SUBSTANTIVE COMMUNICATION OF SPACE MATHEMATICS IN UPPER PRIMARY SCHOOL

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A collaborative action research project with preservice teachers built on Wood's (2003) paper on strategy reporting, inquiry and argumentation and the NSW Department of Education and Training documents called substantive communication. There is little research on argumentation about space mathematics in primary schools so this is the focus of the reported study. A qualitative analysis of the data shows that these teachers took account of students' current knowledge and tried to extend it, acted upon their reflections of their teaching, and provided effective challenges and questions. Within space mathematics, we see incidences of strategy reporting and argumentation reminiscent of Wood's (2003) paper.

INTRODUCTION

Classroom Interaction

In 1980, Bauersfeld wrote "the constitutive power of human interaction (is that) interaction constructs the subjects' various realities. Both teacher and students act according to their actual subjective realities" (p. 30). He alerted us to the fact that the teacher and student may be at cross purposes and over the discussion views of purpose and concepts can change. In these circumstances, disagreements are likely to arise. The context of the conversation, for example, what students notice or have experienced previously and interpersonal relations cannot be forgotten in interpreting a classroom. Perret-Clermont (1980) showed that conflict arose, verbal behaviour changed, and the level of reasoning increased when less able students contributed to conversations. This line of reasoning on classroom interaction has developed 25 years later into how change can be brought about in mathematical thinking through argumentation in classrooms (Yackel, 2002).

Substantive Communication

Wood (1999) and her team carefully analysed a number of sequential lessons for a Year 2 classroom and showed the pattern of interaction that occurred during challenges involved turn-taking and explaining until there was agreement. Wood (2003) compared this interaction to conventional classrooms where thinking involves mere recall. There is no "substantive communication" in the typical "initiate-respond-evaluate" teacher-centred pattern in which the teacher asks a question, a student responds, and the teacher makes an evaluative comment (NSW DET, 2003).

Substantive communication is sustained with logical extension or synthesis where the flow of communication carries a line of reasoning and the dialogue builds on statements or questions of another participant. The communication "is focused on the

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 33-40. Melbourne: PME.

substance of the lesson" (NSW DET, 2003, p. 23). In this framework, the degree of quality is mainly in the proportion of the lesson involving substantive communication.

By contrast, Wood's model gives two contexts that are qualitatively different. First, mathematical thinking was revealed in students' "strategy reporting" as recognising, comprehending, applying, and building with analysing. Explainers told different strategies and clarified solutions while teachers accepted or elaborated these and other students listened to decide if their own strategies were different. Second, inquiry and argument showed students building with "synthetic-analyzing" and "evaluative-analyzing" and by agreeing and constructing through synthesising and evaluating. Explainers were giving reasons and justified or defended solutions while teachers asked questions and made challenges, provided reasons or asked for justification. Listening students asked questions for understanding or clarification or disagreed and gave reasons for their challenges. During strategy reporting teachers might prompt with a variety of statements like "I'm confused. Would you tell us what you thought? How did you decide this? … Are there patterns? Is there a different way you can do this?" Inquiry and argument showed the teacher prompting by questions such as "How are the two things the same? Does this make sense? … Does it always work? Why does this happen?" (Wood, 2003, Vol. 4, p. 440). Questions might be structuring, opening-up, or checking questions (Ainley,1988). Questioning, no matter what type, can be carefully linked with the mathematical thinking and level of responsibility in a classroom (Wood, 2003).

Like Wood (2003), Hufferd-Ackles, Fuson and Sherin (2004) presented a framework of improved classroom interaction which outlines shifts from the teacher to students in questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. At the higher level, teachers expect students to initiate and question. They may ask "why" questions and persist until satisfied with the answer. Teachers will follow students' descriptions of their thinking carefully, encouraging more complete explanations and deeper thinking. Students can defend and justify their answers and are more thorough in explaining. Teachers allow for interruptions from students when explaining in order for students to explain or to own new strategies. While still deciding what is important, the teacher uses students' ideas and methods as the basis of the lesson. Students will spontaneously compare and contrast and build on ideas. Teachers expect students to be responsible for co-evaluation of everyone's work and thinking. They support students helping one another sort out misconceptions and they help when needed. Students may initiate clarifying other students' work and ideas.

Questioning and Teaching Strategies

A broader set of effective teaching strategies than those pivoting around questioning has been identified by the Researching Numeracy Project Team (2004) in Victoria, Australia. The twelve practices are excavating, modelling, collaborating, guiding, convince me, noticing, focussing, probing, orienting, reflecting/reviewing, extending, and apprenticing.

RESEARCH OBJECTIVE

The objective was to describe how preservice teachers were using substantive communication in space mathematics. There has been little action research on argumentation in space mathematics. For this reason, this qualitative study provides data and analysis in teaching and learning space mathematics.

METHODOLOGY

Procedure

Twenty primary school or early childhood pre-service teachers in their third year participated in tutorials for six hours on space mathematics education and substantive communication. During this time, they watched videotapes prepared for the *Count Me Into Space* (CMIS) project in NSW and discussed how students learn space mathematics. They evaluated two videotaped lessons, one on measurement and one space mathematics according to the Quality Teaching framework's section on intellectual quality concentrating on deep knowledge, deep understanding and substantive communication. Their readings included the quality teaching documentation (NSW DET, 2003), Wood's (2003) paper, and excerpts from the paper by Hufferd-Ackles, Fuson and Sherin (2004).

Teachers (cooperating class teachers and preservice teachers) were given a large number of example lessons based on the CMIS project. The lessons covered both two- and three-dimensional space. The strength of these lessons was that they emphasised investigating and visualising as well as describing and classifying. After negotiating with the class teacher, the preservice teachers taught six to ten lessons including their pre- and post-assessment lessons.

Three classes will be discussed in this paper. Class K (Years 5/6) was taught by a primary teacher and an early childhood teacher, Class M (Year 6) by an early childhood teacher and Class P (Year 6) by a primary teacher. All were mature-aged and had received high academic grades but were not particularly confident with mathematics for this Stage, especially the two early childhood teachers.

Data Collection and Analysis

Each preservice teacher kept a journal with anecdotal records and student work samples. They evaluated each lesson using the readings especially the QT in NSW document and prepared a final report. Class K's teachers videotaped and transcribed their lessons. Class M's teacher audiotaped and transcribed her lessons. I observed a lesson in each classroom and viewed the videotapes of Class K. Much of the audible dialogue was from whole class discussions. Other data came from teachers during focus group discussions (preservice and class teachers separately).

This qualitative data was analysed first by marking any recording of interest. Each was annotated with a comment. Many of these comments linked directly back to one of the categories mentioned in the literature review above. From the taped material, I specifically noted how the teachers attempted to extend students' conversations. From these annotations, some perspectives were summarised in order to better understand how beginning teachers can achieve substantive communication in their classrooms.

RESULTS AND DISCUSSION

General Comments

Teachers tended to focus on language and concepts rather than other aspects of mathematical thinking even though they did ask students to report on their thinking or challenges. Teachers noted students' understanding was "uneven" (NSW DET, 2003) meaning it varied between class members or over episodes in the lesson.

Teachers' Analysis of Student Knowledge

Throughout the lessons, preservice teachers realised that students were struggling with three specific concepts – irregular polygons, diagonals and adjacent sides. They were aware of these difficulties in class conversations and from quick quizzes that gave them work samples to look at after the lesson. In Class K, a quiz at the start of the second lesson had most students drawing a triangle when asked to draw a shape with three diagonals indicating that students confused the word diagonal with sloping sides or it was too hard a question. Class P were given a quick quiz of general knowledge of 2D and 3D shapes at the end of the first lesson. The teacher commented

> The students were asked to note if they learned anything today. Surprisingly a large amount [sic] claimed that they knew it all! But their sheets had items which indicated they did not…they said they had forgotten…Only two said something (about properties) … and these were only the number of sides and corners.

Later assessments showed angle size was next to be considered in properties but it seems that the absence of diagonals on diagrams continued to discourage students to mention properties about diagonals in open-ended questions. During lessons, students could use the clues on diagonals when playing the game, what shape am I? In Class P, students made up interesting questions for their peers and some involved angles and diagonals.

Several students in Class P said they had never heard of a polygon with an infinite number of sides, or that a triangle or quadrilateral was a polygon. This was a dissonance that brought about changes in their concepts. Many thought an irregular shape was one that was not common or did not have a name. In Class P, "several students said an irregular shape was not a "real shape" but after a heated argument the class decided irregular shapes with many sides are polygons" (preservice teacher's report). Each class had on the walls the names of shapes and an example for each but they were the stereotypical examples. For example the hexagon and pentagon were regular, the trapezium was isosceles and all had a horizontal side. These diagrams might have helped students with the names but they did not help students to develop a full understanding of different shapes and to know what are essential properties of shapes.

Students did not seem to struggle with enlarging shapes. They knew that the lengths were enlarged but not the angles. However, the majority did not attempt to measure the angles. Deciding on the order of what to measure and draw was not an issue if estimates of angles only were used and just lengths of lines were measured. Hence students were not engaged in reporting different strategies. Lack of time and the need for class control meant the teachers did not persist with deepening understanding. In Class K, most students only doubled the sides even though they were asked to make the shape three times bigger. When reminded to double and measure angles, some students needed assistance with reading the protractor while others had to skip count to work out 6 x 3 or to use repeated addition for 3.5 x 3. Perhaps the multifacets of the problem encouraged some students to take the short cut of estimating angles. The teachers reflected:

> Students were investigating and questioning … engaged with measuring, drawing and discussing how to transfer the information onto another piece of paper. Students were also looking at their peers' work and comparing.

Post-lesson assessment indicated that students had moved in their use of strategies from doing and drawing to static pictorial imagery and some to imagery that contained patterns or required dynamic changes. Class P teacher noted that students could not describe actions of rotation, reflection or slide and felt she may have neglected this area in the lessons.

Challenges

All teachers set students some challenges. In Class M, the teacher gave each group two equal lengths of wool tied together in the centre with cotton. They were to make rectangles. (The wool formed diagonals of the rectangles). In the transcripts, T stands for teacher and other initials for students.

T: What happens when we change the angle in the middle where the cotton is tied? If it's a bigger angle what happens to our rectangle.

M: It gets bigger.

T: The sides get bigger, smaller. Just quickly have a look at your wool and try to make the angle bigger and see whether as the angle gets bigger the rectangle gets bigger or smaller or wider or narrower.

E: Smaller

T: Why do you think its smaller, E.

E: Because the angle makes the ends bigger

T: Will everyone listen to E., when somebody is speaking, everybody else needs to be quiet.

E: When the angles are really really skinny the sides are really, really long and when they move out the sides get shorter and shorter. The angle gets bigger and bigger.

T: Now if we have a piece of wool and either a pencil or piece of chalk, how could we make a circle?

K: Hold one end and draw around (other ideas by students not captured on tape).

Clearly the students were considering different angles and sides of the rectangle but there seemed to be no confusion as they were using the concrete materials (wool) to model their comments. The teacher moved on as the students were unsettled. This was one of the key problems for these beginning teachers in maintaining substantive communication.

Other challenges presented to the classes were:

- how to check whether a piece of paper was square
- making pentomino shapes without repeats,
- deciding why one pentomino shape had a different perimeter than others
- designing and making boxes of different shapes after making a square box
- making cylinders and cones when given a paper-towel roll and a funnel
- deciding on lines of symmetry for the pentomino shapes

In Class M the Z-like pentomino lead to some discussion regarding whether it had one or no lines of symmetry. They grappled with rotational symmetry. The discussion was continued the next week. Students initially did not agree on how many lines of symmetry each shape had but they were able to convince each other by using the concrete cut-outs which the teacher prepared realising this would be an important aid for the discussion. For the same activity, Class P's teacher commented:

> The "magic moment" in this lesson is when a not so bright student argues about the line of symmetry in one of the pentominoes and set about to prove his point or me wrong. He discovered that this shape was symmetrical by rotating the two halves. This student was satisfied because he proved it by himself. Great stuff! (Figure 1 shows) the scrap of paper that the student worked with when he then cut it in half (on the line he thought was a line of symmetry) and placed the two pieces on top of each other and presto they matched.

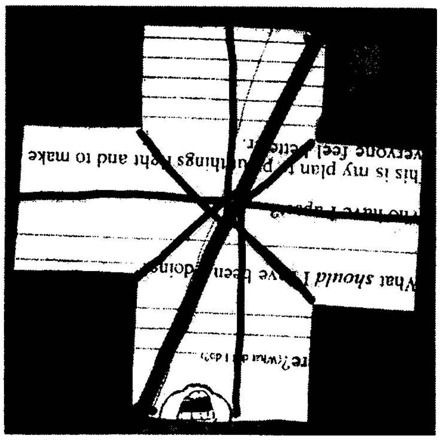

Figure 1: Trying to prove a line of symmetry by cutting and overlapping.

This is a typical example of concrete proof reminiscent of that found by Wood (2003) in strategy reporting in a younger class on number concepts. However, we see that it establishes a new concept. Students in general were reliant on concrete proofs for justifying and explaining although diagrams were used and later some were able to argue verbally and more abstractly. Teachers deliberately provided large examples for whole class discussion to facilitate this kind of proof.

Different Ways of Questioning

The teacher in Class P asked questions in numerous game formats. She noted how these games were both enjoyable but challenging. They made students' question their conceptual knowledge. The games included (a) finding a fellow student for a match of picture and properties, (b) "Celebrity Shapes" in which students ask questions in order to guess the shape drawn or written on the board above their heads, (c) questions provided as chance cards on different self-designed game boards. The best question was one in which the group had to decide if the answer was correct or not. Class K had clues to decide on a shape as a group. These questions were challenging as they focussed on properties other than the number of sides and corners.

Class P's teacher commented that during marking of revision quiz questions there was considerable discussion between neighbouring students and students did not all mark their papers correctly. Were there still areas of disagreement that students needed to discuss rather than quickly marking a quiz? The class teacher noted that questioning by the preservice teacher "is drawing the discovery/information from them rather than giving the answer … the response from students has been great. … The students were beginning to use lesson specific language to describe objects, position etc. It's working!"

Much of the discussion in each class was resulting from direct teacher questioning with some occurring between students during the activities, e.g. from which position the drawing of a 3D model was made or when pentominoes were in different orientations. The videotapes of the whole classroom discussion in the last two lessons of Class K illustrated students' remaining on task throughout the lesson. They may not have talked much but they were thinking and several students were confident to disagree with the teacher or other students' suggestions or to ask their own questions. The students attempted explanations and justifications. The discussion soon moved to students' questions and interests rather than the teacher's initial question.

Teacher Reflection

As a result of reflection, teachers either recognised the effect of having too many students in a small room for group work or re-organised tables to allow closer communication for small groups. They also recognised missed opportunities for group work or sustaining communication and then deliberately allowed themselves more time for discussion and asking more questions to try to encourage discussion.

Classroom teachers pointed out specific lessons in which the students were thinking more mathematically. For example, the lessons on "nets of cubes and making boxes and taking different perspectives for shapes and models made them start to think about nets and shapes and discuss why more than before." "Before, students just gave shape names but now they are discussing them with the correct terminology." There was "far more on tangrams and using more concepts of space than before. Before they made shapes but now they are thinking deeper about concepts of space." "(The lessons were) fun, productive, discovery." "(The preservice teachers) had clear ideas of what outcomes they wanted without restricting it." "They modelled how they wanted them (the students) to ask questions. They talked about lesson expectation and students knew they will be expected to explain and if they were not confident then they got better." "The teachers allowed for incidentals and allowed that deviation to take place. They encouraged and set an atmosphere for taking risk."

CONCLUSION

This paper highlights the importance of the challenge in inquiry/argumentation as Wood (1999) had shown but this paper illustrates the nature of some of the challenges in space mathematics in upper primary school rather than with number in lower primary school. The questioning utilised by the teachers varied. There were (a) quick quizzes with a few challenge questions which the teachers wanted mainly for accessing students' knowledge, (b) planned questions that were strengthened by their reading, reflection, and practice, (c) spontaneous questions as they listened to the students, and (d) questions in game formats. While some of the quizzes and games were followed by whole class discussions, others were left for individuals perhaps talking with their neighbours to resolve conflicts and develop concepts.

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