THE TACIT-EXPLICIT NATURE OF STUDENTS' KNOWLEDGE: A CASE STUDY ON AREA MEASUREMENT

Cristina Frade

Universidade Federal de Minas Gerais – UFMG – Brazil

This paper reports on case study that investigated the development of mainly tacit and mainly explicit components of knowledge of area measurement of a student-pair. The research covered two terms or periods of the students' learning of the subject: when they were aged 11 to 12 and when they were aged 12 to 13. The data analysis was based on Ernest's model of mathematical knowledge, with reference to its mainly tacit and mainly explicit components, and Kitcher's ideas about the development of mathematics practice. The results of the research reinforced our hypothesis that students' mathematical knowledge displays a very similar structure to that of the mathematical knowledge of the mathematicians.

INTRODUCTION

The concept of tacit knowledge does not have a single meaning. As discussed in Frade's (2004) work some researchers address what can be called *Polanyi's psychological version of tacit knowledge*: knowledge that functions as subsidiary to the acquisition of other knowledge. Other researchers use the words *tacit* and *explicit* as opposites to refer to different, but complementary ontological dimensions of the same component of a certain practice. Whatever meaning we choose – psychological or ontological – the researchers quoted by Frade (*ibid*) share in some way Polanyi's (1969) epistemological thesis that all knowledge is tacit or constructed from tacit knowledge: put it in another way, language alone is not enough to render knowledge explicit.

We used the two above-mentioned meanings of tacit knowledge in a research to investigate its manifestation in empirical data. Our research was carried out in a mathematics classroom of a Brazilian secondary school and consisted of two sequential studies. In the first study (see Frade, 2004) we analysed an episode related to a class discussion about the difference between plane figures and spatial figures. The aim of this study was to identify how the mainly tacit and mainly explicit components of students' knowledge (see Ernest, 1998) could manifest in learning processes or in a subsidiary way, from Polanyi's (1962, 1969) perspective. The results of the research strongly pointed to a perspective of cognition not necessarily restricted to and coincident with language, but seen as a situated social practice, moving between the poles of the tacit – effective action – and the explicit – intersubjective projection of such an action – dimensions.

In the second study a student-pair (2 boys) in the same class was investigated as they undertook different mathematical tasks on area measurement. Here, the research covered two terms or periods of the students' learning of the subject: when they were aged 11 to 12 and when they were aged 12 to 13. The aim of this study was twofold:

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1) to observe the development of the mainly tacit and mainly explicit components of the student-pair's area measurement knowledge; 2) to provide more information on how the tacit and the explicit interact during tasks involving conversation. The study in question and its results are presented in this paper. In particular, we highlight the first aspect of our analysis, for the later was intensively discussed in the first study.

THEORETICAL FRAMEWORK

Ernest (1998) uses the ontological meaning of tacit knowledge to classify the nature of the components of his model of mathematical knowledge. To the author, mainly explicit mathematical knowledge is related to those types of knowledge that can be communicated through propositional language or other symbolic representation, as for instance: 1) accepted propositions and statements (e.g. definitions, hypotheses, conjectures, axioms, theorems); 2) accepted reasoning and proofs (all types of proofs including the less formal ones, inductive and analogical reasoning, problem solution including all analysis and computing); 3) problems and questions relevant to be solved by the mathematicians (e.g. Hilbert's problems, Last Fermat's theorem). Alternatively, mainly tacit mathematical knowledge is related to the ways in which the mathematicians use their knowledge, as well as how they appropriate mathematical experiences, values, beliefs through their participation in mathematics practice. And this, says Ernest, cannot be fully communicated explicitly. As mainly tacit components of mathematical knowledge he cites: 4) knowledge-use of mathematical language and symbolism; 5) meta-mathematical views, that is, views of proof and definition, scope and structure of mathematics as a whole; 6) knowledge-use of a set of procedures, methods, techniques and strategies; 7) aesthetics and personal values regarding mathematics[1].

From this perspective, we hypothesized that the students' mathematical knowledge could display a similar structure to that of the mathematical knowledge of the mathematicians. Thus, the above-mentioned components were those ones which we search to identify in the case study. To this end, we proposed an adaptation of Ernest's model of mathematical knowledge to the students' knowledge. Such adaptation is illustrated in the next section, and accounted for the fact that the students are learners and part of the learning process consists in a gradual improvement of their understanding and procedures, which in their initial manifestation may seem mistaken from the viewpoint of the discipline. In particular, the component *aesthetics and values* was associated with the students' predisposition, motivation and participation in classroom practices, or else to the students' mathematical identity as, for example, Boaler (2002) and Winbourne (2002) put it. Therefore, this component has a macro character in the sense that it is a necessary

[1] In this presentation of Ernest's model, the first five components were proposed by Kitcher (1984) whereas the last two ones were proposed by Ernest. Due to lack of space we opted not to present the arguments used by Ernest to classify the model's components as mainly explicit or mainly tacit. These arguments are very insightful and can be seen in Ernest (1998).

condition for the development of the remaining components. The component *problems and questions* was not investigated, as it was difficult to adapt it adequately to the students' knowledge.

THE CASE STUDY

This study consisted of a set of short sequential episodes – seventeen in total – constructed to identify the student-pair's stages of development in terms of the mainly tacit and mainly explicit components of area measurement. The data were collected from their work on mathematical tasks (e.g, oral and written exercises, problem solving, individual tests, interviews) and from audio and video recording of the student's class work. In all episodes that involved mathematical conversation we also examined the internal articulations that preceded the students' utterances, applying the categories presented in the first study (see Frade, 2004) : priority of tacit, tacit on the borderline with the explicit, tacit coincides with explicit, explicit separate from tacit, explicit under check. Bellow we provide a description of an episode to exemplify how the data were treated in this study.

Episode 1: Student 1 confuses a counting, and student 2 discovers the multiplication formula length× width.

During the course of classes 1 and 2, student 1 and student 2 were doing some exercises proposed by their textbook. They were trying to calculate how many ceramic tiles covered the floor of a rectangular room. The book displayed the drawing of the room, which facilitated the students in counting the tiles. Calculating area by counting the units of measurement was the only procedure worked at class, until then. While student 1 finds apparent meaningless numbers, student 2 discovers the multiplication formula length×width. Let us see what happened in the protocol below transcribed from audio tapes:

Student 2: What is the result?

Student 1: 71 and 57.

Student 1: 178.

Student 2: 15, 1, 2, 3...15 times 12. 170? Because here, look 1, 2, 3 ... 16. (Student 2 multiplication is incorrect: 15×12 = 180)

Student 2: 1, 2, 3... 16. 16 times 12. 192.

Student 2: It's 16. 1, 2...16.

Student 2: The result is 192, isn't it? Because here, look, 16, we have to count the width of the tiles.

Student 1: 178

Student 2: 178?

Student 1: Then write it there: 178. (...) There are 178, 178 tiles on the floor.

Student 2: On the floor, on the floor.

The underlined utterances show that, instead of counting the number of tiles that covered the floor, student 2 chooses to multiply the number of tiles in each row by the number of tiles in each column. However, the utterances in italics suggest that he gave up of that procedure, probably influenced by student 1's insistence to give the final result: 178. Searching for a better understanding of the students' calculations, we have analyzed their written registers of the exercises. Both students wrote: '178 tiles', but they did not record any calculation or reasoning.

Student 2's utterances seemed to be good external representations of what he was thinking while solving the problem, as it was possible to infer about his reasoning. The same cannot be said in relation to student 1's utterances. When he refers to numbers 71 and 57 it is possible that the tacit could be prevailing over the explicit: the clues gave by him were extremely vague. And this would only support an equally vague hypothesis about his reasoning, which could not be publicly checked.

In short, this episode captures a moment of strategy choice by student 2: the above-mentioned multiplication. This strategy choice was identified with a manifestation of the model's component *knowledge-use of a set of procedures, methods, techniques and strategies* by student 2. The component *reasoning and proof*, which includes the students' argumentations and computations, is also identifiable in this episode, for example, in the stated computation '1, 2, 3... 16. 16 times 12. 192' made by student 2. It is interesting to note that this component manifests when student 2's strategy choice looses its 'tacitness', or else when this strategy choice become explicit through the stated computation. The internal articulations identified were: *tacit coincides with explicit* for student 2, and *priority of tacit* for student 1 (see Frade, 2004).

End of episode 1

As exemplified in episode 1, we identified all components of Ernest's adapted model (see table 1) in the episodes with variable intensity and visibility. Further, the analysis showed that, throughout the students' learning, some components of the model predominated over others. The components *statements, proofs and reasoning, language and symbolism, methods, procedures and strategies* appeared more clearly and more often than, for example, the components *propositions* and *aesthetics and values*. It is possible that the component *propositions* was not so evident in the analysis because of the way the study of area measurement was approached during the two stages of the research: propositions was not stressed as an objective of teaching at this level of the course. Yet the identification of the component *aesthetics and values* demanded more effort in terms of reflection and interpretation (probably due to the macro feature attached to it as argued previously). On the other hand, the criteria (motivation, interest, high level of interaction between students and between the students and the teacher) used to select the student-pair for the study directed us towards students who had already shown some identity as participants in mathematics practice in school as well as some taste for mathematics, some sense of aesthetics and values concerning the discipline or some of its aspects.

Component/ Nature	Activity	Example
Propositions and statements/ Mainly explicit	The teacher and the students discussing the following proposition for K=3: if the sides of a rectangle are multiplied by K (K > 1), then its area grows K^2 times.	'It's, its side is 2 and D is 6… Then I saw that the area of the square C equals 4 and that of square D equals 36, which is 9 times bigger. Then after my mother gave me another example that it did not matter, that it was only that the number should be the triple, she put it here side 3 and the area 9. In the other 9, that is, if the side is 9 then it's 81, it's the same thing.'
	Answering a written questionnaire question: what do you mean by 'area' in mathematics?	'I remember that area represents a certain space or place. Based on that we can find that area is used to calculate the size of a space or place. Take the example of a piece of land, how many square meters it has. This is already a way to use area as measure.'
Reasoning and proofs/ Mainly explicit	One of the students of the pair explaining to a classmate how they solved the following problem: a wall with height of 2.30 meters and length of 8.76 meters built with square tiles having sides measuring 2 centimetres. Calculate the number of tiles on the wall.	'We found out that the area of the wall is, we multiply length times width and to obtain the area of the tile we multiply side times side. When we find the result of the two, we divide the area of the wall by the area of the tile. The result was…'
	All the activities in which the students proved a proposition or displayed calculations or any other form of computation.	Area calculation of rectangles by counting the units of measure or using the formula: length × width.
Language, symbolism/ Mainly tacit	All the activities.	Oral and written language, and mathematical symbolism used by the students to communicate their area measurement knowledge in class.
Meta-mathematics views/	Written report in which the students had to reflect and to express the general	Student: …the spatial figures that can have volume, seem to be real. Teacher: Okay, but what does this mean,

Mainly tacit	view they constructed on area measurement and excerpts of conversations where the students made some ontological reference to a mathematical entity.	why did you say that is seems real? Student: This thing [spatial figure that can have a volume] here looks like an egg. Teacher: Oh, yes. Student: This thing [plane figure] here seems to be kind of a drawing. Teacher: Oh, yes, this one here seems to be a concrete object; what about that one? Student: No.
Methods, procedures, techniques and strategies/ Mainly tacit	Problem solving. For example, the students were asked to calculate the perimeter of a rectangular piece of land with an area of 450m2 and 25m in length.	'Now we have to find which number that is. The length is 25, we already know. And what about this one here? 25 times 20. Wait, I understood. 25 times 21. Oh, oh, no God, this is too much.'
Aesthetics and values Mainly tacit	All the activities.	This component was observed in terms of students' curiosity, interest, motivation and participation in classroom practices. In all the episodes that involved conversation we found a high degree of interaction between students of the pair, and, in many cases, between them and the teacher. This was interpreted as indicating the students had some affective components in relation to mathematics.

Table 1 – Components of Ernest's model identified

To explain the development of mainly tacit and mainly explicit components of the student-pair's knowledge of area measurement we found support in Kitcher's ideas (1984) about the development of mathematics practice. First, we could see that many episodes evidenced that the students of the pair had built new statements or rebuilt previously known statements. According to Kitcher, this action results necessarily in the development or in the change of mathematical language and symbolism. For example, a diagnostic questionnaire at the beginning of the first stage of the research showed that student 1 had some previous concept of area as a physical geographic space: 'The word area means a certain location or piece of land, or space.' Another questionnaire at the end of the second stage of research showed that student 1's concept of area had evolved and a specific measure had been incorporated into it: 'I think that area is a place or space…used to calculate the size of a place or space'.

Kitcher says that the component *proofs and reasoning* develops or changes, for example, when new reasoning is added. As this component was identified in various episodes when the students had to work on new concepts and procedures, it seems reasonable to say that both the students developed this component in general. In relation to *proofs*, the analysis demonstrated that both students improved their knowledge: they started their area calculation by counting the units; at a later stage they found, although at different times, that such counting relates to a multiplication (length×width, in case of rectangles); at a third stage, they have this multiplication as a formula. Although the development of the component *methods*, *techniques and procedures* had been identified in many episodes, how the other components affected it was not clear. This could be due to the fact that this component can be said to be the most tacit of all, as we are not given privileged access to mental processes to know when or how a technique or strategy is chosen.

Kitcher argues that the changes in the component *meta-mathematics views* are rooted in the changes of other components. We had evidence that ontological mathematical entities such as plane and spatial figures and area, for example, were created by the students as shown in table 1. The analysis of the component *aesthetics and values* was limited in what concerns how much the other components were linked to it. What we have emphasized is that the component *aesthetics* and *values* involves affective components and thus has an impact on the development of the other components of the model. These affective components certainly depend on factors, which are external to mathematics per se. Boaler and Greeno (2000) show how the way a mathematics class is conducted impacts on the mathematical identity of the students. Once this identity is seen as linked to the component *aesthetics* and *values*, that influence may affect the development of this and of the other components of the model with more or less intensity.

Another result was that the development of the components as a whole was not harmonious. In many episodes one of the students of the pair showed difficulty in expressing his ideas or procedures in mathematical language, producing utterances identified as *explicit separate from tacit*. Despite this difficulty, student 1 was able to develop, for example, the ability to use the rectangle area formula adequately and the 'know how' to solve a number of problems. This may indicate that one element of oral mathematical language – the social communication of mathematical knowledge – can be expressed somehow independently from the 'know how' factor. And, as in the first study (see Frade, 2004), this independence seemed to be directly related to the manner in which the tacit interacts with the explicit in the process of articulation. Here, once more the teacher can play a crucial role in the student's development of that component by promoting conversational practices (see Lerman, 2001).

FINAL COMMENTS

The theoretical perspective developed in this paper is innovative in the field of mathematics education in some important ways. Our research offers a contribution to the debate on the theory-practice divide, as it was possible to investigate deep theoretical constructs in practice. The research used Ernest's model of mathematical knowledge and Kitcher's ideas about the development of mathematics practice to open up a line of investigation into the nature and development of students' mathematical knowledge in formal schooling contexts. Although the model does not account for cognitive/sociocultural processes involved in mathematical learning, it helped us understand the types of knowledge – concepts, procedures, attitudes or dispositions – that are presently valued in mathematics curricula. In other words, Ernest's model is a model of scientific mathematical knowledge, and therefore, requires adaptation of the kind suggested in this paper to be applicable to the school context. However, the model is closer to school-acquired mathematical knowledge in the following sense: by the end of a period of learning, and for each level of teaching, learners are expected to have acquired knowledge of a set of statements and propositions; be able to use mathematical reasoning and justify it; use mathematical language and symbolism in individual and social contexts; develop a certain view of the scope and structure of mathematics as a whole; and be able to decide which methods, strategies or procedures are more adequate to the resolution of problems and when to use those methods, strategies or procedures. Moreover, and probably most importantly, learners are expected to have developed a favorable disposition towards mathematical investigation. We believe that such disposition originates mostly from individual experiences with, and values and beliefs about mathematics.

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