Journal of Lie Theory
Volume **6** (1996) 69–146

On Hyperbolic Cones and Mixed Symmetric Spaces

Bernhard Krötz and Karl-Hermann Neeb

Communicated by J. Hilgert

Abstract. A real semisimple Lie algebra $\mathfrak{g}$ admits a Cartan involution, θ, for which the corresponding eigenspace decomposition $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ has the property that all operators $\operatorname{ad} X$, $X\in\mathfrak{p}$ are diagonalizable over $\mathbb{R}$. We call such elements hyperbolic, and the elements $X\in\mathfrak{k}$ are elliptic in the sense that $\operatorname{ad} X$ is semisimple with purely imaginary eigenvalues. The pairs $(\mathfrak{g},\theta)$ are examples of symmetric Lie algebras, i.e., Lie algebras endowed with an involutive automorphism, such that the -1-eigenspace of θ contains only hyperbolic elements. Let $(\mathfrak{g},\tau)$ be a symmetric Lie algebra and $\mathfrak{g}=\mathfrak{h}+\mathfrak{q}$ the corresponding eigenspace decomposition for τ. The existence of "enough" hyperbolic elements in $\mathfrak{q}$ is important for the structural analysis of symmetric Lie algebras in terms of root decompositions with respect to abelian subspaces of $\mathfrak{q}$ consisting of hyperbolic elements. We study the convexity properties of the action of $\operatorname{Inn}_{\mathfrak{g}}(\mathfrak{h})$ on the space $\mathfrak{q}$. The key role will be played by those invariant convex subsets of $\mathfrak{q}$ whose interior points are hyperbolic.

B. Krötz, K. - H. Neeb
Mathematisches Institut
Universität Erlangen-Nürnberg
Bismarckstr. $1\frac{1}{2}$
D-91054 Erlangen
Germany

Received May 5, 1996
and in final form June 11, 1996

ISSN 0949–5932 / \$2.50