ON EMBODIMENT, ARTIFACTS, AND SIGNS: A SEMIOTIC-CULTURAL PERSPECTIVE ON MATHEMATICAL THINKING

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The cognitive significance of the body has become one of the major topics in current psychology. However, it is our contention that claims about the embodied nature of thinking must come to terms with the problem of the relationship between the body as a locus for the constitution of students' subjective mathematical meanings and the historical cultural system of mathematical meanings conveyed by school instruction. Referring to episodes from a Grade 9 mathematics lesson, we here sketch a theoretical account of the aforementioned relationship that emphasizes the social and cultural nature of thinking and the cognitive role played by body, signs, and artifacts.

INTRODUCTION: THE EMBODIED MIND

The cognitive significance of the body has become one of the major topics in current psychology. Thus, it is now often claimed that human concepts are crucially shaped by our bodies and our senses (see e.g. Lakoff and Johnson, 1999; Lakoff and Núñez, 2000). The empiricists of the 18th century could not have agreed more. For the empiricists, who asserted that nothing is in the intellect which was not first in the senses, both body and sensuous impressions were indeed the gate to knowledge. For the rationalists, such as Leibniz and Descartes, the gate to knowledge was the realm of innate logic, a realm from which body and sense data were excluded. They continued a long tradition going back to Plato who said that "if we are ever to have pure knowledge of anything, we must get rid of the body and contemplate things by themselves with the soul" (*Phaedo*, 66b-67b).

Current claims concerning the cognitive significance of the body, however, do not necessarily entail a return to empiricism. They should rather be seen as an attempt to relocate the role of the sensual in the realm of the conceptual. Now, was it not precisely the problem of the sensual and the conceptual that Piaget wanted to unravel with his genetic epistemology? Did he not insist that knowledge starts with body actions? In a recent PME research forum, Nemirovsky suggested a list of guiding questions as a basis for a research agenda, among which are the following two: "What are the roles of perceptuo-motor activity, by which we mean bodily actions, gestures, manipulation of materials, acts of drawing, etc., in the learning of mathematics? How does bodily activity become part of imagining the motion and shape of mathematical entities?" (Nemirovsky, 2003). One of the audience's impressions was that Nemirovsky's questions had already been answered by Piaget's epistemology. In a certain sense, this is the case. However, as a good Kantian and follower of 18th century Enlightenment philosophy, Piaget sought to answer the riddle

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of the sensual and the conceptual through a combination of empiricism and rationalism: for Piaget, knowledge starts with sense data but knowledge and ideas cannot be reduced to a combination of impressions distinguished by their intensity, as empiricists like Hume proposed; for Piaget, it is the logic-mathematical structures which will pick sensual knowledge up and transform it into abstract thinking. Nemirovky's questions make sense only in the current context of psychological and educational theories that seek to place the cognitive relevance of the body in a context larger than the limiting one of the sensorimotor stage that marks the beginning of the conceptual development of the child in Piaget's account. Despite their different perspectives, what these theories are claiming is that sensorimotor activity is not merely a stage of development that fades away in more advanced stages, but rather is thoroughly present in thinking and conceptualizing.

This claim for the full insertion of the body in the act of knowing is an indication, we think, of a turning point in contemporary views of knowledge formation. While mainstream 20th century psychology was subsumed in the empire of the written tradition –a tradition that since the invention of printing in the 15th century endowed the written word with an unprecedented degree of authority, a tradition that justified the use of tests and questionnaires to investigate the depths of the mind– we now live at a time when new forms of knowledge representations (such as digital ones) have made a definite incursion in all the spheres of life. Of course, to be fruitful, the plea for an embodied mind has still to be accompanied by theoretical elaborations about knowledge formation. If the *télos* (i.e. the *end* or *final cause*) of conceptual development can no longer be imputed to universally valid logic-mathematical structures only —as was the case in Piaget's genetic epistemology— how then are we to account for it? Piaget and Husserl were certainly right in asserting that the body is a locus for the production of meaning and the first opening of intentionality towards the world. Nevertheless, the world that the body encounters is a cultural world populated by other bodies, objects, signs, and meanings, a world already endowed with ethical, aesthetical, scientific and other values. These values provide the world with specific configurations that, instead of being neutral, qualify the body with the historicity of events and concepts deposited in language, artifacts, and institutions (Foucault, 2001, p. 1011). Hence, there are some theoretical problems that the paradigm of the embodied mind needs to tackle, in order to avoid a theory where the body is considered as the refuge of the transcendental "*I*" of idealism and to overcome critics like Eagleton who sees in the claims of the embodied perspective a theory that in the end is no more than "the return in a more sophisticated register of the old organicism" or else a token of "the post-modern cult of pleasure" and love for the concrete (Eagleton, 1998, pp. 157-58).

It is our contention that an account of the embodied nature of thinking must come to terms with the problem of the relationship between the body as a locus for the constitution of an individual's subjective meanings and the historically constituted cultural system of meanings and concepts that exists prior to that particular

individual's actions. In what follows we sketch, from a semiotic-cultural perspective, some general points that might be seen as a preliminary contribution to the problem at hand. Since our main interest is the understanding of the learning and teaching of mathematics, we shall focus on the meaning of thinking and learning and the role of body, signs and objects therein. Our theoretical discussion will be intermingled with comments on a Grade 9 mathematics lesson.

THINKING AS SOCIAL PRAXIS

While traditional cognitive psychology considered thinking as the mental processing of information carried out by an individual, for the semiotic-cultural perspective here advocated, thinking is a form of *social praxis* (Wartofsky, 1979).

Thinking as a form of *social praxis* means that thinking is an active mode of social participation in which *what we know and the way we come to know it is framed by cultural forms of rationality (i.e., by cultural forms of understanding and acting in the world) out of which specific kinds of questions and problems are posed*. To illustrate this idea, let us turn to a passage of a Grade 9 lesson in our ongoing 6-year longitudinal classroom-based research program. The lesson was about the interpretation of graphs in a technological environment based on a graphic calculator TI 83+ and a probe –a Calculator Based Ranger or CBR. The students were familiar with the calculator graph environment, whereas the CBR was new for them. Prior to the passage that we shall discuss ("Pierre and Marthe's walk"), the teacher explained to the students how the CBR worked (its wave sending-receiving mechanism to measure the distance between itself and the target, distance limitations, distortions, etc.). One student was chosen to walk towards a door, holding the CBR and pointing it towards the door, to stop a few seconds and then to walk backwards from the door (the target). The class discussed the graph obtained on the calculator through the CBR. After this introductory activity, the students, in small groups of three, had to describe the motion to be performed by a student walking with the CBR in order to match a given graph (not shown here). Later, the students verified their hypothesis by doing the experiment. Right after this verification, the students worked on the interpretation of a graph related to a story in which two children, Pierre and Marthe walked in a straight line, the latter pointing a CBR to the former. The graph showed the relationship between the elapsed time (horizontal axis) and the distance between the children (vertical axis) as measured by the CBR (see Figure 1). The mathematical problem was intended as a way to introduce the students to a particular mode of thinking –a cultural kind of reflexion about the world conveyed by contemporary school instruction. Through this problem, the students became involved in a *social praxis* that goes back to the 15th century investigation of body motions, a historical revolutionary form of thinking about natural phenomena different from the previous

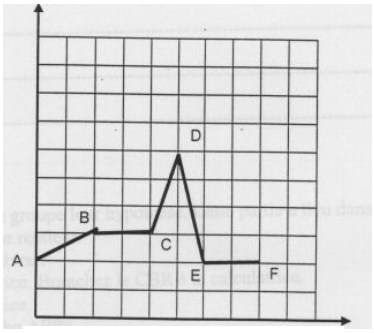

Figure 1. Pierre and Marthe's graph walk.

Aristotelian paradigm of efficient causes. Naturally, we are not asserting that the students are aware of this fact. This is not the point. The point is that, from the outset, the problem on which the students had to reflect was framed by a cultural kind of rationality that legitimizes the problem and endows it with meaning. The relevance of this remark for our understanding of the students' mathematical thinking resides in the fact that what the students will know as a result of their participation in the classroom activity and the way they will come to know it is framed by a historically constituted mode of thinking. However, this remark does not mean that the students are a clean slate or *tabula rasa* on which the classroom activity impresses knowledge. Indeed, what marks the distinctiveness of thinking is its *reflexive* nature, something to be understood not as the individuals' passive reception of the external reality but as a *re- flection*, i.e. a dialectic process between individuals and their reality, a socio-cultural process of active and creative efforts to align subjective meanings with cultural ones. The following excerpt suggests the non-transparency of the kind of mathematical reflexions that were required from the students. The excerpt, that comes from one of the small groups in which the class was divided, starts with an intervention by the researcher to clarify some basic meanings of the graph that were difficult for the students.

1 Researcher: What is important is that, here (*pointing to segment AB*), it went up … the probe (CBR) is there (*pointing to Pierre, the child on the left on the accompanying drawing; see Figure 2*). So, what does that (*pointing to segment AB*) mean in relation to that? (*indicating the distance between the children in the accompanying drawing*).

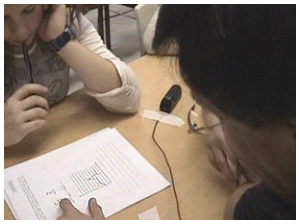

Figure 2. The child with the CBR is indicated through a gesture.

2 Karla: They were closer over there? (inaudible)

3 Researcher: Here (*points to (0,0)*) they were at a certain distance. Here (*points to t =1*) were they further apart?

4 Karla : (*with hesitation and doubt*) further apart…?

5 Cindy : closer…? OK, hold on… […]

Although the three students in this group were able to successfully interpret the graph of the previous problem, here the difficulty arose from the fact that the problem required the students to make sense of a graph involving *relative positions*. Thinking mathematically, that is to say, entering here into this *social praxis* of graph interpretation requires an imaginative endeavor aimed at aligning the subjective meaning of signs with their cultural, objective meanings. Signs, indeed, are both social objects and subjective products. Signs in general and mathematical signs in particular (like the graph in our example) are social objects in that they are bearers of culturally objective facts in the world that *transcend the will of the individual*. They are subjective products in that in using them, the individual expresses *subjective and personal intentions*. This double role of signs should not be understood as a

dichotomous division. On the contrary, the use of signs rests on *understanding*, that is the transformation or interpretation of a sign into a previous sign (e.g. an interiorized one, in Vygotsky's terms) for which the individual has attained a more or less stable cultural meaning. Let us turn to the next part of the previous excerpt to observe the genesis of understanding.

THE LANGUAGE OF SPACE

Cindy continued her utterance:

5 Cindy: […] OK, hold on. Here (*indicating with her pen the point (0,0)*), when they start at zero … (they) are … closer, right? … They start at zero… Well, there they start here (*she indicates the point A*) … No … that wouldn't make sense.

6 Karla: They're further! (*gesture indicating the segment AB*)

7 Cindy: No, I think that they are closer here (*segment AB; see Figure 3, first picture*), and there (*gesture indicating BC; see second picture*) they are closer, and there (*gesture indicating CD; see third picture*) they move away, and they move away here (*indicating segment DE; fourth picture*) and there (*pointing to segment EF; fifth picture*) they arrive together.

Figure 3. Pointing gestures indicating the segment *AB*, *BC*, *CD*, *DE*, and *EF*.

As we can see from the excerpt, segments were interpreted in accordance to their perceptual inclination as signifying proximity or separation between Pierre and Marthe. The linguistic adverbs "closer", "near", as well as verbs and expressions such as "to move away" and "to arrive together" permitted the students to elaborate an initial understanding. This linguistic-based understanding consisted in the transformation of the graph sign into a verbal sign (made up of verbs, adverbs, etc. See line 7). Language offers each one of us a way to objectify the space in which we live and move. With its rich arsenal of deictics (e.g. "here", "there"), spatial adverbs (e.g. "closer", "near"), etc. language enables us to express and shape our intimate experience of space. By casting our experience in the linguistic categories of our culture, language allows us to do what colors allow the painter to do: to create, so to speak, a personal "linguistic painting". Like the painting of the artist, which carries the historical-cultural experience of colors, the "linguistic paintings" of the students carry deposited sediments of the spatial, historical experience of previous generations who used and refined the language of space that we have come to use. Language makes both our experience of space and our understanding of it simultaneously intimate and cultural –or better still, language makes them culturally intimate. However, it would be inaccurate to say that the students' understanding was linguistic-based only. In the next section, we elaborate upon this idea.

THINKING AS MEDIATED BY, AND LOCATED IN, BODY, ARTIFACTS, AND SIGNS

In the previous excerpt we saw that, along with key linguistic signs (adverbs, deictics, etc.), understanding was achieved through pointing gestures and the kinesthetic action of moving a pen along the graph, all of this synchronized with parts of the students' linguistic utterances. Thus, language (in its various dimensions: lexical, syntactic, pragmatic –e.g., pauses, modes to express doubt, exclamations), the Cartesian graph, the pen, and the students' hands were *mediating tools*. We want to suggest that, because thinking cannot be reduced to mental-cerebral activity, thinking is not only *mediated by*, but also *located in*, body, artifacts, and signs. As the anthropologist C. Geertz remarked after discussing the pitfalls of reducing thinking to an essentially mental intracerebral process, "the human brain is thoroughly dependent upon cultural resources for its very operation; and those resources are, consequently, not adjuncts to, but constituents of, mental activity" Geertz (1973, p. 76).

The most important point, however, is not to acknowledge our cognitive dependence on cultural resources, but to realize that they are an integral part of thinking and that in learning how to use them, the natural-biological line of development of our central psychological functions, such as attention, memory and symbolization are *altered*. Referring to these cultural resources as psychological tools, Vygotsky wrote:

> By being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act just as a technical tool alters the process of a natural adaptation by determining the form of labor operations. (Vygotsky, 1981, 137)

In terms of our discussion of thinking as *social praxis*, the previous remarks add an important element. The Cartesian graph, for instance, is not merely something to learn to read, something to make sense of. The graph, the calculator, the CBR, are cultural resources which bear an embodied intelligence (Pea, 1993) and carry in themselves, in a compressed way, socio-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1984). This embodied intelligence and compressed historical cognitive experience offer an orientation to our cognitive activity.

But again, this orientation presented in the cultural system of ideas and the embodied intelligence carried by the artifacts that the body encounters as it moves in the world, are an overture towards possible paths of conceptual development. Since the child cannot merely be carried along by its social environment and since the latter cannot determine the child's individual thinking, as Vygotsky argued against the behaviorists of his time, the problem, from an educational viewpoint, is to present the child with the occasions to align his or her subjective meanings with cultural ones. Bearing this in mind, let us go back to our classroom episode.

REFINING MEANING

8 Karla: (*after a pause of ± 3 sec of reflection*) But Cindy, this (CD) goes up [and] this (AB) goes up … that means it is the same thing! […]

9	Celeste:	Huuuh? … It's the same thing?! Then what? (*in a tone denoting confusion)* […] Yeah! They are maybe closer here (*pointing to AB*), closer (*pointing to BC*), further (*pointing to CD*), further (*pointing to DE*), closer (*pointing to EF*).
10	Karla:	But they can't be closer here (*pointing to CD*) […] It (*indicating CD*) is going up, and so is this one (*pointing to AB*)! […]
11	Cindy:	They move away!! So, that (CD), it's you move back; that (DE), you move forward!
12	Karla:	(after a pause of ±3 s) I don't get it!

From the beginning AB was associated with 'being close'. And since AB and CD both were said to 'go up', CD should also mean 'being close'. And this, Karla claims (line 10), is not right. Celeste then adds a new element: the children moved back! Their final written interpretation reads: "At the beginning "A" they moved back slowly and equally together at a distance of 1m. After, they stopped and then began to move back again, but faster. Then they moved forward again with the same speed at which they moved back the second time. After they stopped and they were finished."

Continuing on with our artist metaphor we can say that, by introducing the idea of moving back, the students added a new color to their mathematical painting. Unfortunately, neither the introduction of this idea nor the experiment with the CBR that they carried out later to test their hypothesis were enough to close the gap between the students' subjective meaning and the cultural one. The students still had to refine their manner of thinking about relative motion in a deeper way and to insert the role of the CBR technological artifact into their reflexions. The general learning achievement of the class was still far from the one determined by curriculum expectations. It took the teacher several lessons and general discussions to make apparent for the students the targeted kind of mathematical reflexion.

CONCLUDING REMARKS

Drawing from Wartofsky's work, in this paper we suggested that thinking is an interpretative and transformative *reflexive social praxis* encompassed by a cultural rationality and oriented from the outset towards a cultural objective system of ideas. In contrast to other social praxes, mathematical thinking is characterized by its orientation towards a theoretical, practical, and aesthetical understanding of historical-cultural reality. Thinking, we proposed, is not only mediated by, but also located in signs, artifacts, and body. If it makes sense to talk about embodiment, it is not because we have discovered that the "thinking substances" that we are (to use Descartes' term) have –notwithstanding Plato and the rationalists– a body which is a source of cognition. Body certainly is a locus for the production of meaning and the first opening of intentionality towards the world. But the object that the body encounters is more than a mere *thing*: it is a cultural object. From an educational viewpoint, the problem is that the cultural conceptuality embodied in the object is not necessarily apparent for the students. The cultural object is seen or perceived *in a certain way*, as interpreted through the students' mediated reflexion and as refracted

by their particular subjective meanings. Where Grade 12 students see a Cartesian graph, Kindergarten students see a bunch of lines with no connection to a relationship between mathematical variables.

The apprehension of the object in its cultural dimension –i.e. the apprehension of the cultural conceptual content and meaning of the object– requires students to engage in an interpretative and imaginative process whose outcome is an alignment of subjective and cultural meanings. Thus, in the example discussed here, the students dealt with a Cartesian graph – a complex mathematical sign whose objective cultural meaning was elaborated in the course of centuries. The alignment of subjective and cultural meanings involved a profound active re-interpretation of signs by the students, framed by the teacher and the particular context of the classroom, leading to a progressive awareness of significations and conceptual relations that remained, in the beginning, tenaciously inaccessible to the students (e.g. that the segment AB may mean a part of the children's walk in which Marthe moved faster than Pierre or Pierre moved slower than Marthe). The aforementioned alignment of meaning should not be understood, however, as the absorption of the students into their culture: it is only one step in the positioning of the students in this distinctive social praxis that we here called *thinking*. But because of its own reflexive, interpretative, and imaginative nature, thinking also means transformation, a going-beyond, an outdoing of what is given. The subjective dimension of thinking, as something accomplished by historically-situated and unique individuals, makes possible the overcoming of the actual and the expansion and modification of knowledge and culture.

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