Calculus Poster Session Abstracts

San Antonio, January 1993

This booklet of abstracts was compiled by Paul Zorn from copy provided by many authors. They appear in no particular order. All information is subject to change.

Calculus: What Constitutes Appropriate Use of Technology?

David Lomen
University of Arizona, Department of Mathematics, Tucson AZ 85721
Email: lomen@math.arizona.edu

Computer technology may be used to:

- Emphasize local and global behavior of functions;
- Explain the interplay between continuous and discrete reasoning;
- Encourage formation of conjectures;
- Provide a means to check an answer (allow alternative approaches);
- Help implement problem solving strategies;
- Promote critical thinking;
- Allow students to judge their preparedness for a course and aid them in taking appropriate remedial action.

Examples of the above uses will be given, and also include the use of "mini-projects" as well as use of IBM's "Personal Science Laboratory."

All of the University of Arizona's twenty-six "Mathematical Toolkits," six "Are You Ready for . . . ?", and eight "Slide Shows" will be available for inspection. These are public domain software packages (MS-DOS) that are available via file transfer protocol. A manual is also available. These programs are used extensively with the calculus materials produced by the Calculus Consortium based at Harvard.

A Calculus Course for Students of Mathematics, Science, and Engineering

Auburn University, Division of Mathematics, Auburn AL 36849-5300
Phillip Zenor, Donald Thaxton and Edward E. Slaminka
Email: slam@ducvax.auburn.edu

This calculus course, developed and taught over the past 18 years, is designed for the student who will take physics either concurrently or subsequently. Its purpose is to help the students learn to use calculus as a powerful and elegant tool to model and solve problems in Newtonian physics. The course was team-taught as a combined calculus/physics course for seven years. Then the calculus part was extracted, maintaining the organization and enough physics to motivate the mathematical tools being developed. The course begins by developing a sense of geometry in two and three dimensions. Then multivariate calculus is developed simultaneously with the calculus of real-valued functions. Position, velocity, acceleration, and force functions (as vector-valued functions) are the motivating ideas in the introduction of the derivative. The course is structured so that by the end of the 4-quarter sequence, line integrals, surface integrals, and volume integrals and the related change of variables theorems, the divergence and curl operators, conservative forces and potential energy are carefully and thoroughly developed and applied.

Project CALC and *Mathematica*

Bowdoin College, Department of Mathematics, Brunswick ME 04011
William H. Barker
Email: wbarker@bowdoin.edu

Project CALC ("Calculus As a Laboratory Course"), developed at Duke University with support from the National Science Foundation, is part of a nationwide effort to revitalize the teaching of calculus. The project represents a major revision of the calculus curriculum, built around (1) selected problem areas in the natural and social sciences that motivate the introduction of calculus techniques, (2) a discovery-based computer laboratory, and (3) writing assignments which place the emphasis of the course on conceptual understanding, not solely on computational skills.

At Bowdoin, the Project CALC materials have been reformulated around *Mathematica*, a sophisticated integration of text-processing, numeric and symbolic computation, graphics, and programming. Grants from both the National Science Foundation and the Alden Trust have supported this conversion, which began in the fall of 1990. *Mathematica* notebooks for over forty laboratories currently exist, covering all three semesters of calculus. These notebooks have been designed for use with Macintosh computers equipped with 13-inch color monitors. Versions of the notebooks have also been produced for NeXT computers.

This semester there are two sections of Project CALC at the College—one each of Calculus I and Calculus II—with a total enrollment of forty-five students. Next semester one section each of Calculus II and Calculus III will be offered. Each course has a weekly two-hour computer laboratory session. Current plans call for the use of the laboratories in all sections of calculus starting in the fall of 1993.

Printed versions of the laboratory notebooks are published by D.C. Heath. Electronic versions of all the materials are currently distributed for free via FTP transfer. Dissemination via disks is also done, although a small fee is charged to cover costs.

For more information contact: William H. Barker, Bowdoin College, Brunswick, Maine 04011, 207-725-3571, FAX 207-725-3123. (Electronic mail is the preferred method of communication.)

HBCU Project to Retain Minority Students in Science and Engineering

Bowie State University, Department of Mathematics, Bowie, MD 20715
Rebecca Lee

The purpose of this project is two-fold. First, it is to encourage students to remain in the sciences by focusing on calculus. As we all know, success in calculus is required in order to pursue a career in mathematics, chemistry, computer science, physics, and other sciences. Secondly, it is to use modern technology in the classroom to aid in the understanding of calculus. This is a two-year project funded by the Minority Science Improvement Program, grant number P120B10033. The funding level is $49,676.

This project involves about sixty Calculus I and Calculus II students a semester. The students have bi-weekly projects using *MicroCalc* as an aid. Some of the projects are discovery projects and others are practice and verification projects. The majority of the students have responded positively to *MicroCalc* as it is very user-friendly. Also, I have been using *MicroCalc* in the classroom to emphasize certain concepts in my lessons. I have included in-class group worksheets which reinforce the current topics.

Calculus Reform at Christian Brothers University

Christian Brothers University, Department of Mathematics, Memphis TN 38104
Cathy W. Carter

Christian Brothers University shared the concern of the mathematical community in revitalizing calculus instruction. We sought to incorporate critical thinking exercises and the advanced technology to increase the understanding of concepts.

The Calculus Project at Christian Brothers University offers a model for schools who have limited funding. In 1989, we added an 80-minute laboratory component to Calculus I. We shared computer facilities with other departments on campus and used relatively inexpensive software. Realizing that one software package would not meet our specific

goals, we developed special software, CalcGraphs, for use in some of the lessons. The combination of CalcGraphs and *MicroCalc* allowed us to meet our educational objectives without spending excessive amounts of time teaching software. We wrote an interactive lab manual to guide students through sessions.

Some computer lessons stress a discovery approach to calculus. Other lessons use the computer to perform algebraic computations so that students can concentrate on the implications of the mathematics. The personal computer format combined with the interactive manual requires that all members of the class become involved. In addition to the computer experiences, some lab sessions use group interaction and problem solving to promote learning. At the end of each lesson is an assessment where writing is an important tool.

In 1990, the National Science Foundation funded an ILI grant. With the help of this grant we equipped a computer classroom dedicated to calculus instruction. In Spring 1991, we extended the lab experiences into the Calculus II course with the production of a second lab manual.

In 1992–93, Christian Brothers will have approximately 180 students in Calculus I and a similar number of students in Calculus II. We feel that the lab experience helps students understand calculus. We will gladly share our experiences and CalcGraphs with those interested.

Project CALC: Calculus as a Laboratory Course

Duke University, Department of Mathematics, Box 90320, Durham NC 27708-0320
Lawrence Moore and David Smith, (919) 660-2825 (voice) 660-2821 (FAX)
Email: `das@math.duke.edu or lang@math.duke.edu`

Project CALC is an award-winning curriculum development project funded by the National Science Foundation. It has produced materials for a three-semester reformed calculus course that emphasizes real-world problems, hands-on activities, discovery learning, writing and revision of writing, teamwork, intelligent use of available tools, and high expectations of students.

Materials: Text and lab materials are in their fourth year of testing and revision at Duke, where the Project CALC course is the only course offered to students who place into Calculus I. The materials are also in use at 25 other schools across the country (2- and 4-year colleges, universities, and one high school). Preliminary editions of The Calculus Reader, lab manuals, and Instructor's Guide are available from D. C. Heath and Co., 125 Spring St., Lexington, MA 02173, (800) 235-3565 or (617) 862-6650 – or request them at the Heath exhibit, Booths 104-106 – no charge to prospective adopters.

Goals: Students should (1) be able to use mathematics to structure their understanding of and investigate questions in the world around them; (2) be able to use calculus to formulate problems, to solve problems, and to communicate the solution of problems to others; (3) be able to use technology as an integral part of this process of formulation, solution, and communication; (4) learn to work and learn cooperatively.

Integrated lab experience: Students working in pairs in a computer or calculator lab explore real-world problems with real data, conjecture and test their conjectures, discuss their work with each other, and write up their results and conclusions. This experience drives the rest of the course, and it shapes the content and approach of the text and the classroom activities.

Pedagogy: There is a difference between TEACHING and LEARNING. Instead of concentrating on the teacher (What makes a good lecture? How can we cover all the topics packed into the syllabus?), we concentrate on the student: What activities will help students construct the mathematical knowledge they need? We now lecture less and devote more time to student activities in groups – gathering data on periods of pendulums, balancing plywood cutouts to locate centers of mass, calculating the rate of approach of two planes and deciding whether they will collide.

The course at Duke: Our course meets for three 50-minute periods in a classroom equipped with one demonstration computer. Each section (maximum of 32) splits into two lab groups; each group has a weekly two-hour lab. Each lab team of two submits a written report almost every week; three or four of these each semester are "formal": submission, instructor review, and resubmission for a grade. The remaining reports are "fill in the paragraph," often finished during the lab period. In the classroom, teams of four work on various activities. Each student has a week-long homework assignment, mostly routine computations and exercises embedded in the reading. One class period each week is "group office hours"; the instructor responds to student problems but does not initiate new material.

Lab versions: Lab materials exist now for MathCAD 2.x/Derive and for Mathematica. Versions for Mathcad 3.x (Windows), for HP-48SX and TI-85 calculators, and for Maple V are under development. Other hardware/software platforms are under consideration, and offers to convert to other platforms are welcome.

Workshop opportunities: (1) June 14-18, 1993, we will offer our second workshop for faculty interested in using our materials. Our NSF grant covers room, board, and all materials; travel to Durham is the responsibility of the attendees or their institutions. To apply, contact the authors at the address above. (2) We offer a workshop every August for our own faculty and graduate students, and these workshops have had attendees from other schools. No support is available for the August workshop. For more information, use the address above. (3) July 18-23, we will present one of the 1993 NSF Calculus Reform Workshops at Pennsylvania State University. For information, contact Prof. David Bressoud, Department of Mathematics, Penn State Univ., University Park, PA 16802 (814-865-4061).

Franklin and Marshall Calculus Reform Project

Franklin and Marshall College, Department of Mathematics, Lancaster PA 17604
Alan Levine and George M. Rosenstein
Email: a_levine@acad.fandm.edu or g_rosenstein@acad.fandm.edu

Background: Franklin and Marshall College is a liberal arts college with 1850 students located in Lancaster, PA. Calculus is a three-semester sequence taught in sections of 25–30 students by full-time faculty members.

Objectives: The project has three objectives. The first is to place more emphasis on the concepts and applications of calculus. The second is to decrease the emphasis on rote mechanical procedures. The third is to change the pedagogical approach to make the students more active participants in the learning process.

Textbook: *Discovering Calculus* by Alan Levine and George M. Rosenstein, Jr. The book is under contract to be published by McGraw-Hill. Preliminary versions are available from the authors or the publisher. The book is intended for a three-semester course.

Significant features include:

Motivation: Each chapter is motivated by one or more interesting problems.

Readability: If students are to participate in the discussion, they must prepare by reading the text in advance. Our text is written for students to read. It includes numerous Test Your Understanding questions designed to ensure that the reader understands a given concept before going on to the next.

Differential equations: First-order differential equations are introduced in Chapter 4 and used to motivate exponential functions. Second-order differential equations are introduced in Chapter 7 and used to motivate trigonometric functions. Infinite series are motivated in Chapter 11 by the need to solve differential equations that cannot be handled by other techniques. Numerous applications to physical problems are interspersed throughout the text.

Projects: The book is supplemented by a set of projects that extend the concepts and/or applications in the book or introduce other interesting topics not vital to the main core concepts.

Technology: While some of the problems are projects may require graphing calculators or computers, the course can be taught without any specific technology.

The Five College *Calculus in Context* Project

Department of Mathematics, Smith College, Northampton, MA 01060
J. Callahan, K. Hoffman, D. O'Shea, H. Pollatsek, L. Senechal
Email: callahan@emmy.smith.edu

The Five College *Calculus in Context* project is a restructuring of the standard three-semester calculus sequence so that the main ideas emerge from context in the life, social, and physical sciences. So, for example, the course begins with the study of a model for the spread of a disease consisting of three (nonlinear) differential equations. By treating the equations as rate equations (and the notion of derivative as distinct from the process of differentiation), and by

using computers from the outset, students are immersed in the heart of the subject from the first day of the course. The project is in its fifth year and now is the standard course at Hampshire, Mount Holyoke, and Smith Colleges. It is also being tried at a number of other institutions.

The course has had a number of benefits which were unanticipated at the outset: for example, students from disadvantaged backgrounds do much better vis à vis students from other backgrounds. Principal investigators are J. Callahan (Smith), D. Cox (Amherst), K. Hoffman (Hampshire), D. O'Shea, H. Pollatsek, and L. Senechal (all of Mount Holyoke). The contact person is Jim Callahan; see address above.

Using Interactive Mathematics Texts as Teaching Tools

Frostburg State University, Department of Mathematics, Frostburg MD 21532
Marcelle Bessman
Email: r2nkbes@fre.towson.edu

The Interactive Mathematics Text (IMT) provides a seamless learning environment wherein a student can explore mathematical ideas, computationally and graphically, as well as obtain needed background information and procedural instructions. Within the context of a particular topic (for example, translation of functions), the user can pose his/her own problems, create or alter equations and functions, and actively observe their mathematical behaviors. The basic textbook can be readily created by the teacher for student use.

Examples of textbooks covering topics in calculus and the mathematics of physics, and their classroom use will be presented. The development of such textbooks will be discussed. In particular, a laboratory manual based on Duke's Project CALC, will be discussed. It is intended that this manual will be used here at Frostburg State in our Project CALC. Site testing is planned for 1993–94.

There are various versions of the IMT. Some are based on using a particular mathematical software, such as *Mathematica* or *MathCAD*. The version used here, however, is based on the symbiotic integration of specially created mathematics clips (utilities) with a Windows-based presentation software package, Toolbook. These utilities can also be used with other Windows-based programs. The software used, MathWindows, is being developed by James E. White of the Institute for Academic Technology in Durham. This software can do two- and three-dimensional graphing, and simple, iterative, or recursive computation. Toolbook has multi-media capabilities that allow the use of sound, video, and animation. The integration of the two programs permits not only multi-media enhanced presentation of mathematical ideas, but also user-controlled experimentation with given data and the real time collection and analysis of experimental data.

The manual is being developed by White and myself, in consultation with the Project CALC authors, David Smith and Lawrence Moore.

The Connecticut Calculus Consortium

University of Hartford, Department of Mathematics, West Hartford CT 06117-1599
Bob Decker
Email: rdecker@hartford

In 1991 the University of Hartford received an NSF Calculus reform grant to implement a laboratory approach to Calculus I and II at both the University and about 25 high schools, community colleges and colleges in the state of Connecticut. In the summer of 1991, teachers from those institutions were trained in workshops at the University of Hartford, and then taught a lab section of calculus at their home schools the following year. In the summer of 1992 more workshops were held, and the second round of participants are now teaching a laboratory version of calculus.

The basis of the program is the laboratory manual *Bringing Calculus to Life*, published by Prentice Hall. The labs involve modeling using real data (some taken by the students themselves) and experimentation with computers or calculators. Either computer algebra systems (such as *Derive*, *Mathematica*, or *Maple*), or graphing calculators (such as the TI-81 or Casio fx7700G) can be used. Currently, about two-thirds of the schools that are participating in the program are using the calculator version of the manual, and about one-third are using the computer version. The manual can be used with a standard calculus book, or a reformed one (though we prefer a reformed one). Both computer software and graphing calculators will be available at the poster table to demonstrate the laboratory activity.

Core Calculus Consortium

Harvard University, Department of Mathematics, 1 Oxford St., Cambridge MA 02138
Deborah Hughes Hallett, (617) 495-5358 or (617) 495-2171

Our philosophy: "The Rule of Three." The most difficult and the most necessary aspect of revitalizing calculus is getting our students to think. The old calculus has become a litany of procedures and template problems which too often results only in giving students some rather mindless algebra practice. In addition, students with weak backgrounds are usually driven away by frustration over the manipulations required, even if they are able to understand the basic ideas of calculus.

Our project is based on our belief that the graphical, numerical, and analytical aspects of calculus should all be emphasized throughout. We call this approach "The Rule of Three" and are working together to design a core curriculum based on this principle. Using the Rule of Three, students will repeatedly be confronted with the graphical and numerical meaning of what they are doing. Besides encouraging understanding, this approach gives students with weak manipulative skills a chance to grasp the concepts behind calculus while strengthening their backgrounds.

The Core Calculus Course: The core calculus curriculum is a two-quarter or two-semester course built around the Rule of Three. The graphical and numerical aspects of each topic are introduced throughout, along with the analytic. For example, consider the traditional introduction of the exponential function as the inverse of lnx, which is itself defined as an integral. This approach is lost on students who don't like logs and are not yet comfortable with integrals. Worse still, it obscures the most important fact about the exponential function—that its derivative is equal to itself. However, using a graphing program or a spreadsheet, it is easy to show students that 'e' is the one value of 'a' making the function of a^x and its derivative equal.

The idea of integration and of solving differential equations comes into our course early. We find that slope fields provide a way of introducing the integral which makes it clear graphically that integration reverses the process of differentiation. In addition, using a slope field makes the Fundamental Theorem of Calculus quite transparent.

Of course, if our good intentions are to have much effect, our tests as well as our presentation must reflect the Rule of Three. In addition, it is important to include some non-routine problems regularly. Even if such problems are not hard or long, we must start to re-establish the idea that such problems are not only part of mathematics, but, indeed, the point of mathematics.

Timetable: During 1990–91, preliminary versions of the Core Calculus Curriculum materials were taught at the consortium institutions. During the 1991–92 academic year, the materials were class-tested at all the consortium institutions and about twenty others; in 1992–93, class testing is taking place at over one-hundred institutions.

A preliminary version of the text is available from John Wiley (212-850-6000).

We welcome input and suggestions from the entire mathematics community, and invite its active participation in testing our approach.

Further Information: Herman Sudholz, Project Coordinator (617-496-5421); address above.

The New Sensible Calculus Program

Humboldt State University, Department of Mathematics, Arcata, CA 95521
Martin Flashman, Phone: 707-826-4950, FAX: 707-826-5555
Email: `flashman@axe.humboldt.edu`

The Sensible Calculus Project (SCP) works to develop curriculum, pedagogy and technology, reforming calculus based on the themes of differential equations and estimation project motivated by modeling situations. This project has been a continuing effort since 1978. Recent work on the project has focused on materials that apply calculus consistently to an understanding of probability beginning early with a treatment of density functions as the derivatives of distribution functions. Other additions have been student journals and oral presentations reporting on outside reading from journals (and other nontraditional sources). Text materials are now more fully developed and a sampling is available for examination upon request with a payment of $10 to help cover costs.

X(PLORE) (previously the Calculus Calculator) by David Meredith, published by Prentice Hall, is the main technological support of SCP materials.

An extension of the calculus project, the New Sensible Calculus Program (NSCP), will be developing precalculus materials that are consistent with the calculus project views. In particular students will have two visual approaches to functions (graphs and transformation figures), as well as reasonable interpretations for functions that help students realize the distinction between the mathematics and the applications/interpretations of the mathematical objects.

Currently SCP materials are the sole text for one section of first-year calculus (about 20 students) and were used for the same number of students last year. Expansion to selected test sites and commercial publication are now under consideration. For more information contact Professor Flashman at Humboldt State University.

A Laboratory-Enriched Calculus Sequence

Lafayette College, Department of Mathematics, Easton, PA 18042-1781
L. Thomas Hill
Email: `hl#2@lafayacs.bitnet`

At Lafayette College, we have integrated a computer-laboratory experience into our three-semester calculus sequence for students of engineering and the mathematical and physical sciences. Meeting in weekly laboratory sessions, students use *Mathematica* to anticipate new ideas and to investigate in greater depth ideas to which they have been introduced in the classroom. Laboratory activities are designed to help students learn to combine effectively the use of graphical, numerical, and symbolic techniques in solving problems and investigating mathematical ideas, to increase students' sense of ownership of mathematical ideas by providing opportunities for exploration and discovery, and to encourage students to develop collaborative learning skills. In addition, the laboratory experience helps our students to acquire a working knowledge of a computational tool that can be of great value to them in their future work in mathematics, science, and engineering.

Our laboratory project began with pilot sections of Calculus I and Calculus II in academic year 1990–91; full implementation of the laboratory experience in the three-semester sequence was completed in 1991–92 when there were more than 450 registrations in the affected courses. In 1992–93, all sections of the calculus sequence will include laboratory experiences, and the total enrollment in those sections will be approximately 600. In addition, by the end of the fall semester of 1992, all eighteen faculty members in the department will have taught laboratory-enhanced calculus courses. Students in intermediate and advanced courses in linear algebra, differential equations, and numerical analysis are already making extensive use of *Mathematica* in their studies, and further expansion of those efforts is anticipated. In addition, we have held workshops for our faculty colleagues in engineering and the sciences to encourage them to take advantage of the laboratory experience that their students are having in calculus.

Partial support for the development of our calculus laboratory was provided by the National Science Foundation through grant number USE-9152827.

Resources for Calculus

Macalester College, Department of Mathematics, St Paul MN 55105-1899
Wayne Roberts, PI
Email: `wroberts@macalstr.edu`

Project Goal: Our goal was to collect resources of the kind that would be useful to anyone, writer, or classroom teacher, wanting to address the criticisms that have been made of the way calculus is being taught in this country. Our intention was not to write text material for students, but to provide resources for the writer or lecturer, and to put them in a form that would make them usable with as little modification as possible. To the extent possible, we wanted these materials in the public domain, easy to copy and use.

The Materials We Produced:

Vol. 1 *Learning by Discovery*, Anita Solow, Editor. Contains 28 laboratory modules, spanning the range of calculus. These modules can be used as lab components in the course, or they can be assigned as independent projects. The labs are written without specific computer commands. Suggestions are provided for implementing these labs on *Derive*, *Maple*, and *Mathematica*. Many can be done on graphics calculators.

Vol. 2 *Calculus Problems for a New Century*, Robert Fraga, Editor. Restates good existing problems in ways so that they are not rendered trivial by a computer or graphing calculator, and introduces problems made newly accessible by technology (though most can still be done by hand). Problems are organized in groups that parallel traditional grouping of ideas, making it possible to use them as supplements to most texts.

Vol. 3 *Applications of Calculus*, Philip Straffin, Editor. Contains 18 applications complete enough to be understood, substantial enough to illustrate the real power of calculus, current enough to engage student interest. Written at a level to be grasped by a lecturer wanting to introduce ideas about which he or she is not an expert, or handed to an interested student.

Vol. 4 *Problems for Student Investigation*, Michael Jackson and John Ramsay, Editors. A collection of problems that require students to draw on multiple sources, to "live with a problem" while thinking and talking about it to peers over a period of time, to exercise creativity in a situation where there is no "way to do these kind," and to defend their answer as reasonable when there is no single right answer.

Vol. 5 *Readings for Calculus*, Underwood Dudley, Editor. Readings selected to give students a sense of the history of the subject, the role it has played in intellectual history, and the contribution it makes to a modern world view. Can be used as collateral readings or handed to an inquiring student.

Availability: These volumes have been published in the MAA Notes, and are available from the Mathematical Association of America, 1529 Eighteenth Street, NW, Washington, DC 20036.

From Projects to Themes: Calculus at New Mexico State University

New Mexico State University, Department of Mathematics, Las Cruces, NM 88003
Douglas S. Kurtz and David J. Pengelley
Email: doug@nmsu.edu or davidp@nmsu.edu

The calculus program for science and engineering students at New Mexico State University has gone through several major evolutionary stages in the past five years, each prompted by input from new groups of faculty. Today essentially all faculty teaching calculus are voluntarily involved in this ongoing process. Our experiences can serve as an example of how widespread change can occur in a department of over 30 research faculty at a state university, teaching each semester about 22 calculus sections of 45 students each.

The introduction of two-week student assignments in 1987 led to the NSF funded development of a "student research projects" program. The goal is to get students to think for themselves on major multistep, take-home problems, working individually or in groups. We hoped to alter fundamentally students' view of what mathematics is all about and simultaneously to build their self-confidence in what they could achieve through imaginative, theoretical thinking. The projects resemble mini-research problems. Most of them require creative thought and all of them engage students' analytic and intuitive faculties, often weaving together ideas from many parts of calculus. While many are couched in a seemingly real-world setting, often with an engaging story line, they are all in a sense theoretical; one cannot do them without an appreciation of the ideas behind the method. Students must decide what the problem is about, what tools from calculus they will use to solve it, find a strategy for its solution and present their findings in a written report. This approach yields an amazing level of sincere questioning, energetic research, dogged persistence, and conscientious communication from students. Moreover, our own opinion of students' capabilities skyrocketed as they rose to these challenges. Typically we assigned two or three major projects per semester. Over 100 of these projects were published in the book *Student Research Projects in Calculus*, by the Mathematical Association of America, which also has chapters detailing the logistics of assigning projects and advice for instructors.

As new faculty became involved, they injected fresh ideas and the projects approach evolved. In 1991 a collection of faculty pioneered a major new emphasis on cooperative self learning both in and out of the classroom. They developed structured in-class group assignments called "themes." A distinct change is that themes are used to introduce core material and much class time is spent working on them, with less time on lecture, whereas the projects were completed

entirely outside class and contained material over and above day-to-day course work. Classes now alternate between a one or two week theme and one or more weeks of more traditional classroom activity, with a total of 4–6 themes per semester. Another recent experiment is mastery based skills exams to ensure command of differentiation and integration skills.

Themes and skills exams have made possible a shift on midterm and final exams, away from rote calculation towards more conceptual and essay questions. Theme assignments have also prompted us to incorporate structured means of improving student writing skills in mathematics. Finally, we are discovering something very encouraging: when students are involved in collaborative self-learning with themes, we are less pressed to "cover" material in class, so the syllabus feels less rushed. The difference may be substantial.

Calculus, Computers, and Commuters

Metropolitan State College of Denver, Department of Mathematical Sciences, Denver, CO 80217-3362
Thomas Kelley

Metropolitan State College of Denver (MSCD) is an urban commuter college located in downtown Denver, Colorado. What happens to the commuting student when calculus reform and computer technology bring about changes in the way in which calculus courses are taught?

The Mathematical Sciences Department at MSCD has tried to take this into account in adopting certain changes in its calculus sequence. Besides taking a hard look at the topics covered (Make calculus LEANER!) the department, with the support of the college and the NSF, have instituted Calculus Modules in *Mathematica* notebooks that the students use on 23 NeXT computers (Make calculus LIVELIER!) located in a lab on campus. The modules serve to introduce, supplement or enhance some of the topics covered in a traditional calculus text. Each module covers a well-defined topic while making extensive use of *Mathematica*'s text and graphics capabilities. There is a corresponding worksheet with each module that the students are required to do on the computer and the results are handed in on a diskette to be graded. The students do one module per week with one 50-minute class session (out of 4) per week in the lab. Students are also required to do "paper & pencil" calculus homework and take in-class exams.

We presently have 4 out of 6 sections of Calculus I using the modules and plans for expansion to sections of Calculus II in the Spring '93. We believe that this approach, although not without difficulties, takes into account the costs of technology for students (additional costs amount to diskettes for students' personal use) and the scheduling constraints of commuters. Students need to schedule lab time while they are on campus, but the lab has generous open hours, the modules and worksheets are intentionally kept brief and hard copy of each module/worksheet is given to each student.

Calculus at Moravian College

Moravian College, Department of Mathematics, Bethlehem, PA 18018-6650
Doris Schattschneider
Email: `schattdo@moravian.edu`

In recent years, the requirement of calculus has become standard not only for the physical and mathematical sciences, but for many other college majors. This has created a serious problem for those who teach calculus: roughly half the students with this requirement are inadequately prepared to take calculus.

The standard response of most institutions is to offer a one-semester course called "Precalculus" (or "College Algebra and Trigonometry") and require such students to successfully complete this course before entering a calculus course. Until 1988, Moravian College followed this standard pattern. The morale of both students and teachers in the Precalculus course was low; the attrition in the course was fairly high; the retention of material needed for Calculus I the next semester seemed extremely low.

In 1988, to address these problems, Moravian introduced a new one-year course that replaced the Precalculus–Calculus I sequence. The course integrates the review of precalculus topics as they are needed within the first calculus course. In the short time it has been offered, we have observed improvement in morale, level of understanding, and numbers completing the course.

Although many Precalculus books are available, there are none that are designed to be used as part of a calculus

course. Our FIPSE-funded project, now in its second year, is to prepare, desk-top publish, and class-test supplemental materials especially designed to integrate the review of algebra skills, functions, graphing, and problem-solving techniques in the first calculus course.

In the first year, faculty members from the Mathematics Departments at Moravian College and Northampton Community College produced a first draft of "A Companion to Calculus" and used it in all sections of Moravian's integrated course "Calculus I with Review" (120 students). It was revised in summer 1992 and is being used at Moravian again (100 students) and Northampton Community College (36 students) this year. The "Companion" is designed to be used with any standard calculus text; it is currently under consideration by several publishers.

Evaluation has been a major aspect of the project. It includes records on student enrollment and completion of the course, comparison of individual student performance on pretests and postests, comparison of performance on selected exam questions with those in the "regular" one-semester course; attitude surveys of those in the integrated course; evaluation of the materials by students and instructors.

On June 18–19, 1993 a conference will be held at Moravian College that focuses on the integration of precalculus with the first course in calculus, and the problems of teaching calculus to underprepared students.

Project Director: Doris Schattschneider, Mathematics Department, Moravian College, 1200 Main St., Bethlehem, PA 18018-6650. Phone: (215) 861-1373; FAX: (215) 861-3919; Email: `schattdo@moravian.edu`

The Northeastern University Excel Mathematics Program

Northeastern University, Department of Mathematics, Boston, MA 02115
Rick Porter (617) 437-2450
Email: `rdp@northeastern.edu`

At Northeastern we are trying to combine the innovative ideas of Uri Treisman with elements of calculus reform and the use of computer labs. Professor Treisman (University of Texas at Austin) discovered that students who worked together in small groups under the supervision of more advanced students ("mentors") could, in a carefully designed program, dramatically increase their ability in mathematics. These "Treisman" programs, designed to improve the mathematics education of African–American and Hispanic students at Berkeley, have proved to be spectacularly successful—there and at several other institutions where they have been tried.

Northeastern's **Excel** program is more than just a copy of Treisman's original ideas. While most Treisman programs have been implemented within a very traditional mathematics curriculum, we decided that we would use his methods in the context of modern calculus "reform." For a number of years we have been shaping our calculus curriculum around the central ideas of differential equations and mathematical modeling. We introduce logarithmic and exponential functions very early in our freshman sequence, and use flow or direction fields soon after to describe differential equations. We accentuate numerical methods throughout our calculus sequence, and all science and engineering students are required to do a project which involves using differential equations to model a physical situation of their choosing; the students have come up with topics such as wind surfing, alcohol fermentation, and aspects of ecology.

Students are expected to use computers as tools in their calculus courses. At present, each of the three quarters of freshman engineering/science calculus requires five computer lab problem sets. Some of these use inexpensive plotting software to study, for example, curves, flow-fields, and Newton's method. More recently, we have been using spreadsheets because of their visual impact, their obvious numerical orientation (with graphs coming from numbers, not from magic), and their flexibility. One of our goals is to have first-year students working in the computer lab once a week on projects of varying length.

Recently we were fortunate enough to obtain funds from NSF, the Northrup Corporation, and the University to build our own computer lab. The **Excel** lab is designed in a way which maximizes student interaction. The computer tables are hexagonal and the chairs are swivel base, with casters, to encourage group and intergroup activity. As with other aspects of Treisman programs, the **Excel** lab is staffed by student mentors. These mentors are specially trained in the use of the software, and they are themselves "graduates" of our calculus sequence and computer labs. They understand the educational content of the exercises, and know how to ask leading questions and suggest lines of attack without giving away answers. Choosing and training our student mentors is one of the most important parts of our program.

Currently, all of our first-year science and engineering students (nearly 500) are involved in the reformed curriculum and computer lab. We also have one (voluntary) section of freshman calculus taught in the Treisman style; in subsequent

years, as more student mentors can be recruited from the program, we hope to teach all sections this way, and expand the entire program beyond the freshman year.

Calculator-Enhanced Instruction in a Consortium of New Jersey and Pennsylvania Institutions

Union County College, 1033 Springfield Ave., Department of Mathematics, Cranford NJ 07016-1599
Jean Lane

This two-year project addresses reform of the mathematics curriculum, particularly in calculus and precalculus, by incorporating the use of graphics and symbolic manipulation calculators to enhance teaching and learning following the Clemson model. Originally funded during the 1991–1992 academic year, the Consortium consisted of five New Jersey community colleges. of which four are continuing in the project. Initial activities targeted precalculus and first-semester calculus; continuing activities include second- and third-semester calculus, trigonometry, and statistics, with implementation planned in linear algebra for 1994. The eleven members of the current Consortium include two-year colleges: Union County College, Essex County College, Ocean County College, Camden County College, Raritan Valley Community College, County College of Morris, and Montgomery County Community College (PA); one private four-year college: Saint Peter's College; and three high schools: Immaculata High School, Linden High School, and Monmouth Regional High School. Both Hewlett-Packard and Texas Instruments are supporting the project with discount prices and donations of equipment. For the new members of the Consortium, at least two participants per school attended intensive, hands-on weekend workshops on the TI-81 and HP-48 calculators in early fall 1992, and will work individually, in pairs, and as a group for the remainder of 1992 to plan implementation of new teaching strategies beginning in spring 1993 and continuing through the 1993–1994 academic year. All participants will convene again for four additional weekend workshops scheduled throughout 1993 and 1994. During the 1992–1993 academic year, we estimate the project will reach over 1000 students. Evaluation of results will be through student surveys of rates in similar courses taught without calculator usage. Results will be disseminated at various conferences including those sponsored by AMATYC, MAA, MATYCNJ, PYSMATYC, NJMATHNet. Participants will give seminars on their experiences at their home institutions. Additionally, two major conferences will be planned for area high schools and college faculty by Union County College and Raritan Valley Community College. It should be pointed out that community college enrollments have soared during the past two years, and that the seven community colleges involved in this project represent a student population in excess of 70,000, with minority enrollments ranging from 8% to 90%.

Project Director is Jean Lane of Union County College; Co-Project Director for the high schools is Elaine Petsu of Immaculata High School.

The Oregon State Calculus Curriculum Project

Thomas Dick
Oregon State University, Department of Mathematics, Corvallis OR 97331-4605
Thomas Dick
Email: `tpdick@math.orst.edu`

Goal: The Oregon State Calculus Project is committed to using technology in a pedagogically sound manner to improve calculus instruction. In particular, the project seeks to take advantage of the unprecedented access to numerical, symbolic, and graphical tools that computers and super calculators allow both students and instructors.

Activities: The major activities of the Oregon State project are: (1) production of calculus text materials appropriate for the equivalent of three semesters of calculus (through multivariate and vector calculus) along with laboratory activities for students, all designed to take full advantage of numerical, graphical, and symbolic tools offered by technology such as supercalculators and computers; (2) class-testing of these materials in a wide variety of instructional settings, including high school, two-year college, four-year college, and universities; and (3) provision of instructional workshops and continuing instructional support for the pilot and field test instructors using the curriculum materials and technology.

Description: *Calculus, Volume I* is based on a multiple representation approach to the concept of function, and has been published by PWS–KENT. *Calculus, Volume II*, covering multivariable and vector calculus, has been completed and became available for use starting Fall 1992. An *Instructor's Resource Manual* is available for use with the

materials. All materials are now being revised, with a new edition available by late 1993. The Student Guide to Using Technology in Calculus is a sourcebook of activities that are designed to (1) anticipate key results through student discovery (examples include discovery lessons on the First and Second Derivative Test, and the Fundamental Theorems of Calculus), or (2) to provide some extension of the material in the text (examples include lessons on the error estimates for numerical techniques of integration and orders of agreement of functions).

Technology: While the curriculum assumes the availability of technology in order to make greater use of numerical approximation strategies and graphic visualization techniques, the materials do not assume any specific hardware/software combination (several different models of graphing calculators and CAS software packages, including *DERIVE*, *MAPLE*, *MicroCalc*, *Calculus T/L*, *Theorist*, and spreadsheets have been used with the materials to date). The text is compatible with the current Advanced Placement syllabus (both AB and BC) and will continue to be as the College Board moves toward a heavier emphasis on visualization in the AP syllabus and technology based testing (graphing calculators to be required by the 1995 examination). Nine high schools using the text in 1991–92 all do so in Advanced Placement courses.

Pilot Test Sites: Through 1991–92, the text has been pilot tested in over 50 high schools, two-year colleges, four-year colleges, and universities. In 1992–93, approximately 100 additional high schools nationwide are using the text in connection with a nationwide super calculator equipment donation program sponsored by Hewlett–Packard and the Oregon State Project. It is anticipated that 125 more high schools will join the program in 1993–94, and that the total number of institutions reached by the project will approach 300. New users of the materials are actively sought, particularly for the revised materials starting in Fall 1993.

For More Information: For an examination copy of the materials, college instructors can call PWS–KENT Publishing Company toll free at 1-800-423-0563, or at 1-617-542-3377. High school instructors can call the Wadsworth school group toll-free at 1-800-831-6996.

Contact person: Thomas Dick, address above. Phone: 503-737-5179; Fax: 503-737-0517.

Calculus from Graphical, Numerical, and Algebraic Points of View

St. Olaf College, Department of Mathematics, 1520 St. Olaf Ave., Northfield MN 55057
Arnold Ostebee and Paul Zorn
Email: ostebee@stolaf.edu or zorn@stolaf.edu

Philosophy: Traditional calculus courses often slight the conceptual foundations of the subject while overemphasizing routine techniques—formal differentiation, antidifferentiation, convergence testing, etc. Moreover, *analytic* objects are almost always represented and manipulated *algebraically*, via symbolic manipulation of explicit elementary functions. For example, textbooks usually treat limits, derivatives, and integrals—all analytic objects—largely as operations on symbolic expressions.

Whether one views calculus as an introduction to pure mathematics or as a foundation for applications, the conclusion is the same—concepts, not techniques, are truly fundamental to the course. The *sine qua non* for a useful command of calculus is a conceptual understanding that is deep and flexible enough to accommodate diverse applications.

Bringing graphical and numerical, as well as algebraic, viewpoints to bear on calculus ideas is the philosophical foundation of our text. By representing and manipulating calculus ideas and objects graphically, numerically, and algebraically, we believe that students gain a better, deeper, and more useful understanding of these concepts.

Technology: Combining graphical, numerical, and symbolic viewpoints is difficult "by hand." Technology helps makes these viewpoints practically accessible; we use it freely. However, we do *not* aim to "automate" or "computerize" calculus. We treat computers and graphing calculators strictly as *tools*—albeit powerful ones—for combining graphical, numerical, and algebraic points of view.

Although it freely uses and refers to numerical and graphical computations, our text is independent of any *particular* technology. Many graphing calculators and software packages provide all the necessary computing power.

Distinctive features of the text: Our text differs from standard treatments in various ways, including:

Graphical, numerical, and algebraic viewpoints Throughout the text we insist that students manipulate and compare graphical, numerical, and algebraic representations of mathematical objects. Graphs or tables of numerical values—or both!—appear on almost every page. Because readers repeatedly confront functions that are *not* defined by algebraic expressions, they must interpret calculus ideas in graphical and numerical terms.

More varied problems and exercises A textbook's problems and exercises generate most of students' mathematical activity and occupy most of their time. Most texts have plenty of routine drills and challenging theoretical problems—so does ours. In addition, though, we aim for more *qualitative* variety, including problems and exercises posed graphically, problems requiring synthesis, and problems that compare results of various solution techniques. Thus, we hope, students will see mathematics as an open-ended, creative activity, not a rigid collection of recipes.

Numerical methods We use basic numerical methods to illustrate and reinforce such *concepts* as integration and convergence. Although we emphasize numerical techniques more than usual, we don't "cover" numerical analysis as a topic in its own right. We choose numerical methods that best illustrate and clarify underlying analytic ideas, not by such strictly numerical considerations as stability and robustness.

Field testing and dissemination activities A preliminary version of our text is being field-tested in 1992–93 at 25 institutions: secondary schools, two- and four-year colleges, and state universities. A revised preliminary version will be field-tested in 1993–94. Additional field-test sites for 1993–94 are now being sought. A summer workshop for field-test participants is planned for July, 1993. For more information, please contact Arnold Ostebee at the address above.

Computer-Integrated Calculus at the University of Connecticut

James F. Hurley
University of Connecticut, Department of Mathematics, U-9, Storrs, CT 06269-3009
Email: hurley@uconnvm.uconn.edu or hurley@uconnvm.bitnet

The project, currently supported by National Science Foundation Grant USE-9153270, aims to realize the potential cited by CRAFTY after a site visit in 1988 to become one of the emerging national models for calculus reform.

It seeks to foster in-depth understanding of calculus ideas and techniques by using the computer as an active component of the learning process. It also strives to promote analytical thinking in dealing with both mathematical problems and quantitative relationships in science and engineering. One hour per week formerly used for lecture has been converted to a computer-laboratory period in which instructors lead the students in activity designed to sharpen their geometric, numerical, and theoretical intuition and to improve their visualization of basic relationships in mathematics and science. Regular assigned computer projects illustrate the usefulness of the computer as a tool for mathematical computation.

The project includes collaborative work on the computer projects, on conventional homework assignments, and at the weekly problem-discussion hour. There students work as teams on worksheets based on the preceding week's new material.

About 90% of the laboratories use the *True BASIC* Language System, the rest use *Theorist*. Code of the locally-written *True BASIC* programs is examined in the belief that its simple instructions to the computer illuminate underlying mathematical ideas at their most essential level, and illustrate their systematic, logical implementation. Limited use of *Theorist* suffices, because numerical and graphical programs are adequate for most topics.

This year, nine sections of freshman calculus are computer-integrated, with a total enrollment of 200–250 students. The rest enroll in five large lectures of 100–150 students each. Next year, there will be no more large-lecture sections, as the Department converts to a new course based on the project's experimental sections that have been offered since the Fall of 1989. The project has also expanded to third-semester calculus, which is incorporating new surface-plotting software, some locally produced and some from Cornell University. This year, two sections of 25–30 students each are class testing new materials for that course.

With support of a new NSF grant (DUE-9252463), materials from the project are now being adapted for use in the Department's high-school cooperative calculus program. The University's high-school cooperative program awards University credit for courses taught at more than 100 Connecticut high schools in cooperation with several University departments, including Mathematics. The program provides both a no-cost alternative to the Advanced Placement

Program for advanced students and a means for the University to work closely with high schools to insure that college-bound students receive modern calculus instruction comparable to that provided by the University's calculus-reform project.

All you need for a graphing calculator- or computer-based calculus course

University of California at San Diego, Department of Mathematics, La Jolla, CA 92093
Al Shenk
Email: ashenk@ucsd.edu

After two-and-a half years of experimenting with graphing calculators and exploratory worksheets in my calculus classes, I have completed my workbook for a graphing calculator or computer based one-variable calculus course and am negotiating a contract with Harper Collins for it to be published.

It contains 270 pages of worksheets and calculator instructions and programs for a TI-81 calculator, a detailed study guide for using it with my calculus book, and course outlines for using it with any text. Substitute pages for other calculators will be available, and there will be an instructor's manual containing answers to all the problems and all graphs students are expected to generate in solving them.

Please contact me by mail, phone, or e-mail for further information or for copies of materials from the workbook.

Al Shenk, (619) 534-2654; see physical and email addresses above.

Calculus Using *Mathematica* at University of Iowa, University of Wisconsin-La Crosse, Brigham Young University, University of Northern Colorado

Departments of Mathematics at: Iowa City, IA 52242; La Crosse, WI 54601; Provo, UT 84602; Greeley, CO 80639
Keith Stroyan
Email: stroyan@stroyan.math.uiowa.edu

The above schools are collaborating to develop a completely new calculus course. In addition to developing students' basic skills and concepts, our course presents calculus as the "language of science" and has students actively doing mathematics in laboratory-type projects. Besides regular classes, dedicated labs associated with our courses use *Mathematica* for once weekly electronic homework and in the larger projects. Our students develop skills and then learn why calculus is important by applying their skills to answer difficult questions of clear contemporary interest such as: (1) What is a sustainable harvest level for Sei Whales? (2) Why was polio eradicated by vaccination, but not measles? (3) Will a bungee diver smash into the bottom of the canyon? (4) How do you calculate the inverse of $y = x^x$ yourself? (5) What is the resonant frequency of an electrical circuit? Carefully selected scientific problems guide our curriculum and motivate the ideas of calculus for our students. Their answers to these large problems are presented in the form of term papers once or twice per semester.

Mathematica programs (called *NoteBooks*) play two main roles in our presentation of calculus. First, we use the computer to illustrate and reinforce basic mathematical concepts. Second, up-to-date scientific computing helps us reduce technicalities and concentrate on the main ideas of calculus. Computing also allows us to deal with more realistic applications that are too "messy" to compute by hand. Modern computing allows us to treat "advanced" topics like phase plane analysis of nonlinear differential equations.

A New Calculus Program at the University of Michigan

University of Michigan, Department of Mathematics, Ann Arbor, MI 48109
Morton Brown, 313-764-9402
Email: mort_brown@um.cc.umich.edu

The University of Michigan is redesigning its first-year calculus curriculum: in content, delivery, and style. The principal features of the new program are cooperative learning, increase in geometric visualization, new syllabus emphasizing problem solving, and quantitative reasoning.

We are developing a program that achieves these goals in the environment of a large research-oriented university.

This year, all new instructors (including non-tenured faculty) had a week-long training program in teaching techniques, including cooperative learning. We also began a pilot program of the new first-semester calculus in ten of fifty-five sections. It is being taught by a mix of senior and junior faculty and teaching assistants. There are about 30–35 students per section. We are using the Harvard Consortium materials and have loaned TI-81 graphing calculators to the students in the pilot sections. Students work in organized homework groups on a regular basis. Most of the sections have incorporated cooperative learning into the classroom.

The instructors meet on a regular basis, to discuss teaching issues, and issues involving cooperative learning. We developed a supplementary handbook to the Consortium handbook, designed for first-time instructors in the program, to assist them in assessing the level of difficulty of each of the exercises in the textbook. We will develop more materials designed to assist new instructors in the use of cooperative learning techniques, including videotapes of classroom experiences of our instructors. We expect to double the number of pilot sections to twenty next year, and be across the board in 1994–1995.

The model we are developing should be applicable to all large research-oriented Universities.

The new program will continue into the second semester with Calculus II. Natural attrition of enrollment will allow us to begin a few sections of the new Calculus I in the winter term. We expect to increase the number of sections to twenty next year, and to all sections in the fall of 1994. At that time, students will provide their own calculators.

For more information, please contact Morton Brown at the address above.

Metrolina Calculus Consortium: Implementing a Technology-Based Calculus Curriculum

University of North Carolina at Charlotte, Department of Mathematics, Charlotte NC 28223
David C. Royster and Mary Kim Prichard
Email: droyster@unccsun.uncc.edu

The Metrolina Calculus Consortium is supported by an NSF-funded project designed to bring about reforms in the teaching of calculus. The Consortium consists of public and private secondary schools, community colleges, and four-year colleges in the Charlotte–Metrolina area. The project provides support and resources for teachers to adapt and implement a technology-based calculus curriculum. Project components include a variety of faculty development activities, research on student learning of calculus, and the development of documents and training materials for use in implementing calculus reform.

Faculty Development: Activities for year-one participants include a one-week summer workshop for calculus instructors (held August 3–7, 1992, at UNC–Charlotte), monthly meetings via the *Vision Carolina* fiber optics network, and two workshops at UNC–Charlotte during the Spring 1993 semester.

The summer 1992 workshop was organized by Drs. Prichard and Royster and conducted by them and Thomas P. Dick (Oregon State University) and Sam Gough (East Mecklenburg High School, Charlotte, NC). The major themes were: (1) to emphasize the multiple representations of calculus concepts; (2) to help instructors plan for the use of computing technology in their classrooms; and (3) to help instructors plan for new teaching and assessment methods. The Oregon State Calculus Project and the material from the Calculus Consortium at Harvard were used as model curricula. This workshop focused on concepts from first-semester calculus: functions, limits, derivatives and integrals.

Monthly meetings are being conducted for consortium members to report on project activities. This will provide a support group for the instructors within which they can discuss issues, problems, and results of the implementation. The instructors are spread over a wide area and the use of the *Vision Carolina* fiber optic network allows them to get together without having to travel too far. The two workshops to be held in the spring of 1993 will continue to discuss the concepts from second- and third-semester calculus.

Research on Student Learning: Research on the learning of calculus is also a critical component of this project. Quantitative and qualitative data are being collected from students to determine their backgrounds in mathematics and to assess their attitudes towards mathematics and their understanding of calculus concepts. Research on student learning will provide direction for our implementation and evaluation of this new curricular approach. Most mathematicians and mathematics educators agree that learning mathematics is a constructive, active process rather than a passive one. "The construction process occurs in the context of the classroom experience, with the learner attempting to make sense out of information by evaluating, connecting, and organizing it relative to his or her prior experiences and the existing

knowledge structure" [1]. This constructivist view will guide the development of research tasks and instruments to investigate students' understanding of calculus.

Quantitative data has been and will be collected from students in the form of pre- and post-treatment questionnaires, surveys, and paper-and-pencil tests. Qualitative data is being collected from a smaller group of students at UNC–Charlotte. These students are individually interviewed two to four times each semester and videotaped as they solve calculus problems. Concept maps are a relatively new research tool for understanding students' knowledge of a subject [2]. Students from selected sections of first-, second-, and third-semester calculus courses were asked to construct their own concept maps for calculus. These maps will provide evidence of how students organize concepts and underlying ideas.

Evaluation: The research component is an important aspect of the project evaluation. This research should provide insight into how the major themes of the calculus reform are influencing students' understanding of calculus. During the summer workshop the first-year participants began keeping a journal in which they recorded their impressions of the workshop goals and activities. Throughout the 1992–1993 school year they are using the journal to document their personal experiences in using materials and ideas from the workshop. They are also using the journal as a tool for reflecting on changes in themselves, their students, and their classrooms.

Calculus in a Real and Complex World and *Multivariable Calculus, Linear Algebra, and ODEs* in a Real and Complex World

University of Massachusetts-Amherst, Department of Mathematics, Amherst MA 01003
Frank Wattenberg
Email: frankw@math.umass.edu

This project includes two one-year courses aimed at math, science and engineering majors. The first course, replacing the first two semesters of the mainstream calculus sequence, is being taught for the fourth year at the University of Massachusetts and at several other sites. A preliminary edition of the textbook *Calculus in a Real and Complex World* is available from PWS-Kent. The second course, replacing Calculus III, Linear Algebra, and Introduction to ODEs, is being taught for the first time this year.

The motivating idea behind these courses is that these subjects should be taught in the context of real and engaging applications for three reasons—mathematics is more accessible in concrete contexts, this approach provides better motivation for students whose primary interest is using mathematics, and students will be better able to use mathematics outside the classroom. Because we emphasize real problems, writing is an essential part of both courses.

Real problems, those which engage our students, are complex and are not feasible without sophisticated computational support. At various times we have used calculators, True BASIC, and *Mathematica*. Although the course has been successful with all three vehicles, a CAS system like *Mathematica* or *Maple* is best suited for these courses, because it enables us to

- Emphasize the interplay between experiment and theory.
- Develop an adventurous and experimental spirit in our students.
- Work with realistic and engaging problems that are typically fairly complex. A CAS system gives students the ability to experiment with variations of their own.
- Include examples and exercises for which computational detail would obscure the point without a CAS system.

First Year Mathematics for Life Science Students: Biocalculus Revisited

University of Tennessee, Department of Mathematics, Knoxville TN 37996-1300
Louis J. Gross
Email: gross@math.utk.edu

Despite the fact that quantitative methods and theoretical developments have become central to modern biology, the quantitative training of undergraduate life science students is generally considerably weaker than that of students in the

physical sciences. A Workshop dealing with the goals and content of a quantitative curriculum for life science students developed guidelines for key concepts and content for both an entry-level course, as well as upper-division courses. Strong arguments were made by quantitative biologists that key components of a first-year mathematics course should include not only basic calculus, but descriptive statistics, linear regression, linear algebra, difference equations, and basic differential equation modeling methods as well. A tenet of this project is that life science students have not been served well by either the standard physical-science oriented calculus sequence or the calculus for business, social and life science courses. A pilot course being taught this year to a class of 30 life science students makes extensive use of MATLAB for several components of the course. I will present results of the Workshop and outline how the pilot course has been organized to allow time for students to investigate key biological topics using various quantitative techniques from difference equations to differential equations. I will also provide evaluations of a variety of software packages potentially useful in the quantitative components of a life science curriculum.

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The Chattanooga Calculus Consortium

University of Tennessee at Chattanooga, Department of Mathematics, Chattanooga TN 37403
Stephen Kuhn
Email: skuhn@utcvm.bitnet

Eight area calculus teachers at six schools, collectively known as the Chattanooga Calculus Consortium, are working together, supported by the National Science Foundation, to improve calculus instruction by adapting to the needs of the students in their own schools the model calculus program being developed at St. Olaf College in Northfield, Minnesota, and which brings graphical, numerical, and algebraic viewpoints, with the help of computers and graphing calculators, to bear on the concepts of calculus to improve students' conceptual understanding.

Since calculus is taught at so many different types of institutions we believe that a collaborative effort on the part of mathematics faculties of public and private high schools and two-year and four-year colleges is needed to thoroughly test and evaluate new dynamic calculus courses for broad effectiveness and portability, and the local consortium was formed to do precisely that in the Chattanooga area. The Consortium activities were funded for two years and its primary intent is the evaluation of the St. Olaf approach, the creation of supplementary materials, and the dissemination of these materials as adapted for local needs and enhanced by increased attention to student writing. To provide easy access for our students to simultaneous numerical, symbolic, and graphical explorations we currently use *MicroCalc*, *Derive*, and TI-81 calculators (our different schools have different needs and tools) and we are in the process of including *Maple*. Our project includes Calculus I and II college courses and AP and non-AP high school courses.

Participating schools in the consortium project, which began in fall 1991, are The University of Tennessee at Chattanooga (Stephen Kuhn and Edward Rozema), Chattanooga State Technical Community College (John Peterson and Marsha Schoonover), Southern College of Seventh-day Adventists (Arthur Richert), Girls Preparatory School (Rita Wilson), Ooltewah High School (Deborah Cantrell), and Hixson High School (Colleen Gurley). In the 1992–93 academic year approximately 250 college students and 50 high school students will participate in the project.

Multi-HBCU Calculus Project

Virginia State University, Department of Mathematics, Petersburg VA 23806
Walter Elias, Jr.

The project began in August 1991. An abbreviated version of the three objectives is as follows:

1. to increase the use of technology in teaching/learning the calculus;
2. to actively engage students in "real-life type" problem solving;
3. to identify and reduce barriers to calculus achievement.

The four primary strategies for accomplishing the objectives include:

1. technology-assisted lectures by the teacher;

2. computer laboratories for students;

3. problem-solving activities individually and in groups;

4. background strengthening.

A computer laboratory component has been added to the course, and some of the "usual" lectures have been changed to computer-assisted presentations. Students are more actively involved in the course and are provided more opportunities to experience calculus concepts through frequent interplay of symbolic, numerical, and graphical representations. Problem solving is receiving more emphasis and is being expanded to include problems that were previously prohibitive because of time constraints and the magnitude of calculations involved. The IBM-compatible technology and the computer software *Derive* that we use render many problems more tractable and permit students to concentrate on concepts involved. Attempts to strengthen backgrounds have included a prerequisite test, self-pretests (specific to textbook sections), item analyses of classroom tests, and individual and group help by the PI, co-PI's, and student assistants.

The first year of the project was restricted to one section of calculus at VSU with an enrollment of thirty students majoring in mathematics or one of the sciences. During the current year, four institutions are participating in the project. Besides VSU, they are Virginia Union University, Bowie State University, and St. Paul's College. Courses include calculus for mathematics/science majors, for non-mathematics/non-science majors, linear algebra, and differential equations. About 200 students and five teachers are directly involved.

The Joy of *Mathematica*: A Point-and-Click Way to Use and Learn *Mathematica*

Wellesley College, Department of Mathematics, Wellesley, MA 02181
Alan Shuchat and Fred Shultz
Email: ashuchat@lucy.wellesley.edu or fshultz@lucy.wellesley.edu

The Joy of Mathematica is a program that has been developed at Wellesley College and tested in calculus classes since 1990. We have used *Joy* with both traditional and nontraditional single and multivariable calculus texts. We anticipate that about 250 Wellesley College students will use *Joy* in their courses in 1992–93.

Joy runs alongside *Mathematica* on the Macintosh and enables users to work with *Mathematica* immediately while simultaneously learning its abilities and syntax. *Joy* makes it easier for faculty to teach with *Mathematica* and makes it easier for everyone who uses mathematics to learn to use *Mathematica*.

In *Joy*, users select menu items ("graph $f(x)$") and fill in dialogs (the equation and domain of $f(x)$) rather than enter *Mathematica* commands directly. *Joy* creates the appropriate commands in the correct syntax and passes them to *Mathematica* for execution. It contains about fifty menu items for common operations and calculus topics. The *Joy* and *Mathematica* windows appear together, and each command that is created can be seen along with its effect. Users can also enter commands directly in the *Mathematica* window as they learn the syntax, and use *Mathematica* notebooks along with *Joy*.

We have found this an effective way to leverage the Macintosh's ease of use while retaining *Mathematica*'s advantages. Students with no computer experience can easily learn the Macintosh, have immediate access to *Mathematica*'s power without knowing its commands, and learn its programming language over a period of time if they wish. *Joy* significantly reduces the overhead of using computers in elementary classes and enables instructors to focus on new ways of teaching with this powerful tool. Faculty and other professionals can learn *Mathematica* as they work with it, and shorten the learning curve by using *The Joy of Mathematica*.

Group Projects and Computer Labs in Freshman Calculus

Worcester Polytechnic Institute, Department of Mathematics, Worcester MA 01609
William Farr, William Hardell, J.J. Malone, Brigitte Servatius
Email: bfarr@wpi.wpi.edu

Group projects involving applications of calculus are the focus of an experimental sequence being taught at WPI. The applications are drawn from science and engineering and are designed to emphasize how calculus is actually used

to solve problems. The students work in teams of two to four students for approximately three weeks on each project. Written reports are required, with emphasis on logical presentation and writing quality. Projects are open-ended so as to give students the opportunity to use knowledge from other courses and disciplines and creatively apply tools from calculus.

Students in this pilot course are also exposed to the computer algebra system *Maple* in weekly laboratory experiences. The emphasis is on student exploration and independent learning. Within each laboratory students are asked to formulate their observations as a conjecture. These student conjectures are then discussed in class. It is interesting to observe the students' interest in defense of their own conjecture and in the proof of related theorems.

Wright State University Calculus Laboratory Project

Wright State University, Department of Mathematics, Dayton OH 45435
Richard Mercer
Email: rmercer@desire.wright.edu

Objectives: The goals of this project are to improve Calculus instruction by

i. providing a student-oriented interactive learning environment that is conducive to experimentation and cooperative learning.

ii. giving more emphasis to concepts and problem solving and less to symbolic manipulation.

iii. emphasizing verbal and written communication of mathematics.

iv. providing exposure to the kind of technology students are likely to be using in a technical career in the 21st century, and training in appropriate use of this technology.

Description: A computer laboratory can help accomplish each of these objectives. Traditional Calculus at Wright State is taught with 5 hours of lecture per week. Students in laboratory sections have 4 hours of lecture and 2 hours lab, or 3 hours lecture and 3 hours lab. In addition there are open lab hours when students can work on assignments not completed during scheduled laboratory classes.

Students know that in any laboratory they will be doing something rather than listening. In our Calculus laboratory, students use *Mathematica* software, a powerful computational and graphics tool that allows instructors to write an interactive text. The notebooks students work with contain narrative, interactive examples, and problems. They can work the problems in the same notebook by entering and executing their own commands. The role of the instructor in the laboratory is to provide assistance and guidance. Graduate teaching assistants and undergraduate lab assistants provide additional help during laboratory classes and staff open lab hours. Because students are required to work in pairs, they must discuss what is happening with their lab partner. They are required to submit lab reports in standard written English. Hence our students do far more talking and writing about mathematics than traditional calculus students. Because *Mathematica* is capable of extensive numeric and symbolic computation, or creating complex color graphics in seconds, students can concentrate more on concepts and problem solving, and less on lengthy hand calculations.

Background: This project has been under way since spring quarter of 1991, and has received substantial funding from the National Science Foundation: a two-year grant for Curriculum Development (awarded August 1991), and a grant for Laboratory Improvement (awarded June 1992). The latter grant was matched by university funds, which allowed us to acquire one of the best-equipped mathematics laboratories in the nation, with 21 networked NeXT color workstations.

Plans: We are now at the end of the experimental stage and working for wider participation. We plan to offer thirteen sections of calculus using the laboratory during the calendar year 1993, and to move toward using the laboratory in all sections in the not-too-distant future.

A "Core Approach" to Calculus

U.S. Military Academy, Department of Mathematics, West Point NY 10996
Don Small
Email: ad5712@usma2.USMA.EDU

The core approach is characterized by:

1. The development of mathematical concepts and applications from a minimal core of objects.
2. The integration of the treatment of one and several variables. Concepts are first developed in R and then generalized to R^n.

The integrated approach and a strong emphasis on the following two processes provide a uniformity and a "stream lining" to the course that allows it to fit nicely into two semesters (approximately 100 lessons).

1. The Basic Approximation Process (approximate-improve approximation-generate sequence of approximations-take limit) is central to the definition formulation for every major concept.
2. The five-step outline for developing all major concepts is: motivation-formulate a definition-apply definition to basic function-develop an algebra-develop applications.

The course integrates several complementary learning activities. Group projects and exploratory computer labs actively engage students in "doing" mathematics as well as encouraging collaboration. Project reports and class presentations are used to address the communication objective. "Gateway" tests (mastery tests) in graphing, differentiation, and integration provide assurance that students master the basic aspects in these areas. Although the content is traditional including multi-variable calculus and differential equations through variation of parameters, the arrangement of topics, approach, and emphasis is not. Major emphasis is placed on

- learning how to learn (mathematics);
- conceptual understanding;
- learning how to analyze functions (graphically, numerically, symbolically);
- approximation and error-bound analysis;
- understanding and appreciating the development process;
- understanding the three basic concepts: nearness, differentiation, and integration.

The integrated approach provides several enriching features, e.g.:

- emphasizes development and generalization, e.g., how to generalize interval (limit, derivative, ...) to higher dimensions;
- places focus on concepts rather than on dimension;
- provides for realistic modeling using several variables.

Text: D.B. Small and J.M Hosack, *Explorations in Calculus with a Computer Algebra System*, McGraw-Hill, 1991.