# CONSTRUCTING PEDAGOGICAL KNOWLEDGE OF PROBLEM SOLVING: PRESERVICE MATHEMATICS TEACHERS

Olive Chapman

University of Calgary

*The paper reports a study of the knowledge preservice secondary school mathematics teachers [PSSMT] hold of problem solving and the role of a reflective-inquiry approach in creating self-awareness of, and in enhancing, this knowledge. The approach included solving problems, narratives, flow charts and observations. The finding shows that the participants were able to construct a deeper understanding of problem solving. It suggests the need for PSSMT to reflect on the learning experiences, not only from the perspective as learner, but also as teacher, in order to be able to construct a meaningful instructional approach for problem solving.*

## IMPORTANCE OF PROBLEM SOLVING

Problem solving is considered central to school mathematics. NCTM (2000) states,

> Instructional programs should enable all students to build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematical problem solving. [p. 52]

Similarly, Kilpatrick et al. (2001, p. 420) explained,

> Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics. Problem-solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Further, problem solving can provide the site for learning new concepts and for practicing learned skills.

Thus, problem solving is important as a way of doing, learning and teaching mathematics. If problem solving should be taught to students, then it should be taught to preservice teachers who are likely to not have been taught it in an explicit way. If it is to form a basis of teaching mathematics, then preservice teachers should understand it from a pedagogical perspective. This paper is intended to contribute to our understanding of these issues for PSSMT. It reports on an investigation of the knowledge PSSMT hold of problem solving and the role of a reflective-inquiry approach in creating self-awareness of, and in enhancing, this knowledge.

## RELATED LITERATURE AND THEORETICAL PERSPECTIVE

Recent studies on PSSMT include investigating their proportional reasoning (Person et al., 2004); pedagogical reasoning on functions (Sánchez & Llinares, 2003); preferred strategies for solving arithmetic and algebra word problems (Van Dooren et al., 2003); reflection on their learning process through collaborative problem solving in geometry (Bjuland, 2004); and deficiencies in specific mathematics concepts, for

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 225-232. Melbourne: PME. 

example, division, operations with integers, functions, and exponents (Ball, 1990; Even, 1993; Kinach, 2002; Wilson, 1994). While these studies do not address problem solving in an explicit way, they imply concerns about how PSSMT may conceptualize it. One study that supports this is Leikin (2003). She found that factors that influenced secondary school mathematics teachers' problem-solving preferences were their tendency to apply a stereotypical solution to a problem and act according to their problem-solving beliefs; the way in which they characterized, and their familiarity with a particular, problem-solving strategy; and a mathematical topic to which the problem belongs. A study, then, of PSSMT's thinking of problem solving could provide insights of their sense-making and how to enhance it.

Jaworski & Gellert (2003) explained that when students enter initial mathematics teacher education they already have extensive knowledge about mathematics teaching and have views on the nature of mathematics. But this knowledge is limited because it is based mainly on their experience as students. Jaworski & Gellert added that since this knowledge serves as a basis of their sense-making, an essential part of preservice teacher education is focusing on their initial personal theories and preconceptions. Reflection has been advocated as a necessary process in facilitating this. The study in this paper addresses the use of reflection as a basis of facilitating PSSMT's awareness of their thinking of problem solving. Theoretically, then, the study is framed in reflection and a social perspective of learning.

The reflective process has a long history as a basis of learning (Dewey, 1916). It is widely accepted as a key factor in facilitating teacher education (Sikula, 1996). It can enable teachers to construct the meanings and knowledge that guide their actions in the classroom and gain understanding of themselves as teachers (Schon, 1987). However, achieving effective reflection can be problematic. For example, as Lerman (1997, p.201) noted, "Reflection on one's own actions presumes a dialogical interaction in which a second voice observes and criticizes. In order to lead to learning it would seem that this must be more than the ongoing observation of one's own actions." This suggests that the reflective process could be enhanced through an interactive process with others.

A social/interactive perspective of learning has been discussed by several people including Dewey, 1916; Lave & Wenger, 1991; and Vygotsky, 1978. Lave and Wenger conceive of learning in terms of participation. Dewey emphasized learning through active personal experience and learning as a social process. In his view, purposeful activity in social settings is the key to genuine learning. Similarly, Vygotsky claimed that individual development and learning are influenced by communication with others in social settings. In his view, interacting with peers in cooperative social settings gives the learner ample opportunity to observe, imitate, and subsequently develop higher mental functions. This theoretical perspective, then, emphasizes human interactions as a key factor to facilitate learning. This formed a basis of the reflective-inquiry approach used in this study.

## RESEARCH PROCESS

The study was framed in a qualitative, naturalistic research perspective (Creswell, 1998) that focused on capturing and interpreting the participants' thinking about a phenomenon, problem solving in this case.

**Participants:** The participants were 26 PSSMT in the second semester of their 2-year post-degree education program. This was their first course in mathematics education, so they had no instruction or theory on problem solving prior to this experience. They also were not taking any other mathematics education course in this semester. They had completed all of their mathematics required for the program in their first degrees.

**Reflective-Inquiry Approach:** Since a goal of the study was to see what the PSSMT knew and what they would learn from this approach, they were not provided with any theory about problem solving before or during it. They worked on problems and in groups without the instructor's intervention. The activities were organized as follows:

***Individual reflection:*** They were required to respond to a list of questions/prompts in sequence that included: What is a problem? Choose a grade and make a mathematics problem that would be a problem for those students. What did you think of to make the problem? Why is it a problem? Is it a 'good' math problem? Why? What process do you go through when you solve a problem? Represent the process with a flowchart

***Inquiry activities:*** This included: (1) They were provided with a list consisting of a non-verbal, algebraic exercise; a simple translation algebraic word problem; a complex translation algebraic word problem; a process [non-routine] word problem; an applied [open] problem, and a puzzle problem. These categories were influenced by Charles &Lester (1982). The categories were not given to the PSSMT. They were asked: Without solving them, how are these problems similar and different? What conclusions can you make about problems? (2) They were required to write narratives of their experiences solving a problem that was assigned to them. The narrative had to be a temporal account not only of the mental and physical activities they engaged in to resolve the problem, but the emotional aspects of the experience. They later analyzed it in terms of 'stuck' and 'aha!' (3) They were required to solve an assigned problem (half got one problem and half a different one) and make notes of their thought processes. They then worked in pairs, with unmatched problems, and took turns to observe each other solve the problem while thinking aloud. They then compared their thought processes. (4) They selected a process problem appropriate for a secondary school student and used it to observe the student solving it while thinking aloud. An example of an assigned problem for item (3)/(4) is:

> Emma was always looking for ways to save money. While in the remnant shop she came across just the material she wanted to make a tablecloth. Unfortunately the piece of material was in the form of a 2m x 5m rectangle and her table was 3m square. She bought it however having decided that the area was more than enough to cover the table. When she got home she decided she had made a mistake because she couldn't see how to cut the material to make a square. But just as she despaired she had a brainwave, and with 3

straight cuts, in no time, she had 5 pieces, which fitted neatly together in a symmetric pattern to form a square using all the material. How did she do it? [Bolt]

***Group reflection:*** This included: (1) Sharing and comparing their individual reflections and their findings from the inquiry activities, preparing summaries of key words of the group thinking to correspond with the questions under individual reflection and a flowchart of the problem solving process. (2) Discussing and summarizing how they would teach problem solving. (3) Whole-class sharing of small-groups' findings.

**Data:** The reflection and inquiry activities served both research and learning purposes. Thus data consisted of copies of all of the PSSMT's written work for all of the activities. There were also field notes of their groups and whole-class discussions.

**Analysis:** The analysis began with open-ended coding (Strauss & Corbin, 1998) of the data. The researcher and a research assistant, working independently, coded the data from the pre-intervention (i.e., the self reflection) activities. The researcher and a different research assistant, who did not have access to the pre-intervention data, working independently, coded data from the post-intervention (i.e., inquiry and group reflection) activities. This allowed for cross checks by research team, elimination of initial assumptions/themes based on disconfirming evidence and validation of the findings. Coding involved, for example, identifying significant statements about their thinking of problems and problem solving and problem solving instruction. The coded information was categorized based on common themes and frequency of occurrence. Changes in the PSSMT's thinking resulting from the activities were determined by comparing the pre- and post-intervention coded information.

## PEDAGOGICAL KNOWLEDGE OF PROBLEM SOLVING

The findings are presented in terms of the PSSMT's initial knowledge of problems and problem solving, the growth in knowledge resulting from the reflective-inquiry approach and the nature of their instructional knowledge of problem solving.

**Initial knowledge:** There was consistency in the nature of the PSSMT's initial knowledge about problems and problem solving. However, there were two categories that emerged as their dominant ways of thinking. Category 1 consisted of 83% of the PSSMT and category 2, 17%. 17% of the PSSMT displayed characteristics of both categories, but leaned much more towards category 1 and were thus included there.

***Category 1:*** These PSSMT initially described a problem as something/situation that requires an answer or needs to be solved, or some variation of this, e.g., "Something, which requires an answer which requires a number of steps to find." Their examples of problems were routine or traditional word problems, e.g., for grade 9, "James is twice as old as Laura. The sum of their ages is 24. How old are they today?" and "A building is 9 meters tall and you are standing 12 meters from the base. At what angle do you have to look to see the top of the building?" In order to make the problem, they thought of the topic, mainly, "I thought of grade 8's doing percentages…" "I thought of frequencies/probabilities …" They viewed these as problems because they provided specific/key/some information to arrive at or help guide to answer, or because answer unknown/some unknown to discover/require answer. All but three of them were definite that theirs were good problems. One explained, "I think so because it is pretty straightforward. The numbers aren't too difficult to work with so they are able to focus on the concept rather than the calculation." Others mentioned: appropriate for the students, plenty information available, contributes to learning, deals with answer and process, allows for many solutions. For the exceptions, one said probably not because straightforward, another said not very interesting problem but still useful to practice application, another said okay problem but wording could be enhanced. The process they described to solve problems focused on identifying the known and unknown or identifying relevant and irrelevant information then trying to solve. For example: "I first read the problem carefully then I mark the known clearly. I then look at what is the unknown part of the problem. I then attempt to find the relationship between the known and the unknown." "First, I figure out what is being asked, then I go back through the question to see what is given, then I remember the process I need to take with the given information, and I follow the procedure." Flowcharts of the process included: (1) Read problem carefully → note known → note outcome → relate the known to the outcome if possible → solve; (2) what is the question → what is given → how do I solve → solve.

***Category 2:*** These PSSMT initially described a problem as, e.g., "A question, a challenge, an opportunity for discovery, a search for an unknown." "To be interesting, should be a question of a type that one has not already learned how to answer." "A challenge, something unknown to some parties and possibly knowable, something which can contribute to knowledge about math." Their examples of problems were more process-oriented and included, for grade 8, a diagram of a map with 6 cities in the province, to find the route(s) that allow(s) a salesman to visit each city exactly once, and how might you find the minimum distance. For grade 12, "A box is to be constructed from a piece of cardboard that is square with squares cut from each corner of length x. When the cardboard is folded into the box, x becomes the height of the box. What length of x will give the area of the box a maximum area?" In order to make the problem, they thought of "curriculum for the grade; specific class ability; relate 'local' experience; open to multiple different methods/ techniques to solve." "Something that would be challenging, intriguing, something that would cause the students to think." They viewed these as problems because the answers were not immediately apparent/obvious and they required thought and a process of struggle to find a solution. They viewed them as good problems. One explained: "Because it requires: creativity to develop a solution; multiple methods to arrive at a solution; provides openings to other related problems; disciplined thought process required." The process they described to solve problems focused on, e.g., "First understand the goal(s); examine constraints; 'play' a bit with ideas that might lead to a solution; develop one or more of ideas into a solution." "Read the problem, understand what is being asked; recall what knowledge I have regarding the topic, decide on a strategy, draw picture, equation, etc. try to solve, check to see if answer is reasonable." Flowcharts for the process included: read problem → pick out important information → decide what is being asked → decide on a strategy → try to solve → check.

**Growth in Knowledge:** The inquiry activities and group reflection served as intervention for the PSSMT to become aware of aspects of problems and problem solving they had either taken for granted, not considered or not been exposed to.

***Comparison of problems:*** This resulted in shift in awareness of problems for most of Category 1. For example: "Problems come in many different forms. They require some thinking on the part of the solver. They can have more than one possible outcome or solution." "The problems are of different types. They require different types of thinking to solve (i.e., logic, spatial, etc.)" "Problems require delving into your thought process and using your skills to sort through information and use that information to find solutions." "They … get students to ponder math in different formats." "Problems can be fun and challenging, but also stressful." From Category 2, "Problems are challenges that require an understanding and application of knowledge. Problems are solved using a variety of strategies and steps. They require thought and often more than one attempt to find a solution."

***Group Summaries:*** The group summaries consisted of their collective thinking resulting from the group-reflection activities. The summaries reflected more depth/ scope in their understanding of problems and the problem solving process. For example, one group's description of a 'good' problem included: "different methods and techniques, focus on problem solving technique - not tedious calculation, students can relate to problem." Another group's: "Should make students think, be challenging." Their description of the problem-solving process was also enhanced, particularly in terms of the flowcharts, which showed the need to move back and forth as opposed to taking a linear path to a solution. The following example of these flowcharts is simplified to fit available space. They actually were drawn with appropriate boxes and arrows. Read the problem → Do you understand the question? [1] → no [arrow to read problem] [1]→ yes → draw a diagram → devise a strategy → does the strategy seem helpful [2]→ no [arrow to devise] [2]→ yes → carry on get an answer → check the answer → is your answer right [3]→ no [arrow to devise] [3] yes → Yay! The whole-class sharing allowed the PSSMT to further extend what their individual groups constructed.

***Instructional Knowledge:*** The groups' responses to what their instructional approach for problem solving would involve focused on what the learner should do. For example, "students should read problem, write down information, determine what is relevant and irrelevant, think of ways to approach problem, write in sentence form." "Kids should learn to: understand the problem, pick out what is important, do not assume there is only one correct solution, relate the problem to what you know, but don't be afraid to try something new, do not worry if you can't see the whole solution at once." However, some groups also noted what the teacher should do. For example, "ask children different ways to do problem and identify wrong ways, demonstrate a couple of ways the children suggested, reflect on which was 'best' way, was there a 'best' way, does it make sense?"

## DISCUSSION AND CONCLUSIONS

The initial knowledge of the PSSMT indicated that most of them made sense of problems in terms of the traditional, routine problems they had experienced, directly or indirectly, prior to entering the teacher education program. They also understood these as genuine problems that require thought and logic to arrive at a solution. They understood the problem-solving process in a way consistent with the traditional classroom way of dealing with these problems. This suggests the need for helping them to become aware of, and to expand, their initial views. The series of questions/ prompts were effective in allowing for more depth in their reflection. Their responses to each question revealed another dimension of their thinking of problems. This suggests the importance of providing more than one prompt to facilitate reflection.

The group activities also enhanced their learning. Collectively, they identified a set of characteristics about problems and problem solving with more depth/scope than individually. This was facilitated by each group, although created randomly, having at least one member from Category 2 and/or one who had some characteristics of Category 2. Each group was able to construct knowledge compatible with formal theory of problem solving. This allowed them to relate to theory in a more meaningful way. They were given theory to read following the reflective-inquiry approach, which they seemed to relate to and assimilate more meaningfully than students I worked with in the past who did not engage in the approach. However, they were unable to conceptualize problem-solving instruction from the approach on their own. Their instructional approach implied teaching by telling or being teacher directed. They did not seem to notice/consider the instructional approach they participated in through the reflective-inquiry approach as a basis of constructing/thinking about their own. It required shifting their perspective of the approach from that of a learner to teacher. This was done by allowing them to engage in reflection on the approach and role-play. Details are not provided here given limitation on space.

The paper provides information about PSSMT's initial knowledge and the type of knowledge they could construct on their own from particular self-inquiry activities. It highlights the need to explicitly address pedagogical problem-solving knowledge in teacher education. It suggests that it is essential to provide constructive and reflective opportunities followed by theory to deepen PSSMT's understanding of problem solving. It suggests the need for them to reflect on learning experiences not only from the perspective as learner, but also as teacher to construct meaningful pedagogical knowledge. It provides a structure/model to make sense of PSSMT's knowledge of problem solving and a practical and effective approach to facilitate their self-reflection and construction of meaningful knowledge about problem solving.

### References

Ball, D.L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21, 132-144.

Bjuland, R. (2004). Student teachers' reflections on their learning process through problem solving in geometry, *Educational Studies in Mathematics,* 55(1-3), 199-225.

Charles, R. & Lester, F. (1982). *Teaching problem solving: What why & How*. Palo Alto, CA: Dale Seymour Publications.

Creswell, J.W. (1998). *Qualitative inquiry and research design.* London: Sage.

Dewey, J. (1916). *Democracy and education.* New York: Macmillan.

Even, R. (1993). Perspective secondary teachers and the function of concept. *Journal for Research in Mathematics Education*, 24 (2), 94-116.

Jaworski, B. & Gellert, U. (2003). Educating new mathematics teachers. In A. Bishop, M. Clements, C. Kietel, J. Kilpatrick, & F. Leung (Eds.) *Second International Hand- book of Mathematics Education*. The Netherlands: Kluwer Academic Publishers.

Kilpatrick, J., Swafford, J., and Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kinach, B. (2002). Understanding and learning-to-explain by representing mathematics, *Journal of Mathematics Teacher Education*, 5, 153-186.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* New York: Cambridge University Press.

Leikin, R. (2003). Problem solving preferences of mathematics teachers: Focusing on symmetry. *Journal of Mathematics Teachers Education,* 6(4), 297-329.

Lerman, S. (1997). The psychology of mathematics teachers' learning: In search of theory. In E. Pehkonen (Ed.), *Proceedings of PME* 23 (vol. 3, 200-207). Lahti, Finland: PME.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Person, A.C., Berenson, S.B., & Greenspan, P.J. (2004). The role of number in proportional reasoning: A prospective teacher's understanding. *Proceedings of PME 28* (vol. 4, 17-24). Bergen, Norway: PME.

Sánchez, V. & Llinares, S. (2003) Four student teachers' pedagogical reasoning on functions. *Journal of Mathematics Teachers Education*, 6(1) 5-25.

Sikula, J. (1996). *Handbook of research on teacher education*. NY: Macmillan.

Strauss, A. & Corbin, J.: (1998) *Basics of qualitative research: Techniques and procedures for developing grounded theory.* Thousand Oaks, California: Sage.

Van Doormen, W., Verschaffel, L., & Onghena, P. (2003). Preservice teachers' preferred strategies for solving Arithmetic and Algebra word problems. *Journal of Mathematics Teacher Education*, 6(1), 27-52.

Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Wilson, M.R. (1994). One pre-service secondary teacher's understanding of function, *Journal for Research in Mathematics Education*, 25(4), 346-370.