# PRESERVICE TEACHERS' UNDERSTANDINGS OF RELATIONAL AND INSTRUMENTAL UNDERSTANDING

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*This paper reports on the responses of a cohort of preservice primary teachers to a statement about the extent to which helping children achieve relational understanding is a realistic expectation. Although the preservice teachers' course had included teaching about understanding a number of misconceptions about the meanings of relational and instrumental understanding were evident in the responses of a sizeable minority, along with evidence that many held beliefs that were likely to result in them teaching instrumentally. The findings highlight the idiosyncratic nature of preservice teachers' knowledge construction and draw attention to a range of disparate meanings that may be attached to the term 'understanding' even when it is qualified with other words such as 'instrumental' or 'relational'.*

## BACKGROUND AND THEORETICAL FRAMEWORK

Ongoing calls for reform in education generally and mathematics education in particular have stressed the importance of teaching for understanding (e.g. NCTM, 2000). In several Australian states, including Tasmania where this study was conducted, significant shifts to values-based curricula that place a heavy emphasis on the development deep understanding are underway (Department of Education, Tasmania, DoET, 2002). It thus behoves mathematics teacher educators to prepare preservice teachers to teach for understanding.

This task is by no means simple, with the difficulty due at least in part to the difficulty of defining exactly what is meant by understanding. Madison (1982) sourced the difficulty in the tendency to equate our understandings with reality, and stressed that understandings can really only be described as beliefs. Much that has been written about understanding, including in the two documents cited above, does not attempt to define the concept, but rather a shared 'understanding' of the meaning is assumed. The danger of such an assumption was highlighted by Skemp (1978) in relation to mathematics when he described the existence of two disparate uses of the term that resulted in, in his view, two quite distinct mathematics curricula. Skemp (1978) labelled these types of understanding instrumental and relational and it is the latter which is implied by authors writing from a reform perspective (e.g. Hannula, Maijala, & Pehkonen, 2004). The term relational implies connections and indeed the development of connections is central to advice on teaching for understanding (Mousley, 2004). Mousley (2004) lists three types of connections that are commonly intended. These are connections between: new and existing knowledge; various mathematical ideas and representations; and mathematics learned in school and everyday life. It was the second of these that Skemp (1978) described.

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 161-168. Melbourne: PME. 

The process by which understanding is achieved (or connections of various kinds are made) has been described by Pirie and Kieren (1989) as recursive in that rather than more sophisticated understandings developing from more primitive ones, there is a need to revisit earlier understandings and view them from a different perspective in order to develop the next level of understanding. Sierpinska (1994) described understanding as emerging in response to difficulties encountered when current knowledge meets new, not readily reconcilable experiences. Wiggins and McTighe (1998), whose work has been influential in the Tasmanian curriculum reforms, provided a framework comprising six not necessarily discrete facets of understanding that they believed could be helpful for teachers in designing learning experiences that fostered the development of understanding. These views have in common that they present the development of understanding as complex, non-linear and unpredictable phenomenon.

All of these perspectives, as well as the underpinning philosophy of calls to reform curricula and specifically mathematics education, are consistent with a constructivist view of learning (Confrey, 1990; Simon, 2000). In describing understandings as beliefs, Madison (1982), is essentially equating understandings with a constructivist view of knowledge in which the distinction between knowledge and beliefs is principally a matter of the degree of consensus attracted by virtue of the amount and quality of information on which they are based, and their powerfulness in terms of explaining and predicting experience (Guba & Lincoln, 1989). Lerman (1997) maintained that researchers should be mindful that theories of learning apply equally to attempts to change the beliefs and practices of teachers. That is, from a constructivist perspective, teachers, including preservice teachers such as those in this study, actively construct knowledge for the purpose of making sense of their experiences (von Glasersfeld, 1990). A further dimension of constructivism derives from the work of Vygotsky (Ernest, 1998) who stressed the critical role of language in social contexts in the development of thinking.

The task of assisting preservice teachers to construct a notion of mathematical understanding as relational (Skemp, 1978) and to value this perspective to the extent that they are likely to teach in ways that foster the development of relational understanding in their students, thus amounts to an effort to change their beliefs about what it means to understand mathematics. Given the established difficulty of influencing beliefs (Lerman, 1997), the strong tendency of teachers to teach in the ways that they were taught (Ball, 1990), the fact that many will have experienced mathematics teaching aimed at achieving instrumental understanding, and the complexities involved in developing understanding of anything, including understanding itself (Pirie & Kieran, 1994; Sierpinska, 1994; Wiggins & McTighe, 1998) this is likely to be a difficult undertaking. In this context it should be remembered that the perception of misunderstanding on the part of a student is also a belief of the teacher. Essentially teachers or educators operating from a constructivist perspective but with particular outcomes for their students in mind are attempting to a greater or lesser extent to replicate their own understandings in their students, with misunderstanding deduced from evidence that students do not share their understanding.

## THE STUDY

The study was motivated by a concern that, almost 30 years after Skemp (1978) articulated the problem, teachers including preservice teachers, still attached differing and conflicting meanings to the term ‘understanding’. It was designed to provide evidence in relation to the extent that this was indeed the case for preservice teachers who had notionally ‘been taught’ about understanding in relation to mathematics and was thus part of ongoing course evaluations.

### Context of the study

At the University of Tasmania, where this study was conducted, students are required to study mathematics curriculum in three semesters of the B. Ed. program - one in each of their second, third and fourth years. Mathematics curriculum studies are combined with English curriculum studies, and so the students study three half units of mathematics curriculum. Each half-unit is conducted over 13 weeks in a single semester, delivered via a weekly one hour lecture and a one hour tutorial in second and third year, and via a two hour weekly tutorial in the fourth year. Tutorials are conducted in groups of 25-30 students. Instruction in this context is designed to be interactive with students working cooperatively on activities designed to illustrate and explore information presented in the lectures. In the tutorials, the lecturers in the program aimed to model an approach to teaching that was consistent with the principles of constructivism. In both lectures and tutorials the emphasis of teaching was on promoting students’ awareness of broad pedagogical ideas for meaningful learning of mathematics, such as the importance of rich mathematical learning environments for conceptual development, a mathematics curriculum that focuses on problem solving and thinking skills, and appropriate materials for concept representation. In lectures and tutorials, it was the lecturers’ intention to communicate these ideas through modelling best practice, using lecture and particularly tutorial times, to engage students in activities designed for such notions to surface. A further objective of the program in total was to promote students’ beliefs in the importance of mathematics and its teaching, whilst enhancing their confidence in their ability to understand basic mathematics, and fostering positive attitudes to the teaching of mathematics.

### Subjects

The subjects were 174 preservice primary teachers enrolled in the first mathematics curriculum half unit, in the second year of the preservice teachers’ study.

### Instrument

The statement to which students were asked to respond was contained in question eight of the examination paper for the unit. The two hour examination was comprised

13 questions requiring short answers in the spaces provided (two lines per mark), accounted for 40% of students' result for the unit, and was designed to assess students' understandings of the material covered in the unit rather than simply their ability to recall information. The specific question was:

> Indicate your agreement or otherwise with the following statement, giving reasons for your choice: "Helping children to achieve relational understanding is too time-consuming. There is so much in the curriculum to cover that it is an unrealistic expectation." (4 marks [of a total of 53])

### Procedure

Teaching mathematics for understanding was a topic of one lecture. The corresponding tutorial included a discussion of the understanding based on a section of the prescribed text, Van de Walle (2002), and Skemp's (1978) article on instrumental and relational understanding. Incidental references to the importance of teaching mathematics for understanding (relational) were made throughout the course and modelled in tutorials.

At the end of the semester students sat the examination and, after the assessment of the unit had been finalised, their responses to question eight were re-examined specifically for evidence of their understandings of understanding. Those that clearly evidenced misunderstandings were further examined in order to identify categories into which these responses could be divided. Some of the responses that demonstrated misunderstandings were allocated to more than one category on the grounds that they showed evidence of more than one type of misunderstanding.

## RESULTS AND DISCUSSION

Of the 174 answers examined 52 (30%) showed evidence of misunderstanding. Table 1 shows the categories of misunderstandings identified, the number of responses falling in each and an example of a response allocated to each category.

Fifteen of the preservice teachers clearly agreed with the statement presented in the question. Given that they responded under examination conditions and that the views of the lecturers who would be marking their papers were likely to have been well known, this figure is likely to be an under-estimation of the numbers who in fact believed that relation understanding was an unrealistic expectation. It seems likely that at least some students in classes taught by these teachers will not be taught with relational understanding of mathematics as the goal.

Categories two to seven all contained responses that presented relational understanding as something additional that should be aimed for, rather than essential, and hence the argument presented in the statement that time is a constraint on teaching for relational understanding is likely to have some merit in the view of these preservice teachers. A likely consequence is that amongst the demands of classroom life the goal of relational understanding will not survive. These preservice teachers may well be among the many who revert to teaching as they were taught (Ball, 1990).

| Category of misunderstanding | | Example | No. of responses (% of 174) |
|---|---|---|---|
| 1. Relational understanding is an unrealistic expectation: | a. For some students | Ideally it would be great to have every student with relational understanding … not every student in the class is going to achieve relational understanding. | 10 (5.7) |
| | b. Under some circumstances | … sometimes there is too much pressure from students, parents and government to allow time for it. | 5 (2.9) |
| 2. Relational understanding follows from instrumental understanding | | … Children need to move from instrumental understanding so that they can see why … | 9 (5.2) |
| 3. Relational/instrumental understanding is a curriculum topic | | A well organised teacher can afford to cover such a topic … | 13 (7.5) |
| 4. Relational understanding is about relating mathematics to other curriculum areas/real life | | … Students should be able to relate mathematics to almost anything as it is ever changing and growing | 7 (4.0) |
| 5. Relational understanding is about knowing the purpose/relevance of mathematics topics | | … if children only have an instrumental understanding then they are merely memorising concepts and not truly understanding what they're learning and why it is learned … | 9 (5.2) |
| 6. Relational understanding is a skill that can be applied to problems in mathematics and other curriculum areas | | … it would save time as students would be able to learn to relate the way to understanding one question to another … | 9 (5.2) |
| 7. Both relational and instrumental understanding are needed | | A child needs to have at least some relational understanding they also need some instrumental understanding … | 3 (1.7) |
| 8. Relational understanding is a teaching technique | | … Although more time consuming this method is far more beneficial than the instrumental method … | 4 (2.3) |

Table 1: Types of misunderstandings of understanding

The idea that relational understanding develops from instrumental understanding (Category 2) is perhaps related to the way in which these preservice teachers have experienced coming to understand mathematics. Brown, McNamara, Hanley and

Jones (1999) reported that many primary preservice teachers are pleasantly surprised by their initial experiences of learning mathematics for teaching and in particular enjoy achieving what could be described as relational understanding of various topics for the first time. For them, and arguably for many teachers who have been taught mathematics instrumentally, relational understanding, if it has been achieved at all, has followed instrumental understanding.

The belief that relational understanding is an additional topic in the mathematics curriculum (Category 3) was conveyed in 7.5% of all responses. It would seem that for these preservice teachers the course has been ineffectual in influencing their beliefs in relation to the nature of mathematical understanding.

Categories four and five contained responses that associated relational understanding with versions of the third kind of connections described by Mousley (2004). These students may have been influenced by the word “relational”. Their views may also have reflected personal experiences of learning mathematics devoid of context, meaning or applicability to their lives. The importance of connecting school mathematics with the lives of students is emphasised in curriculum documents (NCTM, 2000; DoET, 2002) and born out by research that suggests many students cannot see any use for the mathematics they learn at school beyond passing tests and achieving qualifications (Onion, 2004). While having merit, this view of understanding is neither complete nor that described by Skemp (1978).

Pre-service teachers whose responses fell in Category six saw relational understanding as a skill rather than a quality of understanding. It is possible that at least some of these preservice teachers in fact saw relational understanding in terms of the development of connections between mathematics topics which consequently enhanced students’ ability to apply mathematics in a range of contexts. To the extent to which this was the case, and this is not clear, this category is unproblematic and in fact would not represent a misunderstanding.

The view that instrumental and relational understanding are both necessary (Category 7) may be based on the characterisation provided by Skemp (1978) of these types of understanding as respectively knowing ‘what’ and ‘how’, and knowing ‘why’. As Hannula et al. (2004) pointed out knowledge (what) and skill (how) are inherent in mathematical understanding. The extent to which these preservice teachers regarded instrumental understanding as included within relational understanding is not clear but none articulated this view.

Responses in Category eight conveyed a belief that relational and instrumental understandings are teaching methods. These preservice teachers may have focussed on the descriptions by Skemp (1978) of instrumental and relational teaching. The emphasis on how to teach is consistent with Brown et al.’s (1999) observation that preservice primary teachers wanted to be told how to teach.

## CONCLUSION

Up to one third of the 174 preservice teachers in this cohort held some kind of misunderstanding about understanding at the end of a semester in which the topic had been approached in a variety of ways. It is recognised that the use of lectures is neither pedagogically desirable nor effective for many students, as this study attests, but they are sometimes fiscally necessary. There is a need for research on how the effectiveness of courses that are constrained to operate in non ideal modes can be maximised. In the particular context of this study, the findings have lead to the use of an electronic discussion board on which understanding is one of the topics and a variety of questions, similar in nature to that discussed in this paper, are provided to stimulate the discussion. There are also plans to modify the assessment of the unit to facilitate, to the limited extent possible, preservice teachers working with primary school students with a focus on analysing the understandings that students display.

The findings of this study add weight to calls to increase the integration of teacher education in on-campus settings and in schools (Ball & Bass, 2000). Preservice teachers need to experience examples of ‘unlikely’ students achieving relational understanding. They need powerful evidence that their own experience is not the only possible experience of learning mathematics. Mathematics educators approaching their task from a constructivist perspective should not of course be surprised that their students construct idiosyncratic understandings. Findings such as these highlight the inherent difficulty of teaching from such a stance and remind us of the challenging task for which we are preparing preservice teachers. Despite the prominence of the notion of understanding over several decades there clearly remains a need to carefully unpack the meaning attached to it by various users of the term.

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