the whole class in learning	Mr. Koyama put a graph on the board and then used rapid questioning of most students in the class to complete the problem and illustrate difficult sections or confusing parts of the problem
2) Focused curriculum guidelines; aim for mastery	Mr. Nomura's 4th grade used the class period to review the ordering of operations using a worksheet to evaluate mastery by each student
3) In-depth coverage of topics	Ms. Yoshino's 8th grade covered only four problems; she pushed students to think of factoring concretely by tossing her chalk into the air
4) Teachers as facilitators or guides	Mr. Endo urged his 12th grade class to ask questions of the presenting students and prodded the presenters to explain more fully and carefully
5) Problem solving for the development of inductive reasoning	Ms. Tanaka's 4th grade class was pushed to go from the concrete example of dividing packages of sticks among themselves to the abstract problem of remainders in division

Each teacher in the TIMSS Case Study Project in Japan took an individual approach to presenting mathematics to their students. Not every class proceeded exactly as Monbusho guidelines suggest because students' needs, weather, or looming examinations reshaped lesson plans. Yet, in interviews teachers repeatedly stressed their goals of facilitating conceptual understanding, promoting multiple problem solving strategies, and linking classroom learning to general life experience. As expected, teachers were thoroughly versed in the Course of Study; however they also approached each classroom and student with flexibility. Teachers relied on textbooks to structure the lessons and provide a conceptual approach to materials but all also regularly supplemented lessons with their own handouts and sometimes with mathematics workbooks from other sources.

As described, the Japanese mathematics curriculum is structured to develop mastery of concepts at each grade level. Instruction is thorough and in-depth. In each lesson described here, except to some degree for Mr. Endo's third year high school class, one mathematics topic was reviewed and a variation of that topic was introduced. In Mr. Nomura's third grade class, using a review worksheet, he developed the idea of order of operations, taking care to emphasize the importance of order when multiple operations are used in the same problem. Mr. Koyama introduced first a review of the quadratic formula, and then factoring of similar problems, while Ms. Tanaka began with a review of division without remainders and continued to teach about remainders.

The mastery of extremely focused concepts is also evaluated by teachers through worksheets, tests and discussion. A follow-up to teaching for mastery is evaluation of understanding; in the mathematics department meeting at Meiji High School, test scores were used to evaluate which lessons students had not mastered and Mr. Endo immediately used those results to emphasize certain problems missed on practice examinations. Students' mastery seemed to be seen by teachers as the responsibility of the instructor.

In Ms. Yoshino's class on graphing, she covered only four problems during the entire class period, and pushed students to think of the rather abstract lesson using procedures for factoring in a way that related to life. Her goal was conceptual understanding of the problems. The text, by spending a whole page on the first problem, emphasized higher order problem solving skills. Likewise, Ms. Tanaka began with the concept of remainders in division through the concrete example of boxes of sticks, and then worked relatively slowly through problems with remainders, asking students to describe each step's answer.

Most of the lessons described in this paper were reviews of already learned material. As a result, students participated less in these lessons than in some descriptions of model Japanese mathematics lessons. Nevertheless, teachers still tried to involve students in the lesson by asking them to provide comments, explanations, and evaluations of their own work as well as the work of their classmates. As Ms. Yoshino described, she often puts students' work from their notebooks on the television screen and has students compare it to their own and explain and evaluate each others' work. Sometimes evaluation was overt, as when Mr. Endo pushed students to explain the problem they presented while pushing other students to ask for further elucidation. Ms. Yoshino had one student put a problem on the board and another student explain it to the class. In a teacher-directed example, Mr. Nomura put one student's solution on the board, and asked how many others had used a similar method. In all classes observed, students were adept at putting problems on the board and then explaining them to their classmates; possibly more importantly, students asked questions and challenged each others' and sometimes the teacher's work.

Although not every class example in this paper emphasized the inductive approach, all teachers related the problems to real-life experiences, as Ms. Yoshino did when she got students' attention by tossing chalk into the air, thus emphasizing the connection between abstract factoring and concrete experience. In junior high and high school mathematics, making the link to everyday experience can be more difficult; one of Mr. Koyama's colleagues, in another class on series, spent time making an analogy to strategies in the game of Mahjong.

The examples in this paper show both how varied the practical application of model teaching strategies is and how basic values of Japanese mathematics education pervade classrooms, even when teachers rush to complete material before an exam or when the weather makes it hard for students to concentrate. We have also attempted to present a portrait of Japanese teachers as individuals who approach their classrooms through their own perspectives and values. Normal daily activities, Monbusho guidelines and a diverse group of teachers combine to allow innovative teaching practices to emerge and shape learning. The TIMSS Case Study Project, while not comprehensive enough to provide a definitive policy approach to mathematics teaching, complements other studies and suggests how Japanese educational policy and research is actually implemented at the classroom level.

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