STUDENTS' REFLECTION ON THEIR SOCIOMATHEMATICAL SMALL-GROUP INTERACTION: A CASE STUDY

Petros Chaviaris and Sonia Kafoussi,

University of the Aegean, Greece

In this paper we present a case study of a small group of two 11 years old students who participated in a research program whose the purpose was to investigate the way that students can be actively involved in a reform of their own behavior as they cooperate in small-groups to solve mathematical problems. We study the opportunities that were offered for the development of the small-group students' interaction in mathematics in two alternative environments: a) the students' observation and discussion on their videotaped cooperation and b) their participation in dramatic role-play. The results of the research showed that both environments gave the group members the opportunity to reflect on their actions and the consequences of their actions during their cooperation and to achieve the development of new effective social rules.

THEORETICAL BACKGROUND

The investigation on social interaction that takes place in classrooms' microculture continues to be an issue of great interest among the mathematics educators researchers. This is a consequence of the acceptance that in order to make sense of students' learning of mathematics, classroom life has to be interpreted not only from a psychological perspective but from a sociological perspective as well (Cobb & Bauersfeld, 1995; Lerman, 2001). Towards this effort many researchers have developed theoretical constructs for the study of the relation between student's cognitive development and social interactions in the classroom. For example, interpretative constructs for this purpose are the social and sociomathematical norms (Yackel & Cobb, 1996), the thematic patterns of interaction (Voigt, 1995), the meta-discursive rules (Sfard, 2001). It is widely accepted that the way that the members of the classroom develop rules that guide their social behavior determine the evolution of their mathematical discourse. Moreover, there is a reflexive relationship between the sociomathematical interaction and students' beliefs and values about their own role, others' role, the general nature and the goals of mathematical activity (Yackel et al., 2000).

In this tradition the research has mainly focused on teacher's role of initiating and guiding the formation of the rules of sociomathematical interaction (McClain & Cobb, 2001). However, little research has been done on students' role in the development of their social behavior in mathematics classroom (Hershkowitz & Schwarz, 1999). The investigation of the role of different environments that give opportunities for students' reflection on their mathematical discourse is a critical question.

In this paper we present a case study of a small group of two 11-year-old students who participated in a broader research program whose purpose was to investigate the way that students can be actively involved in a reform of their own behavior as they cooperate in small-groups to solve mathematical problems. More specifically, we study the opportunities that were offered for the development of small-group students' interaction in mathematics in two alternative environments: a) the students' observation and discussion on their videotaped cooperation and b) their participation in dramatic role-play.

METHOD

The two students were participated in a research program realized in a fifth grade of a typical public school of Athens in autumn of 2003, which lasted four months. Initially, in order to construct the students' profile, they were interviewed about their beliefs for their own role, others' role, the general nature and the goals of mathematical activity. Furthermore, we recorded their parents' beliefs about the mathematical activity of their children in school as well as in home. During their mathematical activity in the classroom, the two students worked in group and their cooperation was videotaped once a week. The mathematical topic they discussed during the research program concerned the concept and the operations of fractions. After a session of cooperation the members of the group participated in a meeting with the researcher. During this meeting, the students observed and discussed on issues concerning their videotaped cooperation. These discussions were tape-recorded. Moreover, the students of the group were obliged to organize and to present drama role-plays in the classroom based on the experiences of their cooperation. These role-plays were videotaped. At the end of the program, the members of the group were interviewed about their own role and the others' role in mathematics. So, the data consisted of the videotaped recordings of the small-group's work in mathematical lessons, the tape recorded students' discussions about their own videotaped cooperation, the videotaped recordings of the students' role-play and the protocols of tape recorded clinical interviews conducted with each student at the beginning and at the end of the program.

The discourse analysis of the group's engagement in classroom mathematical activities was based on interactivity flowcharts that Kieran and Sfard have developed (Sfard & Kieran, 2001). The mathematical discussion of the group was analyzed according to the way that the members negotiated their mathematical activity (who offered the solution, what kind of solution offered, how explained their thinking, how every member of the group was influenced by the other, etc.). The tape recorded students' discussions about their own videotaped mathematical cooperation were analyzed according to: a) the way that the students assessed their cooperation, b) the critical moments of their interaction and c) the targets they put for their next cooperation. The role-plays were analyzed according to: a) the roles that the students chose to play, b) the relationship between drama text and their cooperation in mathematics and c) their comments for this experience.

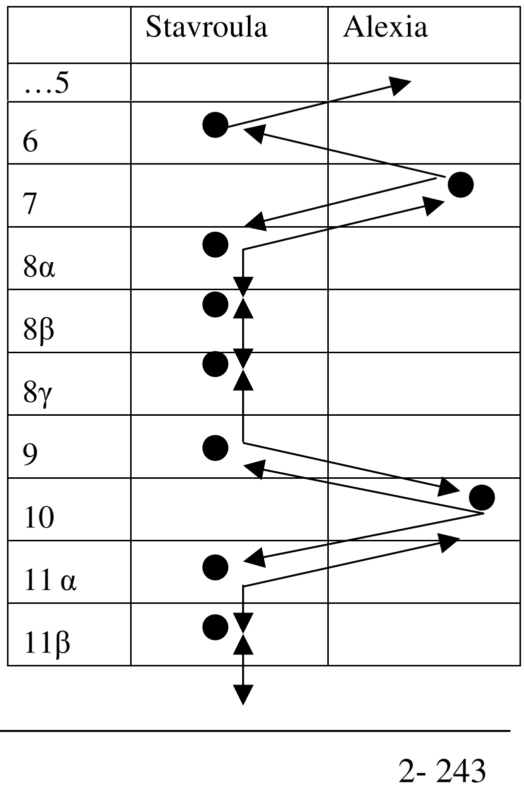
RESULTS

We chose to present the work of this group (Stavroula and Alexia) because these two students had different beliefs about the role of cooperation in mathematics and they had different capacities on this lesson. Firstly, we present the students' profile before their cooperation and then the development of their reflection on their cooperation in mathematics through the two alternative environments.

The children's profile

Both students had developed their beliefs in a traditional context of mathematics teaching. The goals posed by both students for their mathematical activity concerned the result of their effort (right or wrong) and not the process. Nevertheless, they had different conceptions about the role of cooperation in mathematics. Stavroula considered the cooperation to be an obstacle in the understanding of mathematics, because she believed that *"if someone doesn't work on his own, he cannot understand mathematics"*. On contrary, Alexia believed that cooperation could help her to control her thoughts before she announced them in the classroom and so she could *"avoid mistakes"*. Moreover, we should mention that the students' parents attributed to the cooperation in school mathematics a social role and not a cognitive one, that is they conceived the cooperation as a means for students' socialization. As for the two students abilities in mathematics, Stavroula was a student that managed to find solutions on mathematical problems on her own and Alexia was a student that, most of the times, need some help to complete a mathematical activity.

At the beginning of their cooperation Stavroula and Alexia worked individually and they didn't negotiate their ideas. Most of Stavroula's utterances were addressed to herself, revealing, this way, a private discourse and very few utterances indicated a challenge for reaction from her interlocutor's part. The few utterances of Alexia had mostly the form of a challenge for reaction from Stavroula's part and were related to her effort to understand her classmate's solution. Stavroula was guiding the dialogues that were developed by presenting her solution to Alexia without arguments or explanations about it, while Alexia didn't challenge her classmate to explain her solutions. The interactivity flowcharts of their initial cooperation had a form like the next one.



Students' reflection as they observed their videotaped cooperation

Concerning the way that the two students assessed their cooperation, we can notice that the two students experienced it in different ways. Initially, Stavroula assessed the evolution of their cooperation mostly based on the solution (wrong or right) of their mathematical activity. On the contrary, Alexia was based on the type of their interaction, that is if her interlocutor gave her some help. However, both of them had an awareness of the quality of their cooperation (productive or not). For example, after an unproductive cooperation the students commented:

Stavroula: I wanted to write on my own as I was used to, but afterwards I thought that we must cooperate and so sometimes we discussed.

Alexia: We tried to cooperate, I asked Stavroula to discuss the problem, but we sometimes managed it.

The critical moments of the cooperation that they gave the group members the opportunity to reflect on their actions were related to the existence of conflicts. These conflicts were connected to: a) the existence of different ideas and the failure of investigating them and b) the type of explanations that each member offered and the lack of understanding from the partner.

For example, at the beginning of the program, the children discussed about their cooperation in which they had to solve the following problem: In Alexandra's Avenue, public works are being made by 3 different firm of constructors. The works are being made at three different points. The first firm of constructors makes works at a point corresponding to the 1/3 of the avenue, if we count from its beginning. In the ¾ of the avenue there are works of the second firm of constructors and in the 5/6 of the avenue there are works of the third firm of constructors. Note in the following schema where the works are being made. Use red color for the first point, green for the second one and blue for the third one .

beginning ____ end

When they observed their videotaped cooperation, they had the opportunity to reflect on their failure to negotiate Alexia's idea:

Researcher: Did you have different ideas about the solution of the problem?

Stavroula: I said to count with a rule and to put centimeters, but Alexia said to divide it in small pieces.

Researcher: What did you do after your conflict?

Stavroula: I tried to do what I said.

Alexia: Me too, I tried to do what Stavroula said, but I did not manage it.

Researcher: Did you discuss your different ideas, let's say who had right and why?

Alexia: No.

Researcher: Let's observe at the video the solution given in the class… The solution at the blackboard with whose idea does it matche? Stavroula's or Alexia's?

Stavroula: With Alexia's idea…

Observing another video of their cooperation, the students had the opportunity to reflect on the value of explanation for the development of a productive mathematical discussion. For example, the following dialogue took place:

Researcher: How could you understand Alexia's thinking?

Stavroula: She had to explain to me her solution better.

Researcher: Alexia, did you try to solve the problem as Stavroula proposed and you said that you didn't manage it?

Alexia: I had not understand what she said.

Researcher. What could you do then?

Alexia: I could ask her to explain to me her solution again.

About the way by which the two students put goals for the evolution of their next cooperation, we observed the following: At the beginning, the goals posed by the students were common and general (e.g. "*to cooperate more", "to solve the problem together*"). Afterwards, their goals concerned concrete actions that they were addressed to their interlocutor (e.g. *"Stavroula must explain to me her solution*"). Finally, the goals became personal and concerned their own actions about their interaction (e.g. *"I have to think more about what Alexia wants", "I have to listen what Stavroula says").* At this phase, the goals reveal the mutual responsibility that the students managed to develop concerning their cooperation in mathematics.

Students' reflection as they participated in dramatic role-play

Alexia and Stavroula chose to represent a discussion between two students in the classroom, as they tried to solve a problem that was difficult for the one student. The scenario that they designed and played was the following:

[1]Alexia: A fruit-bowl contained 21 apples. George ate 2/3 of the apples. How many apples did they remain?

[2]Stavroula: Ah! It seems difficult!

[3]Alexia: Think again about it. It is easy.

[4]Stavroula: Help me a little.

[5]Alexia: What do you mean "a little"?

[6]Stavroula: Such a little! (She shows with her hands.)

[7]Alexia: If I help you such a little, the problem will be solved by myself and not by yourself !

[8]Stavroula: It doesn't matter at all!

[9]Alexia: It doesn't matter at all? It matters a lot, because you will not learn it.

[10]Stavroula: Oh! You talk like my mother! She told me the same things.

[11]Alexia: She has right! You should solve it alone.

[12]Stavroula: Come here now!

[13] Alexia: What do you want?

[14]Stavroula: Do you want to solve it together, to discuss about it?

[15] Alexia: OK.

[16]Stavroula: How did you solve it?

[17] Alexia: Look here, we can divide the apples in three parts.

[18]Stavroula: Ah! Three times seven …21, every part has 7 apples.

[19] Alexia: Yes, what about the 2/3?

[20] Stavroula: 7 plus 7… Ah! It ill be remained 7.

[21] Alexia: OK.

The above scenario developed in two scenes: at the first scene [1-13] the students chose to represent difficult moments of their cooperation and at the second scene [14-21] represented a productive cooperation. Concerning the choice of the roles, we should mention that Alexia and Stavroula decided to play the opposite roles in relation to those that they experienced at their cooperation in mathematics. Alexia played the student who managed to solve the problem alone and Stavroula chose to play the student who need help. Their experiences during their cooperation were impressed on their play. More specifically, Stavroula's belief that mathematical learning is only an individual process was mentioned by Alexia at the phrases [7, 9, 11]. The continual efforts of Alexia to challenge Stavroula's cooperation printed on Stavroula's phrases [4,8]. The change of Alexia's behavior during the dramatic play prints the evolution of their cooperation in mathematics. Moreover, an interesting fact is the comments made by the students for their parents' beliefs about their mathematical activity [10, 11].

The following discussion took place in the classroom after the dramatic play:

[1] Researcher: Do you want to talk about the roles? How do you feel about your role?

[2] Alexia: I don't think that I am egoist because I finally helped Stavroula. I felt nice because I helped her, I did not solve her the problem, I just helped her.

[3] Stavroula: I felt a little upset at the beginning, when I asked her to help me. When I persuade her to discuss, I felt nice.

[4]Researcher: Very good. Who want to talk about the cooperation that your classmates showed to us?

[5]Student 1: At the point where she told her "solve it alone", she felt upset. Then they began to discuss and they solved the problem together, it was good. I said Kostas the same thing. (Kostas was his partner)

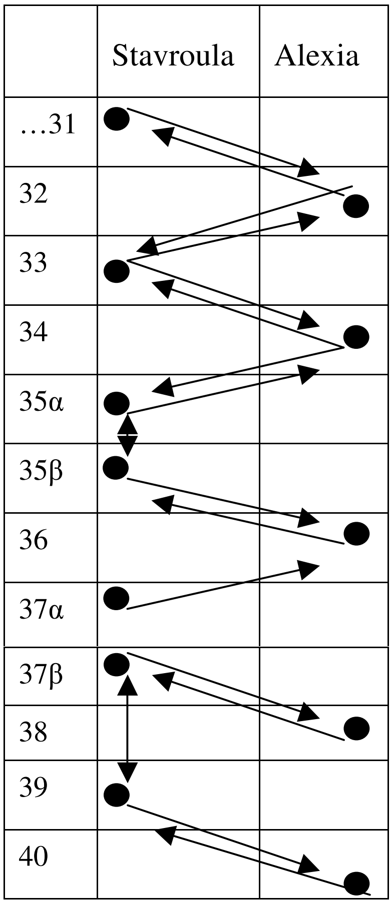
[6] Stavroula: Do you want to explain why I chose this role?

[7] Researcher: Yes.

[8] Stavroula: I chose this role because I usually solve the problems quickly and then I help Alexia, but I did not know how it is if someone does not understand the problem.

This environment gave the students the opportunity to think and to express their feelings about the social actions that take place by themselves as well as by their partners [2,3]. The presentation of the dramatic play in the classroom gave the opportunity to the other members of the classroom to reflect on their own behavior during their cooperation in mathematics.

During the last month of their cooperation, the two students developed productive cooperation as, most of the time, each interlocutor challenged the other's participation. Most of Stavroula's utterances were addressed to her interlocutor. These proactions had mostly the form of questions (request for approval of a suggested mathematical action, request for explanation). Furthermore, she seems to take account of Alexia's reactions in several moments of their discussion. Alexia participated more actively, she did not only tried to understand Stavroula's solutions, but many of her utterances related to the production of a solution and not to the request of an explanation. The interactivity flowcharts of their cooperation had a form like the next one.



Based on the previous analysis, we noticed that the members of this group formed social rules that allowed the development of productive cooperation in mathematics. More specifically, the students explain their thoughts without prompting, they try to make sense to their interlocutor's explanations and justifications, to express their disagreements and to share the responsibility of their actions.

At the final interview about their beliefs for mathematical cooperation, the students said: "It is beautiful to cooperate in mathematics, I can listen other opinions, sometimes better from mine, I don't feel alone when we have to solve a difficult problem…" (Stavroula), " I like to work together because everyone talks about its opinion and we can find a better solution …" (Alexia).

CONCLUSIONS

These environments gave the group members the opportunity to reflect on their cooperation, to evaluate it and to pose goals about the improvement of their mathematical discussion. Although the two students had considerable differences in their beliefs about the role of cooperation in mathematics, they managed to achieve the development of new effective social rules. Both environments (observation of their videotaped cooperation and dramatic role-play) allowed them to reflect on their actions and on the consequences of their actions and to feel the necessity of new rules in their cooperation.

The first environment gave them the opportunity to focus their attention on these moments that were obstacles for the development of a productive cooperation (the consequences of the non exploitation of an effective idea and the lack of understanding of partners' explanation of the mathematical solutions). The second environment offered them the opportunity to experience the role of the other member of the group, to experience the whole history of their cooperation and to express their thinking about it without prompting in front of the other members of the class.

An open question that arises from this research concerns the way that these environments which promote students' reflection on their sociomathematical interaction can be incorporated in teaching practice of mathematics.

References

Cobb, P. & Bauersfeld, H. (1995). Introduction: The coordination of psychological and sociological perspectives in Mathematics Education. In P. Cobb and H. Bauersfeld (eds.), *The emergence of mathematical meaning: Interaction in Classroom Cultures*, pp. 1-16. NJ: LEA.

Hershkowitz, R. & Schwarz, B. (1999). The Emergent Perspective in Rich Learning Environments: Some Roles of Tools and Activities in the Construction of Sociomathematical Norms. *Educational Studies in Mathematics*, 39, 149-166.

Lerman, S. (2001). Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics. *Educational Studies in Mathematics*, 46, 87-113.

McClain, K. & Cobb, P. (2001). An Analysis of Development of Sociomathematical Norms in One First –Grade Classroom. *Journal for Research in Mathematics Education,* 32(3), 236-266.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46, 13-57.

Sfard, A. & Kieran, C. (2001) Cognition as communication, Rethinking learning by talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture and Activity*, 8(1), 42-76.

Voigt, J. (1995). Thematic Patterns of Interaction and Sociomathematical Norms. In P. Cobb & H. Bauersfeld (eds.), *The emergence of mathematical meaning: Interaction in Classroom Cultures,* pp.163-201. LEA.

Yackel, E. $ Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education,* 27, 458-477.

Yackel, E., Rasmusen.C & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *Journal of Mathematical Behavior*, 19, 275-287.