TEACHERS' BELIEFS ABOUT STUDENTS´ DEVELOPMENT OF THE PRE-ALGEBRAIC CONCEPT OF EQUATION

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Pre-algebraic content has recently been introduced in primary education. One question that is worth examining is to what extend does the teachers' views of the complexity of algebraic tasks match the actual students' difficulty. In this report we focus on teachers` beliefs about the students` difficulty to handle simple equations. Ninety-three 6th grade students completed a test with 14 tasks, while 50 teachers rated the items according to difficulty. It was found that teachers could only partially predict students understanding and reasoning. Contrary to teachers` perceptions, the students could manage word equations and story problems easier than they could handle tasks represented by pictures and diagrams. This mismatch needs to be addressed to help teachers organize productive learning activities.

Introduction

Beliefs constitute one of the three basic components of the affective domain, the other two being emotions and attitudes (McLeod, 1992). Beliefs might be defined as one's personal views, conceptions and theories (Thompson, 1992). The importance of the construct lies in findings that teachers' behavior is primarily determined by their belief system rather than by their own knowledge. Experience and prior knowledge are also important, but beliefs act as the "driving forces" in shaping the structure and content of their practices in the classroom.

The teachers' beliefs shape the type, content and representation format of the activities used in the classroom. As Hersh (1986) put it, "one's conceptions of what mathematics is affects one's conceptions of how it should be presented" (p.13). As Nathan & Koedinger (2000a) mention, "teachers' beliefs about students' ability and learning greatly influence their instructional practices" (p. 168). More specifically, their previous study of teachers' beliefs has revealed that teachers consider students' ability to be the characteristic, which has the greatest influence on their planning decisions. Furthermore, Borko & Shavelson (1990) have found that teachers generally report that information about students is the most important factor in their instructional planning. Raymond (1997) presented a visual model depicting the relationships between mathematics beliefs and practice. She found a direct relationship between mathematics beliefs and mathematics teaching practice.

Recently, the mathematics education community has given more emphasis on studying the teachers' beliefs on specific aspects of mathematics teaching, while little attention has been paid on studying beliefs about the students' ability on developing specific mathematical content. The *Professional Standards for the Teaching of Mathematics* (NCTM, 1991) proposes that teachers must be more proficient in

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selecting mathematical tasks to engage students' interests and intellect. For successful learning outcomes, it is necessary for mathematics teachers to have strong mastery of mathematics content, mathematics pedagogy and knowledge of children's mathematical thinking. Thus, it is important to study how the teachers' beliefs guide them to take into consideration these variables during instructional decision making.

In this study we examine teachers' beliefs about the ways students' develop the concept of equation in the elementary school. This concept has traditionally been taught at middle and high schools. Elementary school teachers' preparation did not include any training on the teaching of pre-algebraic concepts.

The early development of algebra concepts

During the last decade, there has been an effort internationally, to "algebrafy" the mathematics curriculum from as early as the pre-kindergarten years. That is, to introduce algebra content into the elementary school curriculum. According to the National Council of Mathematics (NCTM, 2000):

> by viewing algebra as a strand in the curriculum from pre-kindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more-sophisticated work in algebra in the middle grades and high school (p. 36).

The question though is "What might mean to suggest that algebra should start that early?" Kieran and Chalouh (1993) "consider as pre-algebraic the area of mathematical learning in which students construct their algebra from their arithmetic" (p. 179).

The difference between arithmetic and algebra is in the way questions and problems are expressed. The position of the unknown quantity in a problem statement determines the type of the equation and the required procedure for its calculation. Therefore, we consider the position of the unknown quantity to have a significant effect on the difficulty level of mathematical problems in an early algebra curriculum. For the purposes of this study, we consider as arithmetic equations those that the unknown quantity is the result (at the end), i.e. 32 + 25 = أ and as algebraic equations those that the unknown quantity is at the start, i.e. أ + 25 = 57.

Additionally, another factor that influences the difficulty level of mathematical problems is the representation format. Specifically, according to the developmental theories of Piaget, for each new concept studied, students use concrete objects to solve problems, next they use pictures, icons or diagrams and finally they use abstract symbols. This sequence of learning must be used for each new major concept that is introduced to elementary school children.

Furthermore, we anticipate that another factor that influences the difficulty level of mathematical tasks is the number of quantities in a problem situation; the difficulty level increases with the number of quantities. Thus, a problem with two quantities (i.e., the number of beads Mary and John have) is less difficult than a problem with three quantities (i.e., the number of beads Mary, John and Peter have).

This study has focused on teachers' beliefs about a specific topic of the mathematics curriculum for the elementary grades. In particular, we studied the teachers' beliefs on the development of an early algebra concept. Since, MacGregor and Stacey (1999) suggest that one of the five aspects of number knowledge that are essential for algebra learning is understanding equality, this study has given emphasis on the ability of 6th graders to solve arithmetic and algebraic equations in different problem contexts and on the teachers' beliefs about the factors that affect the difficulty level of the equations.

The research questions were: (1) Which factors do teachers' of 5th and 6th grades believe that influence the difficulty level of arithmetic and algebraic equations that 6th graders are expected to solve? (2) How do teachers' beliefs compare to students' responses on different types of arithmetic and algebraic equations?

METHODOLOGY

The student questionnaire was made up of 14 mathematical tasks and students were asked to complete it in 40 minutes. The tasks included were designed according to the factors considered to affect the difficulty level of problems, as mentioned in the previous section. Table 1 refers to the specifications of each type of task used with a sample from each type. The first factor considered was the position of unknown quantity. Problems with the result as unknown quantity are considered arithmetic equations, whereas those with start unknown quantity are considered algebraic equations. The second factor considered was the representation format of the equations. Five formats were used: pictures, diagrams, word equations (verbal equations with no context), story problems (verbal equations with context) and symbolic equations where a geometrical shape represented the unknown quantity. The third factor considered was the number of quantities/variables in the equation. Problems were designed with either two or three known quantities and one unknown.

The teacher questionnaire was made up of the same mathematical tasks that were included in the student questionnaire. Teachers were asked to sequence them by giving a value of 1 to 14 (1 for the easiest and 14 for the most difficult) in order to evaluate the level of difficulty of each task. They were given a week's time to complete the questionnaire at their own time.

The student sample consisted of 93 grade 6 students from two urban schools in Nicosia and the teacher sample consisted of 50 grade 5 and 6 teachers in urban and rural elementary schools in Nicosia district. Their teachers administered the student questionnaires. The teachers read aloud the directions to them, supervised the completion of the questionnaires without giving any additional information, collected them and returned them to the researchers the next day.

The data were analyzed using the statistical package SPSS. The 14 mathematical tasks were ordered according to the percentage of students who successfully answered the problem. The same tasks were also ordered according to the average value of level of difficulty teachers had given in the questionnaire.

RESULTS

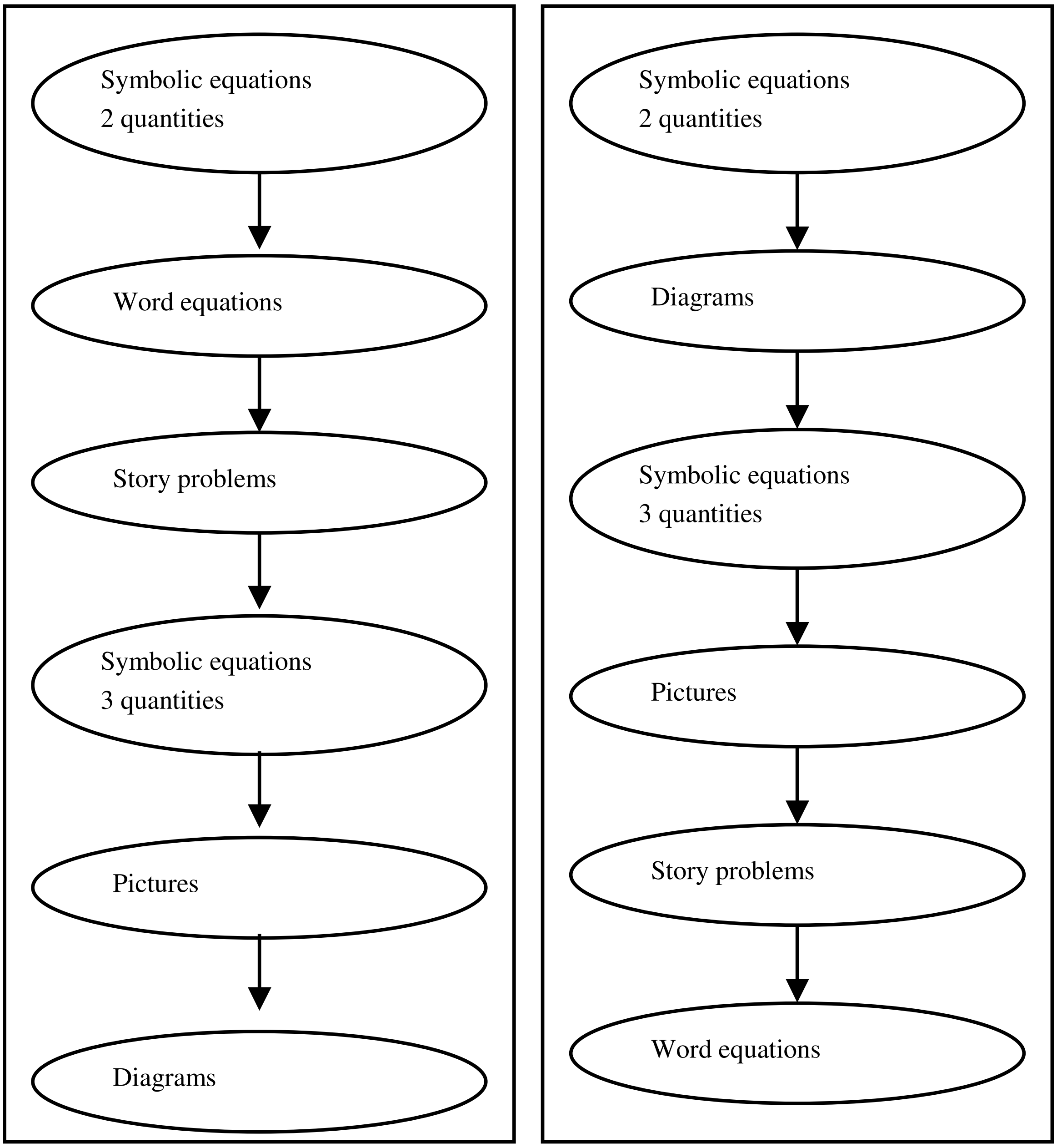
The reliability indices (Cronbach Alpha) for student and teacher questionnaires were 0,67 and 0,83 respectively. Both exploratory factor analyses for each of the student and teacher data confirmed the factors-variables used to design the mathematical tasks included in the two questionnaires.

The students' performance in the early algebra problems showed that none of the problems was very difficult for them. The percentages of students' successful responses to the problems were from 98% to 61%. The easiest problems for them were the symbolic equations with 2 quantities, the word equations with 3 quantities, the start unknown story problem with 3 quantities and the result unknown picture with 2 quantities. The problems with medium difficulty for the students were the symbolic equations with 3 quantities, the result unknown story problem with 3 quantities and the start unknown picture with 3 quantities. More difficult tasks were the four result and start unknown diagrams with 2 or 3 quantities.

Overall, teachers' believed that the algebraic equations were more difficult than the arithmetic ones. They systematically ordered problems with result unknown quantity with a smaller value of difficulty level than those with start unknown quantity, for each representation format of the problems. Additionally, they ordered problems with 2 quantities with a smaller value of difficulty level than those with 3 quantities for each representation format. As for the representation format of the problems, teachers believed that the easiest tasks for the students were the symbolic equations. Next, they believed that diagrams with 2 quantities were more difficult, along with the result unknown diagram with 3 quantities, the symbolic equations with 3 quantities and the result unknown word equation and story problem with 3 quantities. Finally, teachers believed that the most difficult problems for 6^{th} graders were the start unknown diagram, picture and word equation with 3 quantities.

Figure 1 presents the way students performed, considering the representation format of the problems, starting from the ones that students found the easiest. They were able to successfully complete symbolic equations with 2 quantities more easily than word equations. Those were easier than story problems and next were the symbolic equations with 3 quantities. Students faced difficulties solving the equations, which were presented pictorially and diagrammatically.

Figure 2 presents the way teachers ordered the problems according to how difficult they believed they were, starting from the easiest ones. They believed that symbolic equations were the easiest tasks for the students. Next were the diagrams and the symbolic equations with 3 quantities and the pictures. Teachers believed that the most difficult tasks were those represented verbally, either in a word equation format or in a story problem format.



Comparing the above two figures, one can understand that there is an agreement on the level of difficulty of the symbolic equations with 2 and 3 quantities, whether they are arithmetic or algebraic ones. Next though one can notice a disagreement between the students' performance and the teachers' beliefs on the level of difficulty for diagrams, word equations and story problems. Although teachers believed that diagrams and pictures were easier than story problems and word equations, the students' performance manifested the opposite direction. Word equations and story problems were less difficult for them than pictures and diagrams. This finding shows that students were able to respond in a better way to equations at the pre-algebraic level, which were represented verbally than pictorially.

CONCLUSIONS

Teachers' beliefs about the difficulty level of early algebra problems indicate their conceptions on the ways that their students are able to respond to them. Teacher decision making about planning and structuring the content of their teaching is greatly influenced by their beliefs on the difficulty of the activities they include in the classroom. The tasks need to be according to the cognitive developmental stage of the students. For these reasons, the accuracy and relevance of teachers' beliefs on a specific topic of the curriculum influence the ways of teaching and, consequently, the learning outcomes.

The results showed that 5th and 6th grade teachers were able to correctly predict the level of difficulty between arithmetic and algebraic equations in different representation formats. This finding is in agreement with previous research outcomes (Carpenter, Fennena & Franke, 1994; De Corte, Greer & Verschaffel, 1996) that problems with result unknown quantities are easier than problems with start unknown quantities.

On the contrary, teachers' beliefs have been found to be discrepant from the students' performance about the level of difficulty of differently represented equations. The representation format is a very important factor to consider when selecting tasks and activities for teaching concepts. As the data showed, students were able to perform better at verbal problems overall, whereas teachers believed that these tasks were harder than pictorial ones. This finding is in line with Nathan & Koedinger (2000a) who mention, "these differences have a significant role on how teachers perceive students' reasoning and learning" (p. 184). Consequently, when tasks are not in accordance with the cognitive level of the students, they are not able to respond successfully to the requirements of the lessons. This may affect their interest, participation, performance and attitude toward mathematics and their mathematical ability.

Teachers' beliefs have been found to follow the ways that this particular mathematical content is presented in the textbooks used in Cypriot schools today. This finding verifies what Nathan and Koedinger (2000b) had concluded. Consequently, it seems essential to include tasks and activities in the mathematics textbooks that are represented in pictorial, diagrammatical and verbal formats. Thus, students will be able to develop the concept of algebraic equation in a natural way as early as the elementary school, in such a way that will help them extend their knowledge later on to the symbolic formats required for further algebra study.

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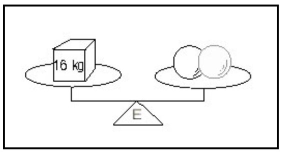
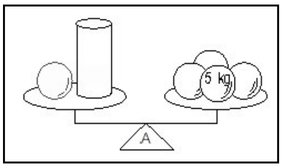
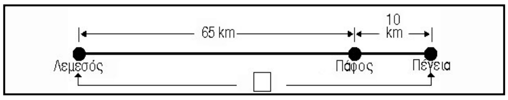
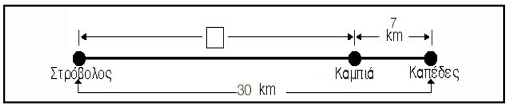
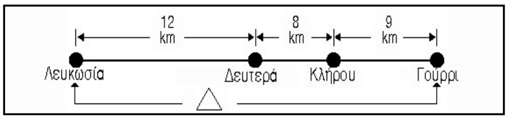
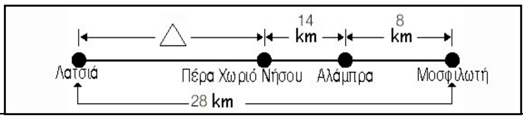
Appendix 1

Mathematical task	Position of unknown	Representation format	No of quantities
Find the value of ◯. 	Result	Picture	2
Find the value of ▯. 	Start	Picture	3
Find the value of Ì. 	Result	Diagram	2
Find the value of Ì. 	Start	Diagram	2
Find the value of Δ. 	Result	Diagram	3
Find the value of Δ. 	Start	Diagram	3
Chris played with his taws. At the beginning, he had 32 taws. At game 1 he lost 12. At game 2 he won 8. How many did he have at the end?	Result	Story problem	3
Steve bought a cheese-pie from the school canteen for 30 cents and an orange juice for 25 cents. He was left with 45 cents in his pocket. How much did he have at the beginning?	Start	Story problem	3
When I multiply 5 by 4 and add 3, what number do I get?	Result	Word equation	3
I think of a number A and multiply it by 3. Next I add 2 and I get 14. What is number A?	Start	Word equation	3
Find the value of Ì. 24 + 8 = Ì	Result	Symbolic	2
Find the value of ◊. ◊ + 7 = 30	Start	Symbolic	2
Find the value of Δ. 28 – 16 + 8 = Δ	Result	Symbolic	3
Find the value of ○. ○ + 25 – 12 = 33	Start	Symbolic	3