A COMPARISON OF HOW TEXTBOOKS TEACH MULTIPLICATION OF FRACTIONS AND DIVISION OF FRACTIONS IN KOREA AND IN THE U.S.

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To illuminate the cross-national similarities and differences in ways of teaching multiplication of fractions and division of fractions, this study compared the lessons on fractions in Korean mathematics textbooks and accompanying teacher's manuals with the corresponding lessons on fractions in one U.S. reform mathematics textbook series and accompanying teachers' manuals [Everyday Mathematics]. This study found that there is a gap between learning goal [intended curriculum] and problems presented in textbooks [potentially intended curriculum].

INTRODUCTION

It is well known that across country, students' learning is highly correlated with curricular treatment of related topics (Ball & Cohen, 1996; Garner, 1992; McKnight et al., 1987; Olson, 1997; Schmidt, McKnight, & Raizen, 1996; Schmidt et al, 2002). A lot of cross-national comparisons of mathematics textbooks including the TIMSS study have reported that U.S textbooks constitute a de facto national curriculum, which has been characterized as superficial, underachieving, and diffuse in content coverage (Fuson, Stigler, and Bartch, 1988; Schmidt, McKnight, Cogan, Jakwerth, and Houang, 1999; Mayer, Sims, & Tajika, 1995; National Council of Teachers of Mathematics, 1989; Schmidt, Houang, and Cogan, 2002).

About the time the TIMSS study was underway, three reform curricula were developed with support from the National Science Foundation: *Everyday Mathematics, Investigations,* and *Trailblazers*. Among them, it has been often reported that *Everyday Mathematics* is used most widely in America. It is reported that *Everyday Mathematics* increases both the depth and the breath of the mathematics taught, focuses on students' mathematical solutions and the examination of alternative strategies, and encouraging students to develop, use, and discuss their own methods for solving problems (Carroll, 1998).

It is well known that many students and adults have difficulty with understanding multiplication of fractions and division of fractions. Algorithms for multiplication of fractions and division of fractions are deceptively easy for teachers to teach and for children to use, but their meanings are elusive (Kennedy & Tipps, 1997). However, students should learn mathematics with understanding (NCTM, 2001). "Instructional programs should enable all students to understand meanings of operations" [with fractions] and how they relate to one another; compute fluently and make reasonable estimates (NCTM, 2001, p. 214).

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This study examined how multiplication of fractions and division of fractions are taught in the reform curriculum presently being used in Korea and *Everyday Mathematics.*

In the TIMSS study, Korean students showed high achievement, ranking number two. Yet, there is little research of how Korean students learn mathematics. Recently, Grow-Maienza and Beal (2003) studied Korean mathematics curriculum. Yet, they focused on the traditional 6^{th} mathematics curriculum. Korean mathematics has been recently changed. There is little research addressing on how the Korean reform curriculum 7^{th} mathematics teaches mathematics and on how problems in mathematics are presented in textbooks.

The purpose of this study is to illuminate the cross-national similarities and differences in ways of conceptualizing and presenting multiplication and division of fractions in Korean reform textbooks with the corresponding lessons on fractions in Everyday Mathematics. According to previous researches, a lot of studies on textbook analysis have focused on either content analysis or problem analysis. They recommend that combining two types of analysis--content analysis and problem analysis--provide richer promises for revealing potential effects of textbooks on students' mathematics achievement. This study focused on two aspects of textbook analysis: content analysis and problem analysis. This study has three research questions:

(a) What are the learning goals related to multiplication of fractions and division of fractions in each curriculum?

(b) When and how are multiplication of fraction and division of fractions introduced and developed in each curriculum?

(c) How many and what types of problems in multiplication of fractions and division of fractions are presented in each curriculum?

METHODOLOGY

This study conducted content analysis and problem analysis. Content analysis is focused on two research questions. Problem analysis is conducted focusing on problems presented in the textbooks.

Textbooks and the Mathematical Problems analysed

EM provides three textbooks (Student Journal 1, 2, and Student reference book) and Korean mathematics provides four textbook (Student Mathematics Ga, Na, and Mathematics workbook ([1]Ga, Na). All textbooks in 5^{th} and 6^{th} are analyzed.

Analysis Plan

Content analysis is conducted focusing on two research questions (a) and (b). In content analysis, both teacher's manuals and student's book were used. First, this study referred to teacher' manuals in order to identify learning goals of multiplication of fractions and division of fractions. Based on the learning goals stated in the teacher's manual, this study explored their emphasis on learning about multiplication of fractions and division of fractions. Second, this study examined when and how multiplication of fractions and division of fractions are introduced and developed in the each textbook series.

Problem analysis is conducted focusing on problems presented in the textbooks. In this study, problem is identified as those mathematical problems or problem components that do not have accompanying solutions or answers presented. Previous studies identified three important dimensions for analyzing mathematical problems: mathematics feature, contextual feature, performance requirement (Li, 1998; Stigler et al., 1986; Tabachneck, Koedinger, & Nathan, 1995). Based on previous studies, by adding some other factors, three-dimensional frameworks were developed in this study. Table 1 shows the dimension of problems analysis: (a) mathematics feature; (b) contextual feature; (c) performance requirement.

Dimensions of Problem Analysis	
1. Mathematics Feature	Single step required (S)
	Multi-step required (M)
2. Contextual Feature	Purely mathematical context in numerical or word form (PM)
	Illustrative context such as visual representation (IC)
3.Performance Requirement	
(1) Response type	Numerical answer only (A)
	Numerical expression required (E)
	Explanation or solution required (ES)
(2) Cognitive requirement	Conceptual understanding (CU)
	Procedural knowledge (P)
	Mathematical Reasoning (MR)
	Representation (R)
	Problem solving (PS)

Table 1. Conceptual Framework for problem analysis

Each problem in all textbooks was coded in terms of the three dimensions stated above. In order to avoid the researcher's subjectivity, a second independent rater who is literate in both English and Korean languages coded problems in textbooks. The interrater agreement was 98 %.

RESULTS

Research Question 1: What are the learning goals related to multiplication of fractions and division of fractions in EM and KM

While EM emphasizes understanding the meaning of multiplication of fractions and division of fractions more than the algorithms for them, KM emphasizes both conceptual understanding and mathematical fluency. EM first provides folding the paper and area model to address multiplication of fractions and then introduces the algorithm of multiplication of fractions. In division of fractions, EM expects students to understand a common denominator method for division of fractions and an algorithm for the division of fractions. In contrast, through the whole learning goals, KM emphasizes both understanding and using algorithms effectively. In addition, KM expects students to understand and formulate various type of multiplication of fractions and division of fractions.

Research Question 2: When and how are multiplication of fractions and division of fractions introduced and developed?

Content organization

First, while EM introduces and develops multiplication of fractions and division of fractions at the same time, KM introduces multiplication of fractions and division of fractions separately. KM first introduces multiplication of fractions and develops it intensively in one unit. Then, it introduces division of fractions and develops it intensively in two units across two grades.

In addition, in EM, several topics are covered in one unit. Almost each lesson has different topics. For instance, 5^{th} graders learn multiplication of fractions and division of fraction with comparing fractions, addition and subtraction of fractions, and percent. In contrast, Korean mathematics curriculum is much more sequentially organized, with almost no repetition. Different topics are taught in different grads. While the sixth grade text in Korea does not duplicate fifth grade topics, the typical EM often duplicates most of the content.

Third, KM devotes more time to developing multiplication of fractions and division of fractions for students to master it. KM devote as twice time as EM does to develop multiplication of fractions and division of fractions. While EM covers multiplication of fractions and division of fractions in a total of 9 lessons, KM covers them in a total of 17 lessons. In addition, there is different intensity of multiplication of fractions and division of fractions.

Content Presentation

First, EM emphasizes understanding first and then algorithm. In particular, EM does not emphasize the algorithm of multiplication of fractions until 6^{th} grade. EM first introduces “many of” and “part of “as indicators of multiplication. Before introducing the algorithm for multiplication of fractions, EM give concrete meaning to finding a fractional part of a fraction part by providing the paper-folding exercise and area-model diagram. However, KM emphasizes understanding and algorithm of multiplication of fractions at the same time. Different lessons teach different types of multiplication. KM, through whole lesson, introduces three activities; understanding the multiplication of fractions, knowing the algorithm of multiplication of fractions in different types, and practice.

Second, EM and KM introduce the algorithm of multiplication of fractions with two same two strategies. However, in the problems of (whole number) × (fractions) or (fractions) × (whole number), EM uses common denominator strategies, which KM does not use. EM asks students to rewrite each fraction in the form.

In division of fractions, both curricula introduce division of fractions from whole number division. However, while EM introduces two strategies of division of fractions—common denominator and invert and multiply method, KM only relies on invert and multiply method.

Research Question 3: How many and what types of problems in multiplication of fractions and division of fractions are presented?

It was found that EM provides more problems in multiplication of fractions than KM in terms of the total number (EM: 251, KM: 190). However, KM provided more problems in division of fractions than EM (EM: 58, KM: 400). Because problem analysis results of division of fractions are similar to those of multiplication of fractions, this study reports the results of multiplication of fractions.

Mathematical Feature

Table 2 and figure 1 show mathematical feature in fractions multiplication.

	Simple computation	Multiple computation	Total
Everyday Math	246 (98%)	5 (2%)	251
Korean 7^{th} Math	158 (83%)	32 (17%)	190
	403	38	

Table 8. Distribution of problem in KM and EM by mathematical feature

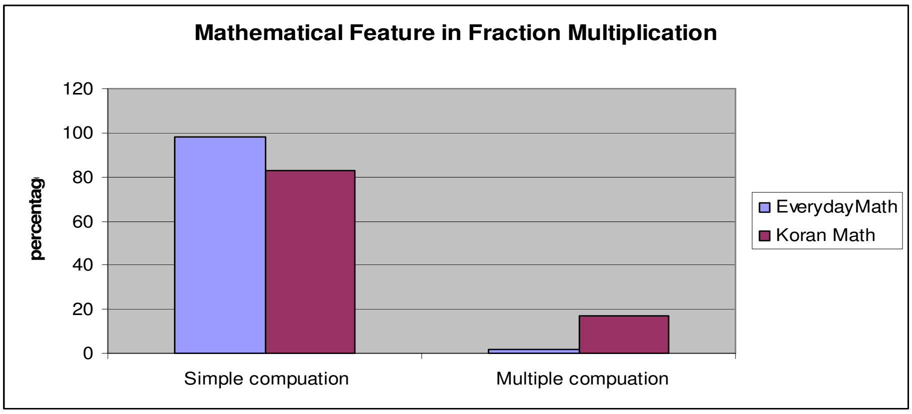

Figure 1. Distribution of problem in KM and EM by mathematical feature

In terms of the number of steps required in the solutions to multiplication of fractions problems, this study revealed that problems in the KM are more challenging than those in EM. It was found that 17 % of the problems in KM needed multi-step to solve, whereas such problems in the EM were around 2%. The less frequent exposure to multiple-step problems for U.S. students might be one reason why they performed not so well on this type of problems, as found in many studies (Carpenter et al., 1980).

Contextual Feature

Table 3 and figure 2 show contextual feature in fractions multiplication.

	Purely Math context	Illustrative context	Total
Everyday Math	175 (70%)	76 (30%)	251
Korean 7th Math	136 (71%)	54 (29%)	190
	311	130	

Table 3. Distribution of problems in KM and EM by different representation forms

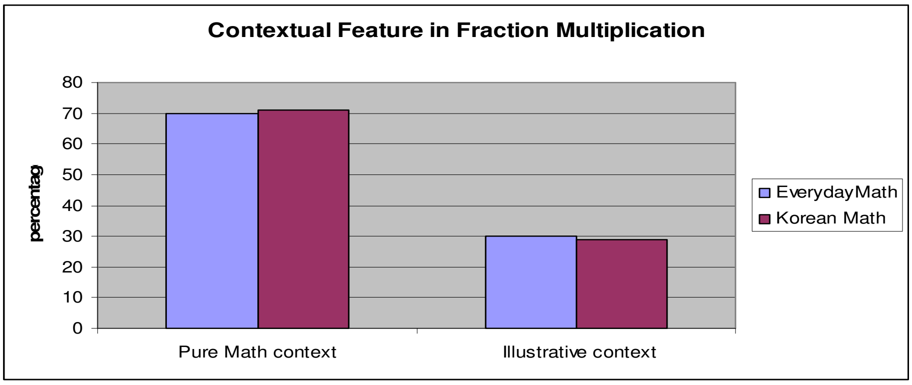

Figure 2. Distribution of problems in KM and EM by different representation forms

It was found that contexts of problems in both Korea and EM textbook were all most same. The majority of problems in all the books were presented in symbolic forms, including mathematical expressions, written words, or a combination of the above two forms (Korea: 71 %, US: 70%).

Performance Requirement

A. Response Type

Table 4 and Figure 3 show the results.

	Numerical answer	Explanation required	Total
Everyday Math	179 (71%)	30 (12%)	251
Korean 7th Math	119 (63%)	27 (14%)	190
	298	57	

Table 4. Distribution of problems in KM and EM by response type

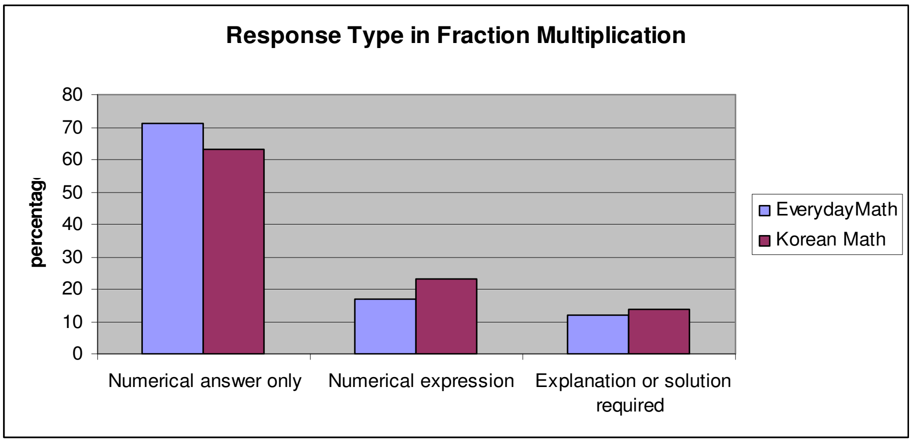

Figure 3. Distribution of problems in KM and EM by response type

It was found that a lot of problems in both Korea and EM textbook were required numerical answer only, numerical expression, and explanation or solution required in order. The distribution of problems-response type in the KM textbooks is more balanced than that in the EM. Clearly, the majority of problems from both textbooks were found to require a numerical answer. However, fewer problems in KM required a numerical answer only (EM: 71%, KM 63%) and more problems required numerical expressions, or explanations or solutions (EM: 29%, KM: 37%).

Cognitive Requirement

Table 5 and figure 4 show the results.

	Conc. Know.	Proc. Fluen.	Math. Reas.	Repre.	Prob. Solving.
EM	1	198 (79%)	6 (2%)	22 (9%)	24 (10%)
KM	1 (1%)	151 (79%)	13 (7%)	7 (4%)	18 (9%)

Table 5. Distribution of problems in KM and EM by cognitive requirement

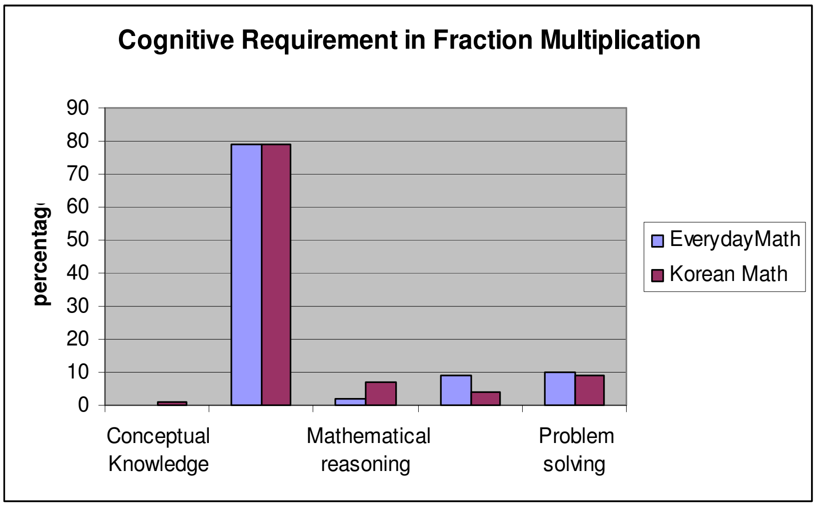

Figure 4. Distribution of problems in KM and EM by cognitive requirement

This study found that procedural knowledge is the most frequent type of knowledge required in the problems about fractions multiplication both curricula. Conceptual knowledge is the least frequent type of knowledge. This result shows that even though both curricula intend to improve conceptual understanding, mathematical reasoning, problem solving, the problems presented in the textbooks ask almost exclusively for procedural knowledge. While more problems in EM require representation and problem solving than KM, more problems in KM require mathematical reasoning.

DISCUSSION

This study examined both Korean 7th mathematics and Everyday Mathematics at 5th and 6th grade. One of important finding in this study is the gap between what is intended and what is presented in textbooks. Both curricula intend to improve students' conceptual understanding of multiplication of fractions and division of fractions. *Everyday Mathematics* seems to provide more opportunities to developing concepts behind algorithms. However, it was revealed that problems in both textbooks are presented in purely mathematical contexts and that a large portion of problems is required single-computational steps and procedural knowledge only. Based on this result, it is not difficult to assume that there is understandably some gap between what described in the textbooks and what actually happen in classrooms. This study has implications to curriculum developers, teachers, and researchers.

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[1]Ga and Na is Korean own language. Ga means one and Na means two. Thus, 5-Ga and 5-Na are 5-1 and 5-2, respectively.